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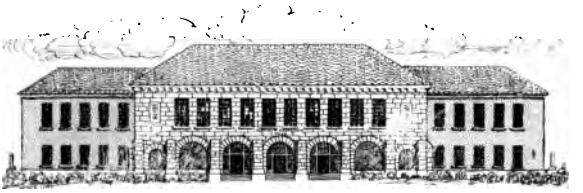

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WILL'S
COMMERCIAL ARITHMETIC

PRESENTING

THE BEST USAGE IN MODERN
BUSINESS PRACTICE

BY

WILLIAM R. WILL

FOR THIRTY-TWO YEARS PRINCIPAL OF THE MATHEMATICAL DEPARTMENT
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SERIES OF COMMERCIAL ARITHMETICS

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PREFACE

IT has been found extremely difficult, if not impossible, for the average student of arithmetic to remember its many topical rules and explanations, and to exercise due discrimination in giving to each its local application. "Will's Commercial Arithmetic" has therefore ignored this unsatisfactory method of treatment, and has substituted the few cardinal principles which underlie all arithmetical processes, and which can be used in the solution and explanation of *all* problems. The learner is thus liberated from slavish adherence to mechanical rules and hackneyed explanations which are soon forgotten, and is properly cultured in the exercise of his reasoning faculties in the solution of problems, the effect of which is permanent.

The only conceivable modifications of any given number in a problem are effected either by increasing it or by decreasing it. The law of increase or decrease is uniform in its application. Addition and multiplication are the only known numerical processes for increasing a number, the former being employed when the increase is by one or more given unequal components, and the latter when by equal components. Conversely, subtraction and division are the only known means for diminishing a number, the former being employed when the decrease is by one or more given unequal components, and the latter when by equal components. Each of the four preceding processes is thus seen to be invariably limited to its own prerequisite conditions; and the proper process to select will depend upon the particular prerequisite conditions which are given. The intelligent solution of a problem can therefore involve nothing further than the identification of its numerical terms, a knowledge of the specific process to which these identified terms invariably belong, and the mechanical performance of that process.

The above method of treatment of all problems, though a departure from the traditional standards, is not an untested theory, but a thoroughly demonstrated success. It is the ripe fruitage of forty-eight years' experience in the class-room, during the last thirty-two of which the author has been engaged in teaching commercial arithmetic exclusively in one of the oldest and most reputable business colleges in the United States.

To his fellow teachers who have been disappointed either in the lack of permanency or of quality in the results hitherto accomplished by the conventional treatment of this most important study, the methods of this volume are confidently submitted as an efficient means for the accomplishment of entirely satisfactory results in the future.

THE AUTHOR

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COMMERCIAL ARITHMETIC

INTRODUCTION

1. Arithmetic is that branch of knowledge which treats of numbers. It explains the possible relations between the numerical expression of one given quantity and that of another given quantity, and how those relations may be utilized to find the numerical expression of a required quantity.

2. Quantity is that property of matter which may be measured, weighed, valued or counted, and which may be expressed by a number.

3. A number is the expression of a quantity by the use of figures.

NOTE 1. (*a*) A number is said to be *abstract* when the kind of quantity is not expressed, but only its *extent*, as three, five times; (*b*) to be *concrete* when the *kind* of quantity is expressed as well as its extent, as five *gallons*, seven *yards;* (*c*) to be *simple* when a concrete quantity is expressed in only *one denomination*, as three *bushels*, twelve *barrels;* (*d*) to be *compound* when *one* concrete quantity is expressed by the use of *two or more different denominations*, as *three pounds and nine ounces* of sugar, *six gallons three quarts and one pint* of water; (*e*) to be *integral* when a quantity is expressed in one or more *whole* units, as *six, nine* pounds; (*f*) to be *fractional* when a quantity is expressed in one or more *parts* of a whole unit, as one-*half*, three-*fourths*.

NOTE 2. When one number is placed in contrast with one or more other numbers it is said to be (*a*) *like* when it expresses a quantity of the *same* name, as five *yards* is a like number to eight *yards;* (*b*) and it is said to be *unlike* when it expresses a quantity of a *different* name, as eight *inches* is an unlike number to five *gallons*.

4. A unit is *one* thing of the quantity expressed by a number. Thus, the unit of eight yards is *one* yard, of twelve miles is *one* mile.

5. A problem in arithmetic is a statement containing numerical terms which are mutually related, and which requires

that relation to be perfected by finding an omitted term, called the *answer*.

NOTE. The separate unequal components and their total are distinctive terms of addition (if a total is required), or of subtraction (if a total is given). One equal component, the number of equal components, and the total of equal components, are distinctive terms of multiplication (if a total is required), or of division (if a total is given). These *five terms* are the alphabet of arithmetic.

6. The **analysis** of a problem is the process of separating it into its given numerical terms, ascertaining their character, discovering their relation to each other, and thus deducing the appropriate operation to find the required term which will complete that relation.

NOTE. A *completed relationship* of arithmetical terms is a numerical total and the several equal or unequal numerical terms which constitute that total.

7. The **solution** of a problem is the process of finding the required term which will complete the relationship which exists between the given terms.

NOTE. A relation is *completed* by finding a required component which will complete a given total, by finding a required total which will include all of its given components, or by finding the number of given equal components, which are contained in a given total.

READING NUMBERS

8. Reading numbers is the mental conception or the vocal expression of the value of a written number.

9. Numbers are commonly expressed by the use of the ten following figures, used singly or in combination:

0	1	2	3	4	5	6	7	8	9
Naught	One	Two	Three	Four	Five	Six	Seven	Eight	Nine

The first of the above figures is called naught or cipher, and it expresses the absence of value. The remaining nine figures are called *significant* figures. (*a*) When written by itself or at the *extreme* right of a number, each figure expresses the value placed underneath it in the above illustration. Numbers greater than nine are expressed by a systematic combination

of the above figures. Thus, (*b*) to increase any of the above figures tenfold, place one figure at the right of it; (*c*) to increase it a hundredfold, place two figures at the right of it; (*d*) to increase it a thousandfold, place three figures at the right of it; etc. (*e*) The local value of a figure is determined by its position, increasing tenfold for each place that it is moved to the left; (*f*) and decreasing tenfold for each place that it is moved to the right. (*g*) The position which a figure occupies in a number, counting from the right, determines its value and is called its *order*.

ILLUSTRATIVE EXERCISE

10. Read 4976.

EXPLANATION. Commencing with the left-hand figure (4) read it and name its local value (4 thousand), as it has three figures at its right [**9**, *d*]; then read the next following figure and name its local value (9 hundred), as it has two figures at its right [**9**, *c*]; then read the next following figure and name its local value (7ty or 7 tens, *ty* being a corruption of ten), as it has one figure at its right [**9**, *b*]; then read the next following and last figure (6), omitting its name (units), as it is understood [**9**, *a*]; thus making the entire expression read "four thousand nine hundred seventy-six."

11. Numbers composed of more than three figures are read in sets of three figures each. (*a*) The first set of three figures, counting from the right, is called the *Units' Period;* (*b*) the second set of three figures from the right is called the *Thousands' Period;* (*c*) the third set of three figures, the *Millions' Period;* (*d*) the fourth set, the *Billions' Period;* (*e*) the fifth set, the *Trillions' Period;* (*f*) the sixth, the *Quadrillions' Period;* etc.

SUGGESTION. To read and write numbers with facility, it is necessary that the above periods be memorized until, without hesitation, they can be repeated in regular order, forward from the highest period, or backward from the lowest.

EXERCISES

Express the following numbers in written or spoken words:

1. 9	**5.** 72	**9.** 426	**13.** 7836	**17.** 8003
2. 14	**6.** 45	**10.** 865	**14.** 5492	**18.** 4600
3. 18	**7.** 97	**11.** 309	**15.** 6285	**19.** 3280
4. 36	**8.** 58	**12.** 200	**16.** 9058	**20.** 7025

ILLUSTRATIVE EXERCISE

12. Read 26463728496219.

EXPLANATION. To discover the name of each period in the given number and the figures which compose it, commence at the right and separate it into periods of three figures each, as follows: 26,463,728,496,219. As the right-hand period (219) is pointed off, think of its name (units); as the second period (496) is pointed off, think of its name (thousands); as the third period (728) is pointed off, think of its name (millions), and so continue until the left-hand period (26) is reached, which is thus found to be trillions.

Then read the number expressed by the left-hand period as if it stood alone, calling its name (twenty-six trillion); then read the number expressed by the next following period as if it stood alone, calling its name (four hundred sixty-three billion); then, similarly, the next following period (seven hundred twenty-eight million); then the next period (four hundred ninety-six thousand); and finally the last period, omitting its name (units) as it is understood (two hundred nineteen).

EXERCISES

Express the following numbers in written or spoken words:

1. 2785	**11.** 678349526	**21.** 51836274813429
2. 16298	**12.** 2817462785	**22.** 200070600083294
3. 34276	**13.** 7132975138	**23.** 56000000400000
4. 416298	**14.** 231678261582	**24.** 290867320000078
5. 728341	**15.** 57281334967864	**25.** 689712000091863
6. 2685913	**16.** 1963475162834748	**26.** 500030007100006
7. 5873265	**17.** 5600328176003452	**27.** 8620053000007008
8. 37264897	**18.** 92086005027316814	**28.** 65000000002009
9. 52819624	**19.** 51378260000009162	**29.** 183400000000710006
10. 83429857	**20.** 60000520080076003	**30.** 2000090000070005

WRITING NUMBERS

13. (*a*) **Writing numbers** is the expression *in figures* of the value of a number when given in words. (*b*) Knowing the names of the several periods and their relative positions [**11**], any number is written by commencing with its highest period and writing it as if it stood alone, then writing its next highest period as if it stood alone, and so continuing with period after period in regular order until the lowest period is written.

NOTE. (*a*) If, after writing the highest period, any subsequent period should be found to contain fewer than three figures, supply the deficiency by

prefixing the requisite number of ciphers. (*b*) If an entire period is omitted, supply three ciphers for its three vacant orders.

ILLUSTRATIVE EXERCISE

Express in figures, twenty-seven quadrillion, nine hundred sixty-eight trillion, seventy-two million, eight thousand forty.

EXPLANATION. Commencing with the highest period of the given number (quadrillions), write it as if it stood alone (27 . . .); then write the next lower period (trillions) as if it stood alone (27,968 . . .) then write the next lower period (billions) and since no billions are expressed in the given number [Note, *b*, **13**], supply three ciphers for its three vacant orders (27,968,000 . . .); then write the next lower period (millions), and as seventy-two requires only two figures to express it and each period except the highest must contain three figures [Note, *a*, **13**], supply the deficiency by prefixing one cipher (27,968,000,-072 . . .); then write the next lower period (thousands), and as eight requires only one figure to express it, and each period except the highest must contain three figures, supply the deficiency by prefixing two ciphers (27,968,000,072,-008 . . .); then write the next lower period (units), and as forty requires only two figures to express it, prefix one cipher, obtaining 27,968,000,072,-008,040 as the complete numerical expression.

EXERCISES

Express in figures the following numbers:

1. Six thousand, three hundred fifty-two.
2. Forty-nine thousand, eight hundred seventy-six.
3. Two hundred nine thousand, seven hundred.
4. Five hundred eight thousand, sixty-three.
5. Ninety thousand, eight.
6. Eighty-three thousand, six hundred ninety-five.
7. Twelve million, seventy-one thousand six.
8. Three million, sixteen thousand, two hundred eight.
9. Nineteen million, three hundred fifty-eight thousand forty.
10. Twenty-five million, five hundred twenty-seven.
11. Four trillion, seventy-six million, two hundred fifty-three thousand, seven.
12. Fifteen quadrillion, two hundred ninety-one trillion, five billion, *seven-two* thousand, sixteen.

Express in figures the following numbers:

13. Ninety-seven quadrillion, one hundred twenty-eight million, four hundred thousand, ninety-two.

14. Six quadrillion, seven hundred trillion, eighty billion, nine hundred fifty thousand, twenty.

ROMAN METHOD OF EXPRESSING NUMBERS

14. The **Roman method** of expressing numbers is dependent upon the following seven capital letters, used as numerals.

I	V	X	L	C	D	M
One	Five	Ten	Fifty	One hundred	Five hundred	One thousand

NOTE 1. Placed by itself, each of the above Roman numerals expresses the value written underneath it.

NOTE 2. Successive repetitions of the above numerals denote repetitions of the value which they express. Thus, II express two ones, or two; III, three ones, or three; XXX, three tens, or thirty; CC, two hundred; etc.

NOTE 3. The combined value when one numeral is placed *at the left* of another of *greater* value expresses the *difference* between their respective values. Thus, IV expresses the difference between one and five, or four; IX expresses nine; XL, forty; XC, ninety; etc.

NOTE 4. The combined value when one numeral is placed *at the right* of another of *greater* value expresses the *sum* of their respective values. Thus, VIII expresses eight; XXV, twenty-five; LXII, sixty-two; etc.

15. The manner in which Roman numerals are combined to express numbers is shown in the following

TABLE OF ROMAN NUMERALS

I	One	XIV	Fourteen	XC	Ninety
II	Two	XV	Fifteen	C	One hundred
III	Three	XVI	Sixteen	CC	Two hundred
IV	Four	XVII	Seventeen	CCC	Three hundred
V	Five	XVIII	Eighteen	CCCC	Four hundred
VI	Six	XIX	Nineteen	D	Five hundred
VII	Seven	XX	Twenty	DC	Six hundred
VIII	Eight	XXX	Thirty	DCC	Seven hundred
IX	Nine	XL	Forty	DCCC	Eight hundred
X	Ten	L	Fifty	DCCCC	Nine hundred
XI	Eleven	LX	Sixty	M	One thousand
XII	Twelve	LXX	Seventy	MM	Two thousand
XIII	Thirteen	LXXX	Eighty	MMM	Three thousand

NOTE 1. No numeral should be repeated more than *three successive times*, except C which may be repeated *four times*, but no more.

NOTE 2. Only *one* numeral of less value can be written at the left of another of greater value.

NOTE 3. A *bar* placed over a Roman numeral, or a combination of such numerals, increases its value a thousandfold. Thus $\overline{V}$ expresses 5000; $\overline{XXIV}$, 24000; $\overline{CCXLVI}$DXII, 246512.

NOTE 4. Vacant orders are not expressed as is found necessary when figures are employed. There is no Roman numeral to express naught.

ILLUSTRATIVE EXERCISE

16. Express 3675 by the use of Roman numerals.

EXPLANATION. First write the highest order, three thousand (MMM); then the next lower order, six hundred (MMMDC); then the next lower order, seven tens or seventy (MMMDCLXX); and finally the lowest order, five units (MMMDCLXXV).

Express the following by the use of Roman numerals:

1. 12	**5.** 92	**9.** 219	**13.** 729	**17.** 1468
2. 27	**6.** 69	**10.** 188	**14.** 942	**18.** 2372
3. 16	**7.** 35	**11.** 347	**15.** 638	**19.** 1090
4. 45	**8.** 74	**12.** 406	**16.** 826	**20.** 3007

Express the following by the use of figures:

21. LXXXVI	**25.** DCCXXVI	**29.** MCXLVIII
22. CCXLIV	**26.** MCCLVII	**30.** MDCCCLV
23. DCLXIX	**27.** DCCCCXXI	**31.** MDCCCCIX
24. CCCCXII	**28.** MMCXXXV	**32.** $\overline{D}$CCCLXXXIX

ADDITION

17. Addition is the name of the process for finding the *total* of two or more *unequal* quantities.

NOTE 1. The *combined value* of all the quantities to be added is called the *sum*, or *total*, or *amount*.

NOTE 2. (*a*) The *sign of addition* is an erect cross (+) and is called *plus*. (*b*) When placed between quantities, it denotes that they are to be added. (*c*) The *sign of equality* is two short horizontal lines (=), and is read equals or equal. (*d*) The sign of equality denotes that the combined expression on the left of it is equal to the combined expression on the right of it.

18. Principles of Addition. 1. *Only like numbers* [Note 2, *a*, **3**], *or like orders* [**9**, g], *can be added.* 2. *The sum expresses a quantity of the same name as the quantities added.*

ILLUSTRATIVE EXAMPLE

19. Add 8167, 59, 374, 2742, 19, 3816, 7 and 34.

SOLUTION

$$\begin{array}{r} 8167 \\ 59 \\ 374 \\ 2742 \\ 19 \\ 3816 \\ 7 \\ 34 \\ \hline 15218 \\ 234 \end{array}$$

EXPLANATION. So place the numbers to be added that figures of the same order [**9**, *g*] shall fall in the same column [Prin. 1, **18**], and draw a horizontal line beneath. Beginning with the right-hand or units' column, find the sum of $4 + 7 + 6 + 9 + 2 + 4 + 9 + 7$, amounting to 48 units, or 4 tens and 8 units. Place the 8 units beneath the horizontal line and under the units' column, and carry the 4 tens to the tens' column, that is, mentally add the 4 carried tens with the figures in the tens' column. Thus, 4 (the carried figure) $+ 3 + 1 + 1 + 4 + 7 + 5 + 6 = 31$ tens, or 3 hundred and 1 ten. Place the 1 ten beneath the line and under the tens' column, and carry the 3 hundred to the hundreds' column, as follows: 3 (the carried figure) $+ 8 + 7 + 3 + 1 =$ 22 hundred, or 2 thousand and 2 hundred. Place the 2 hundred beneath the line and under the hundreds' column, and carry 2 thousand to the thousands' column, as follows: 2 (the carried figure) $+ 3 + 2 + 8 = 15$ thousand. As this is the last column to be added, place the entire result beneath the line in such a manner that the 5 thousand shall fall under the thousands' column.

NOTE 1. *To prove addition,* add each column in the opposite direction to that first employed. If the same results are obtained, the answer is presumed to be correct.

NOTE 2. When adding, write the carrying figure under the column from which it was obtained, as shown in the illustrative example, that it may also be included in the proof.

NOTE 3. In adding columns, acquire the habit of pronouncing results without naming the figures from which they have been obtained. Thus, in adding the units' column of the illustrative solution, it is better *to read* 4, 11, 17, 26, 28, 32, 41, 48, than *to spell* 4 and 7 are 11, 11 and 6 are 17, etc.

EXAMPLES FOR PRACTISE

Find the sum of

1. 58, 36, 75, 92, 85, 36, 27, 48, 62, 59, 23, 37, 56, 18

2. 326, 62, 927, 35, 5268, 76, 248, 9, 628, 3253, 769

3. 8296, 573, 2894, 37, 518, 6275, 93, 235, 58, 617, 75

Find the sum of

4. 5368, 7864, 3295, 48, 2834, 376, 58, 675, 49, 3278

5. 18237, 625, 34296, 5867, 928, 2678, 18326, 754

6. Three thousand two hundred ninety-five; seven hundred sixteen; one thousand forty-eight; thirty-five thousand ninety-seven; eight thousand three hundred five; sixty-nine thousand eight; eighty thousand four; forty thousand one hundred; two hundred six thousand seven hundred three; four hundred thousand five hundred twenty-eight.

20. It is frequently found necessary to add numbers which are written horizontally, or in some other way than in the usual vertical arrangement. To find the total of numbers thus irregularly arranged, first add all the right-hand orders; next add all the second orders from the right; next, all the third orders from the right, etc.; setting down and carrying in the usual manner.

EXAMPLES FOR PRACTISE

Find the total by adding horizontally:

1. 562, 375, 812, 492, 716
2. 283, 826, 749, 514, 328
3. 617, 482, 627, 835, 569
4. 986, 789, 582, 796, 328
5. 826, 9187, 46, 3285, 768
6. 96, 487, 3256, 7834, 279
7. 38, 46, 298, 84, 2637, 526
8. 3682, 597, 2876, 736, 98

SUBTRACTION

21. Subtraction is the name of the process for finding the *difference* between two *unequal* quantities.

NOTE 1. (*a*) Subtraction involves the *reverse use* of the same terms as have been already considered in addition [Note 1, 17]. The *names* of these terms of addition are changed, however, to accord with the reverse process employed in their use. (*b*) The total value of all the unequal components formerly called the sum, is now distinguished as the *minuend*, which means the quantity to be diminished; (*c*) the given component (or the sum of the given components if more than one is given) is now called the *subtrahend*, which means quantity to be subtracted; (*d*) and the required component is now called the *remainder*, which means that (quantity) which remains. Hence,

NOTE 2. (*a*) The *minuend* is the quantity to be diminished; (*b*) the *subtrahend* is the quantity to be taken from the minuend; (*c*) and the *remainder* or *difference* is what remains of the minuend after the subtrahend has been taken from it.

NOTE 3. (*a*) The *sign of subtraction* is a short horizontal line, as follows (−). (*b*) It is called *minus;* (*c*) it means *less;* (*d*) and when placed between two numbers, it indicates that the number at its right is to be subtracted from the number at its left.

22. Principles of subtraction. 1. *The difference can be found only between like numbers* [Note 2, *a*, **3**] *or like orders* [**9**, g]. 2. *The difference between like numbers or like orders will also be like.* 3. *An increase of any order in the subtrahend is equivalent to a corresponding decrease of the similar order in the minuend.* 4. *The difference between two numbers is the difference of the several orders in the subtrahend separately taken from their respective similar orders in the minuend.*

ILLUSTRATIVE EXAMPLE

23. Subtract 482 from 827.

SOLUTION

827
482
345

EXPLANATION. Write the less number under the greater so that each order [**9**, *g*] of the less number shall fall under a similar order of the greater. Commencing with the lowest order, subtract 2 units of the subtrahend from 7 units of the minuend, leaving 5 units to occupy the units' order of the remainder.

Then subtract the next higher order of the subtrahend (8 tens) from the corresponding next higher order of the minuend (2 tens), and since the 8 tens of the subtrahend cannot be taken from the 2 tens in the minuend, "borrow" 1 hundred from the 8 hundred in the minuend, and add the borrowed hundred (or 10 tens) to the 2 tens, thus making 12 tens, and then subtract 8 tens from 12 tens, leaving 4 tens to occupy the tens' order of the remainder.

Lastly, subtract the next higher order of the subtrahend (4 hundred) from the next higher order of the minuend (7 hundred), that is, 8 hundred diminished by the previously borrowed 1 hundred, obtaining 3 hundred as the hundreds' order of the remainder.

NOTE. *To prove subtraction,* add the remainder (the required component, (Note 1, *d*, **21**) to the subtrahend (the given component, Note 1, *c*, **21**). The result should equal the minuend (the given total, Note 1, *b*, **21**).

EXAMPLES FOR PRACTISE

Find the difference between

1. 9426 and 4732
2. 8592 and 5278
3. 92816 and 37284
4. 79218 and 24835
5. 121829 and 72636
6. 187213 and 95847

Find the difference between

7. 68329142 and 41768315
8. 50380674 and 27300407
9. 80000000 and 26040058
10. 918326781 and 40067007
11. 700400305 and 22080026
12. 602005009 and 41030070

13. Subtract three hundred forty-five thousand seven hundred nine from eight hundred four thousand six hundred.

14. Subtract one million eight hundred seven thousand five hundred thirty-six from twenty-five million one hundred thousand ninety.

15. A speculator bought a tract of land for $52786 and afterwards sold it for $71000. What was his gain?

16. A firm's deposits in a bank amounted to $62309 and its withdrawals to $18027. How much did the firm have remaining in the bank?

17. If a man was born in the year 1843 and died in the year 1911, how old was he at the time of his death?

18. A house was sold for $28134 which was $2698 more than the sum paid for it. What did the house cost?

24. To save the time consumed in re-arranging the terms of subtraction, learners should acquire the art of finding the difference between two numbers in whatever position they may have been written. This may be done as follows: Subtract the right-hand order of the less number from the right-hand order of the greater; proceed in a similar manner with the second orders from the right; with the third orders from the right, etc.; until all the orders have been considered, checking off each order as it is subtracted to facilitate the proper identification of the next order to be considered.

EXAMPLES FOR PRACTISE

Subtracting horizontally, find the difference between

1. 92863 and 75392
2. 57029 and 20807
3. 80035 and 62008
4. 3289658 and 1832982
5. 8050027 and 4207009
6. 7200503 and 2830408

MULTIPLICATION

25. **Multiplication** is the name of the process for finding the *total of two or more equal* quantities.

NOTE 1. (*a*) One of the considered equal quantities is called the *multiplicand;* (*b*) the number of equal quantities is called the *multiplier;* and (*c*) the total of all the considered equal quantities is called the *product.*

NOTE 2. The multiplicand and multiplier are called *factors* (producers) of the product.

NOTE 3. (*a*) The *sign of multiplication* is an oblique cross (×). (*b*) It is read *multiplied by;* (*c*) and when placed between numbers, it indicates that they are to be multiplied.

26. Principles of multiplication. 1. *The product expresses a quantity of the same kind as the multiplicand* [Prin. 2, **18**]. 2. *The complete product equals the product of each order of the multiplicand separately multiplied by each order of the multiplier.* 3. *To multiply or divide the multiplier by any number, the multiplicand remaining unchanged, is equivalent to multiplying or dividing the product by that number.* 4. *To multiply or divide the multiplicand by any number, the multiplier remaining unchanged, is equivalent to multiplying or dividing the product by that number.*

MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9	10	11	12	1
2	4	6	8	10	12	14	16	18	20	22	24	2
3	6	9	12	15	18	21	24	27	30	33	36	3
4	8	12	16	20	24	28	32	36	40	44	48	4
5	10	15	20	25	30	35	40	45	50	55	60	5
6	12	18	24	30	36	42	48	54	60	66	72	6
7	14	21	28	35	42	49	56	63	70	77	84	7
8	16	24	32	40	48	56	64	72	80	88	96	8
9	18	27	36	45	54	63	72	81	90	99	108	9
10	20	30	40	50	60	70	80	90	100	110	120	10
11	22	33	44	55	66	77	88	99	110	121	132	11
12	24	36	48	60	72	84	96	108	120	132	144	12
1	2	3	4	5	6	7	8	9	10	11	12	1

ILLUSTRATIVE EXAMPLE

27. Multiply 976 by 8.

SOLUTION

976
8
7808

EXPLANATION. Write the multiplier (8) under the lowest order of the multiplicand (6). Commencing with this lowest order of the multiplicand, separately multiply it (6 units) and each successive higher order (7 tens and 9 hundred) by the multiplier (8), setting down and carrying as usual. Thus, 8 times 6 units equal 48 units, or 4 tens and 8 units. Write the 8 units beneath the line and under the units' order of the multiplicand, and carry the 4 tens to 8 times 7 tens or 56 tens, making 60 tens, or 6 hundred and 0 tens. Write the 0 tens beneath the line and under the tens' order of the multiplicand, and carry the 6 hundred to 8 times 9 hundred or 72 hundred, making 78 hundred. As this completes the multiplication, write this final result in full beneath the line, but in such a manner that the 8 hundred shall fall under the hundreds' order of the multiplicand.

EXAMPLES FOR PRACTISE

Multiply	Multiply
1. 68734258 by 5	**7.** 84037028009 by 8
2. 27893542 by 7	**8.** 70040006897 by 12
3. 72486359 by 9	**9.** 40398002756 by 4
4. 86957423 by 2	**10.** 920007100008 by 6
5. 51627384 by 11	**11.** 53647298307 by 8
6. 91827436 by 10	**12.** 80043090076 by 3

ILLUSTRATIVE EXAMPLE

28. Multiply 6723 by 894.

SOLUTION

6723
894
26892
60507
53784
6010362

EXPLANATION. First, as explained in **27**, multiply the successive orders of the multiplicand (6723) by the units' order of the multiplier (4) as if it stood alone, obtaining 26892 as the first partial product.

Next, similarly multiply the successive orders of the multiplicand (6723) by the next higher order of the multiplier (9 tens) as if it stood alone, obtaining 60507 tens. So place this result that its right-hand order (7 tens) shall fall under the tens' order of the preceding partial product [Prin. 1, **18**].

Next, similarly multiply the successive orders of the multiplicand (6723) by the hundreds' order of the multiplier (8) as if it stood alone, obtaining 53784 hundred. So place this result that its right-hand order (4 hundred) shall fall in the hundreds' column of the preceding products. [Prin. 1, **18**.]

The sum of *these partial* products (6010362) will be the complete product.

NOTE 1. The first figure of each partial product when properly placed will fall directly under that order of the multiplier which was used to produce it. Thus, in the above solution, the figure at the right of the first partial product (26892), which is 2, should fall under that order of the multiplier (4) which produced it. The figure at the right of the second partial product (60507), which is 7, should fall under that order of the multiplier (9) which produced it. The figure at the right of the third partial product (53784), which is 4, should fall under that order of the multiplier (8) which produced it.

NOTE 2. *One proof of multiplication* is to reverse the position of the multiplicand and multiplier, and multiply. If the same final product is obtained, it may be accepted as correct. After division has been learned, another method of proof is to divide the obtained product by either of its factors [Note 2, **25**], and the quotient should equal its other factor.

EXAMPLES FOR PRACTISE

Multiply	Multiply
1. 4768359 by 86	**8.** 683291482 by 8296
2. 96378342 by 74	**9.** 503800209 by 6083
3. 72839457 by 63	**10.** 700680005 by 4009
4. 84976523 by 29	**11.** 829600706 by 9206
5. 58297386 by 629	**12.** 920053008 by 70806
6. 72682943 by 785	**13.** 438007056 by 40075
7. 83795824 by 246	**14.** 320008062 by 32004

15. Find the total weight of 8965 barrels of flour which average 196 pounds per barrel.

16. What is the cost of building 374 miles of railroad at $34-768 per mile?

17. A man earns 135 dollars per month and spends 46 dollars per month. How much does he save in 9 months?

18. If the average speed of a steamer is 365 miles per day, what distance can it traverse in 137 days?

29. Continued multiplication is the successive multiplication of more than two factors. The final result of such a multiplication is called a *continued product.*

Thus, if 816 be multiplied by 6, and the resulting product by 7, the process is called a continued multiplication, and the final result, a continued product. Continued multiplication is employed when an inconvenient multiplier, as 42, can be changed to two or more convenient multipliers, the product of which is equal *to 42, as* 6×7.

ILLUSTRATIVE EXAMPLE

Multiply 986 by 63.

SOLUTION

986
9
8874
7
62118

EXPLANATION. One of the factors (63) is separable into two other and more convenient factors (9×7). Therefore, first multiply the multiplicand (986) by one of these two factors of the multiplier (9), obtaining 8874; and multiply this result by the remaining factor of the multiplier (7), obtaining 62118 as the required product. If 63 = 7 times 9, then 63 times 986 must equal 7 times 9 times 986.

EXAMPLES FOR PRACTISE

Employing continued multiplication, multiply

1. 6783 by 72
2. 52896 by 49
3. 78348 by 54
4. 925687 by 42
5. 758439 by 96
6. 4273869 by 132
7. 3842768 by 144
8. 52783 by 432 ($= 8 \times 9 \times 6$)

30. If a number be multiplied by 1, the resulting product will be found equal to the multiplicand. Hence, if the same number be multiplied by ten times 1, or 10, the resulting product must be ten times the multiplicand, or that multiplicand with *one cipher* at the right of it [**9**, *b*]; or if that number be multiplied by one hundred times 1, or 100, the resulting product must be one hundred times the multiplicand, or that multiplicand with two ciphers at the right of it [**9**, *c*], etc. Hence, to multiply any number by 1, followed by one or more ciphers, simply *place as many ciphers at the right of the multiplicand as are found at the right of* 1 *in the multiplier.*

EXAMPLES FOR PRACTISE

Multiply

1. 8763 by 100
2. 9578 by 10000
3. 75623 by 10

Multiply

4. 62875 by 1000
5. 41329 by 100000
6. 72865 by 10000

31. If a multiplier be greater than 1, the product must be as many times the multiplicand as that multiplier is times 1. Therefore if the multiplier be 8, the product must be eight times the multiplicand, and if the multiplier be ten times 8, or 80, the result-

ing product must be ten times the preceding product by 8, or eight times the multiplicand with one cipher at the right of it [9, *b*]; and if the multiplier be one hundred times 8, or 800, the resulting product must be one hundred times the product by 8, or eight times the multiplicand with two ciphers at the right of it [9, *c*], etc.

ILLUSTRATIVE EXAMPLE

Multiply 876 by 9000.

SOLUTION

876
9000
7884000

EXPLANATION. Multiply the multiplicand (876) by 9, as if it stood alone, obtaining 7884. As the true multiplier is one thousand times 9, the true product must be one thousand times the product by 9, or 7884 with three ciphers at the right of it [9, *d*].

EXAMPLES FOR PRACTISE

Multiply	Multiply
1. 894 by 700	4. 6792 by 57000
2. 7683 by 32000	5. 863 by 437000
3. 5789 by 64900	6. 9187 by 190000

32. If, as shown in Ill. Ex., **31**, the product of 876 multiplied by 9000 is 9 times 876, or 7884, with three ciphers at the right of it (7884000), it must follow that one hundred times 876, or 87600, multiplied by 9000, must be one hundred times the former product, or the former product (7884000) with two additional ciphers at the right of it, that is, 788400000.

ILLUSTRATIVE EXAMPLE

Multiply 27300 by 85000.

SOLUTION

27300
85000
1365
2184
2320500000

EXPLANATION. So arrange the given factors that the right-hand significant figure of the multiplier (5) shall fall under the right-hand significant figure of the multiplicand (3). Then multiply as if there were no ciphers at the right of either factor, obtaining 23205. As, however, the true multiplier is one thousand times 85, the product of 273 multiplied by the true multiplier (85000) must be one thousand times 23205, or 23205000; and as the true multiplicand is one hundred times 273, or 27300, the true product of 27300 multiplied by 85000 must be one hundred times 23205000, *or 2320500000.* Hence,

RULE. To multiply when either or both factors terminate in ciphers: *So place the multiplier under the multiplicand that the right-hand significant figure of the former shall fall under the right-hand significant figure of the latter, and multiply as if there were no ciphers at the right of either. At the right of the resulting product, place as many ciphers as are at the right of both factors.*

EXAMPLES FOR PRACTISE

Multiply	Multiply
1. 468000 by 7200	**5.** 823000 by 47000
2. 67200 by 9000	**6.** 12900 by 680000
3. 5286000 by 30	**7.** 3400000 by 90000
4. 9270000 by 2800	**8.** 7820000 by 5300

DIVISION

33. Division is the name of the process which is used when the total of all the considered *equal* quantities is given, and it is required to find the value of *one* of those equal quantities, or the *number* of those equal quantities.

NOTE 1. Division involves the reverse use of the same terms as have already been considered in multiplication [Notes 1 and 2, **25**]. These terms of multiplication are however changed in name to accord with the reverse process employed in their use. (*a*) The total of all the equal quantities, formerly called *product* [Note 1, *c*, **25**], is now called *dividend* which means a *quantity to be divided*. Hence, one of the equal quantities and the number of such equal quantities must be the factors or producers of the dividend [Note 2, **25**]. (*b*) Either of these two factors, if given, and used to divide the dividend, is distinguishably called the *divisor* which means "that by which a quantity is divided"; and (*c*) the remaining factor, resulting from the division, is distinguishably called the *quotient*, which means "the number of times one quantity is contained in another." Hence,

NOTE 2. (*a*) The *dividend* is the number to be divided; (*b*) the *divisor* is the number by which the dividend is divided; and (*c*) the *quotient* is the result obtained from the division.

NOTE 3. After the last quotient figure has been obtained, the undivided portion of the dividend, if any, is called the *remainder*.

NOTE 4. (*a*) Division is indicated by the sign ÷ which follows the dividend and precedes the divisor. (*b*) It is read *divided by*.

34. Principles of division. 1. *If the divisor and dividend are concrete numbers, they must both represent units of the same*

name to produce the true quotient. 2. *If the dividend be concrete and the divisor abstract, the quotient will be concrete and express units of the same name as the dividend.* 3. *The remainder, being the undivided portion of the dividend* [Note. 3, **33**], *must express units of the same name as the dividend.* 4. *To multiply or divide the dividend by a number is equivalent to multiplying or dividing the quotient by the same number.* 5. *To multiply or divide both divisor and dividend by the same number produces no change in the quotient.* 6. *To multiply the divisor by a number, the dividend remaining the same, is equivalent to dividing the quotient by the same number.* 7. *To divide the divisor by a number, the dividend remaining the same, is equivalent to multiplying the quotient by the same number.* 8. *The complete quotient equals the quotient of each order of the dividend separately divided by the divisor.*

SHORT DIVISION

35. Divide 895 by 5.

SOLUTION

$$5)\overline{895}\quad 179$$

EXPLANATION. Divide the highest order of the dividend (8 hundred) by the divisor (5) to find the highest order of the quotient, obtaining 1 hundred, and leaving an undivided remainder of 3 hundred in the dividend. Write the first quotient figure (1 hundred) underneath the 8 hundred of the dividend from which it was obtained.

Carry the 3 undivided hundred (= 30 tens) of the preceding operation to the next lower order of the dividend (9 tens), and divide the result (39 tens) by the divisor (5), obtaining 7 tens as the next lower order of the quotient, leaving 4 tens of the 39 tens as an undivided remainder. Write this second quotient figure (7 tens) underneath the order of the dividend (9 tens) from which it was obtained.

Carry the 4 undivided tens (= 40 units) to the next lower order of the dividend (5 units), and divide the result (45 units) by the divisor (5), obtaining exactly 9 units, as the next lower order of the quotient. Write this third and final quotient figure (9 units) underneath the units' order of the dividend from which it was obtained.

NOTE 1. The above method of obtaining the successive quotient figures by mental processes is called *short division.* Short division is employed when the divisor is not greater than 12.

NOTE 2. *To prove division,* multiply the quotient by the divisor, and to the product add the remainder, if any. The result should equal the dividend [Note 1, **33**].

EXAMPLES FOR PRACTISE

Divide	Divide
1. 71862496 by 4	**7.** 48329157684 by 12
2. 827359281 by 3	**8.** 25671928347 by 9
3. 952873656 by 6	**9.** 76982763451 by 11
4. 829673845 by 7	**10.** 82534826780 by 10
5. 528379186 by 2	**11.** 50380700608 by 8
6. 734829635 by 5	**12.** 90070008301 by 7

LONG DIVISION

36. Divide 74865 by 58.

SOLUTION

$$58)74865(1290\tfrac{45}{58}$$

58
168
116
526
522
45

EXPLANATION. Divide as many of the highest orders of the dividend as will contain the divisor (74 thousand) by the divisor (58), obtaining 1 thousand as the highest order of the quotient. Write 1 (thousand) as the first quotient figure, multiply the divisor (58) by this quotient figure (1 thousand) and subtract the product (58 thousand) from 74 thousand, thus finding that 16 thousand of the 74 thousand remain undivided.

Carry the 16 undivided thousands (=160 hundred) of the dividend to the next lower order of the dividend (8 hundred), which is most conveniently done by "bringing down" the 8 hundred of the dividend to the right of the 16 undivided thousand, obtaining 168 hundred as the next portion of the dividend to be divided, and 168 hundred ÷ 58 equal 2 hundred, the next quotient figure. Multiply the divisor (58) by this second quotient figure (2 hundred), and subtract the product (116 hundred) from 168 hundred, thus finding that 52 hundred of the 168 hundred remain undivided.

Carry the 52 undivided hundreds (= 520 tens) of the last considered part of the dividend to the next lower order of the dividend (6 tens), by "bringing down" the 6 tens of the dividend to the right of the 52 hundreds, obtaining 526 tens as the next portion of the dividend to be divided; and 526 tens ÷ 58 = 9 tens. Write 9 tens as the next quotient figure, multiply the divisor (58) by this third quotient figure (9 tens) and subtract the product (522 tens) from 526 tens, thus finding that 4 tens of the 526 tens remain undivided.

Carry the 4 undivided tens (= 40 units) of the considered part of the dividend to the next lower order of the dividend (5 units), and divide the result (45 units) by 58, obtaining 0 units times, thus finding that 58 are contained in 74865, 1290 times, and that 45 is the final undivided portion of the dividend. Write the divisor 58 under this undivided part of the dividend to denote this unexecuted part of the division, and place

this fractional part of the quotient ($\frac{45}{58}$) to the right of the integral part (1290), obtaining $1290\frac{45}{58}$ as the complete quotient.

NOTE 1. The above method of obtaining the successive quotient figures by written processes is called *long division*. Long division is usually employed when the divisor is greater than 12.

NOTE 2. (*a*) To find the first quotient figure when the divisor consists of several figures, mentally divide the first figure or the first two figures at the left of the dividend by the first figure at the left of the divisor, or by the first figure plus 1 if the second figure from the left of the divisor is greater than 5. (*b*) To find each succeeding quotient figure, point off from the right of each partial dividend as many figures lacking one as there are figures in the divisor, and divide the remaining figures of the partial dividend by the left-hand figure of the divisor as in (*a*).

NOTE 3. If, after multiplying the divisor by the last obtained quotient figure, the product is found to be greater than the partial dividend above it, the last quotient figure is too great and should be diminished by 1.

NOTE 4. If, after any subtraction from a partial dividend, a remainder is obtained which is greater than the divisor, the last obtained quotient figure is incorrect, and should be increased by 1.

EXAMPLES FOR PRACTISE

Divide	Divide
1. 876349572 by 23	**10.** 4378692573 by 486
2. 924183762 by 41	**11.** 5827394618 by 928
3. 729385647 by 63	**12.** 6537428719 by 4365
4. 682753725 by 37	**13.** 8291736451 by 72849
5. 521837649 by 58	**14.** 5278060072 by 603005
6. 437529183 by 79	**15.** 9428910003 by 412078
7. 621938726 by 48	**16.** 8006000342 by 890065
8. 529486312 by 67	**17.** 7200650004 by 27384
9. 297834263 by 325	**18.** 4120082005 by 728163

37. A continued division is a successive division by two or more divisors. It is usually employed when an inconvenient divisor, as 48, involving long division, can be changed to two or more convenient divisors ($48 = 8 \times 6$), involving successive short divisions.

ILLUSTRATIVE EXAMPLE

Divide 6487 by 336.

SOLUTION

$336 = 8 \times 7 \times 6$

8)6487

7)810 + 7 = 7

6)115 + 5 (× 8) = 40

19 + 1 (× 8 × 7) = 56

103

Quotient $19\frac{103}{336}$

EXPLANATION. If 6487 be divided by 8 or one forty-second of the true divisor, the quotient (810) will be 42 times the true quotient [Prin. 7, **34**], and a remainder of 7. As this remainder is from a partial division of the true dividend it must form a part of the true remainder [Prin. 3, **34**].

If 810 (forty-two times the true quotient) be divided by 42, the result will be the true quotient; but if 810 be divided by the second factor 7, or one-sixth of 42, the resulting quotient (115) must be 6 times the true quotient [Prin. 7, **34**] and a remainder of 5. As this remainder (5) is from a partial division of one-eighth of the true dividend (810 = one-eighth of 6487), it must be one-eighth of the second part of the true remainder [Prin. 3, **34**] and 8 times 5 or 40 will form the second part of the true remainder.

As 115 is 6 times the true quotient, divide 115 by 6, the third factor, to find the true quotient (19), and a remainder of 1. As this remainder (1) is from a division of one-seventh of one-eighth of the true dividend, it must be one-seventh of one-eighth of the true remainder [Prin. 3, **34**], and 7 times 8 times 1, or 56, will form another part of the true remainder. Hence, add these several parts of the true remainder, obtaining 7 + 40 + 56, or 103 as the complete remainder.

SUMMARY. 1. *The successive division by the factors of a divisor is equivalent to a division by that divisor.* 2. *The complete remainder of a successive division by the factors of a divisor equals the first remainder + the product of the second remainder multiplied by the first divisor + the continued product of any succeeding remainder multiplied successively by all the divisors which precede its own divisor.*

EXAMPLES FOR PRACTISE

Divide	Divide
1. 896542 by 28	**7.** 3678251968 by 96
2. 1768329 by 32	**8.** 4826731853 by 84
3. 768296854 by 45	**9.** 2372853 by 168 (8 × 7 × 3)
4. 93725823 by 72	**10.** 7283945 by 504 (9 × 8 × 7)
5. 62834251 by 64	**11.** 5678325 by 125 (5 × 5 × 5)
6. 5162738429 by 54	**12.** 9671847 by 315 (9 × 7 × 5)

38. Since the value of an order [**9**, *g*], is decreased tenfold for each place that it is moved to the right [**9**, *f*], if *all* the orders of a dividend be moved one place to the right, it will be equivalent to dividing that dividend by 10; if moved two places to the right it will be equivalent to dividing that dividend by 100; if three places, by 1000, etc. Now to strike off one figure from the right of any dividend will have the effect of moving its remaining orders one place further to the right than their original position, and thus to dividing that dividend by 10; to strike off two figures will move the remaining orders two places to the right of their original position, and thus to dividing by 100, etc. It also follows that the figure or figures which are taken from the right of the dividend will still retain the same local position and value which they had in the original dividend, and must be the undivided portion of the original dividend, or the true remainder. Hence,

RULE. *To divide by* 10, 100, 1000, *etc., cut off from the right of the dividend as many figures as there are* 0's *at the right of* 1 *in the divisor. The remaining figures of the dividend will constitute the quotient; and the figures cut off will be the remainder.*

EXAMPLES FOR PRACTISE

Divide	Divide
1. 67859 by 10	**5.** 1483762 by 1000
2. 82678 by 100	**6.** 7834106 by 100
3. 92567 by 1000	**7.** 28710015 by 10000
4. 26835 by 10000	**8.** 62520003 by 1000

39. If, as found in **38**, when three figures are stricken from the right of a number, the remaining figures will be the true quotient of that number divided by 1000, and the figures stricken off will be the true remainder, it must follow that striking off three figures from the right of that number will be seven times the true quotient of that number divided by 7 times 1000, or 7000 [Prin. 7, **34**] and, therefore, that striking off three figures from a number and dividing its remaining figures by 7, must be the quotient of that number divided by 7000. Further, if, as also shown in **38**, the three figures stricken from the right of the dividend constitute the true remainder from dividing by 1000, they must also form a part of the true

remainder from dividing by seven times 1000 [Prin. 3, **34**]; and any succeeding remainder from dividing the remaining figures of the dividend (or one-thousandth of the true dividend) by 7 will be one-thousandth of the second part of the true remainder, and hence must have three figures at its right to exhibit its true local value [**9**, *d*]. If, therefore, the three figures cut off from the right of the original dividend, which constitute one part of the true remainder, be placed at the right of the second remainder, the result will be the complete true remainder.

RULE. When any divisor has ciphers at its right: 1. *Cut off from the right of the dividend as many figures as there are* 0's *at the right of the divisor.* 2. *Divide the remaining figures of the dividend by the divisor without the* 0's *at its right, to find the true quotient.* 3. *Place the figures cut off from the original dividend at the right of the final remainder to obtain the complete remainder.*

ILLUSTRATIVE EXAMPLE

Divide 93865 by 3700.

SOLUTION

37ØØ)938ØƷ(25 Quo.
74
198
185
1365 Rem.

EXPLANATION. 3700 = 100 × 37. To divide 93865 by the first factor (100), cut off two figures from the right of the dividend, obtaining 938 as 37 times the true quotient, and 65 as part of the true remainder [**38**]. Then,

Divide 938 by 37, obtaining the true quotient (25) and a second remainder of 13. As this remainder (13) is from dividing one-hundredth of the original dividend (93865), it is one-hundredth of the true remainder [Prin. 3, **34**], and 100 times 13, or 13 hundred must be the second part of the true remainder. Hence, place the first part of the true remainder (65) to the right of the second part (13 hundred), obtaining 1365 as the complete remainder.

EXAMPLES FOR PRACTISE

Divide

1. 8736541 by 800
2. 9183762 by 6000
3. 4275918 by 400
4. 6823475 by 7000
5. 5938764 by 4100

Divide

6. 73289653 by 218000
7. 52671839 by 493000
8. 42835267 by 63000 (1000 × 9 × 7)
9. 84276834 by 4800 (100 × 8 × 6)
10. 64837291 by 350000 (10000 × 7 × 5)

UNITED STATES MONEY

40. United States money is usually expressed in dollars and cents; but in performing calculations in United States money, it has been found convenient in certain cases to be more specific in designating its orders by naming the first figure to the right of the decimal point, *dimes*, and the third figure to the right of the decimal point, *mills*.

NOTE 1. Scale: 10 mills = 1 cent; 10 cents = 1 dime; 10 dimes = 1 dollar.

NOTE 2. In expressing the several orders of United States money, the dollars are written in accordance with **9**. A point (.) is used to separate the dollars from the lower orders. The first order to the right of the point expresses dimes; the first two orders to the right of the point, considered as one number, express cents; and the third order to the right of the point expresses mills.

NOTE 3. To indicate that a certain number expresses dollars, the sign $ is placed at its left; and that a certain number expresses cents, the sign ¢ is placed at its right.

NOTE 4. If the cents to be expressed are less than 10, a cipher should be written after the point to denote the absence of tens of cents or dimes. Thus, 8 dollars 7 cents should be written $8.07.

EXERCISES

Write

1. 78 dollars, 19 cents
2. 267 dollars, 23 cents
3. 59 dollars, 42 cents
4. 167 dollars, 3 cents
5. 5273 dollars, 8 cents
6. 435 dollars, 36 cents
7. 2926 dollars, 5 cents

Write

8. 16 dollars, 8 dimes
9. 341 dollars, 6 mills
10. 472 dollars, 2 cents
11. 673 dollars, 34 cents, 7 mills
12. 9183 dollars, 7 cents, 2 mills
13. 4865 dollars, 5 mills
14. 793 dollars, 1 cent, 9 mills

Express the following by spoken or written words:

15. $35.42	**18.** $500.34	**21.** $86.352	**24.** $80.706
16. $76.28	**19.** $2176.09	**22.** $75.089	**25.** $625.105
17. $95.05	**20.** $513.06	**23.** $54.003	**26.** $532.017

41. (*a*) As 10 dimes, or 100 cents, or 1000 mills equal 1 dollar [Note 1, **40**], dollars may be changed to dimes by annexing *one*

cipher, to cents by annexing *two* ciphers, and to mills by annexing *three* ciphers [30]. (*b*) As 10 cents or 100 mills equal 1 dime, dimes may be changed to cents by annexing *one* cipher, and to mills by annexing *two* ciphers. (*c*) Similarly cents may be changed to mills by annexing *one* cipher. (*d*) The lower orders may be changed to dollars by pointing off one figure from the right of the dimes, two figures from the right of the cents, or three figures from the right of the mills [38], prefixing ciphers if necessary.

EXERCISES

Change	Change
1. $748 to cents	6. 8 cents to mills
2. $57 to mills	7. 5278 cents to dollars
3. $382 to cents	8. 6175 mills to cents
4. 7 dimes to cents	9. 69183 cents to dollars
5. 3 dimes to mills	10. 5368 dimes to dollars

ADDITION OF UNITED STATES MONEY

42. The processes for adding, subtracting, or multiplying United States money are the same as for abstract numbers, both being expressed upon a scale in which 10 units of any order equal 1 unit of the next higher order [compare Note 1, 40 with *e* and *f*, 9]. Any expression in United States money may therefore be considered a simple integral number of the same name as its right-hand order, and added as such, and the resulting sum changed back to dollars by *d*, 41.

ILLUSTRATIVE EXAMPLE

43. Add $372, $631.18, $9352.605, and $43.

SOLUTION

$ 372.
631.18
9352.605
43.
$10398.785

EXPLANATION. So arrange the numbers to be added that the points of each shall fall in an erect column, thus causing mills to fall under mills, cents under cents, dimes under dimes, dollars under dollars [18]. Add and carry as in ordinary addition [19]. As the sum of the dimes column (6 + 1) must be 7 dimes, and the sum of the dollars' column (3 + 2 + 1 + 2) must be 8 dollars, a point should be placed in the sum between the 7 dimes and 8 dollars to denote their respective values [Note 2, 40], that is, *the point in the sum should fall directly under the column of points in the numbers added.*

EXAMPLES FOR PRACTISE

1. Add \$573.25, \$6187.92, \$48.34, \$7283.16, \$8.64, and \$58.25.
2. Add \$589.18, \$39.42, \$643.28, \$7.24, \$826.32, and \$65.58.
3. Add \$82.74, \$9.36, \$628.15, \$97.87, \$527.62, and \$4.50.
4. Add 3 dollars, 4 cents; 28 dollars, thirty cents; 7 dollars, 2 mills; 45 dollars, 9 mills; 19 dollars, 5 cents; 356 dollars, 8 cents.
5. Add 9 cents; 2 dollars, 18 cents; 74 cents; 3 cents, 5 mills; 4 dollars, 25 cents; 2 cents; 34 cents; 4 dollars; 15 dollars, 5 cents.

SUBTRACTION OF UNITED STATES MONEY

ILLUSTRATIVE EXAMPLE

44. Subtract \$8, 7¢ from \$22, 15¢.

SOLUTION

\$22.15
8.07
\$14.08

EXPLANATION. So place the subtrahend (8.07) that the point at the right of its dollars shall fall under the point at the right of the dollars in the minuend (22.15), thus causing similar orders to fall in the same column. Subtract, borrow, and carry as in ordinary subtraction [23], and place a point in the *remainder* directly under those of the minuend and subtrahend, as already explained in pointing off the *sum* in Ill. Ex., 43.

EXAMPLES FOR PRACTISE

Subtract	Subtract
1. \$518.25 from \$838.13	5. \$358, 5¢ from \$500
2. \$28.32 from \$47.157	6. \$67 from \$285, 80¢
3. \$813 from \$1826.45	7. \$5, 4¢ from \$12, 3 mills
4. \$534.68 from \$913	8. \$248 from \$326, 5¢

MULTIPLICATION OF UNITED STATES MONEY

ILLUSTRATIVE EXAMPLE

45. Multiply \$873.42 by 53.

SOLUTION

\$873.42
53
262026
436710
\$46291.26

EXPLANATION. Write the multiplier under the multiplicand so that the right-hand significant figure of the multiplier shall fall under the right-hand significant figure of the multiplicand. Multiply and carry as in ordinary multiplication [28]. As the multiplicand is equivalent to 87342 cents, 53 times 87342 cents must also be cents [Prin. 1, 26]. Hence, change the resulting product 4629126 cents to dollars as shown in *d*, 41, that is, *point off from the right of the product as many figures as are pointed off from the right of the multiplicand.*

EXAMPLES FOR PRACTISE

Multiply	Multiply
1. \$6875 by 253	**6.** \$357 and 16¢ by 72 (8 × 9)
2. \$768.25 by 48 (8 × 6)	**7.** \$213 and 8¢ by 258
3. \$27.53 by 37	**8.** \$562, 3¢, 4 mills by 24
4. \$827.50 by 468	**9.** \$65, 9 mills by 900
5. \$91 and 3¢ by 53	**10.** \$8, 15¢, 7 mills by 32000

DIVISION OF UNITED STATES MONEY BY AN ABSTRACT DIVISOR

46. In division of United States money, the dividend may be regarded as a simple integer of the same name as its right-hand or lowest order. Thus, \$5.26 may be regarded as 526 cents, and \$2.065 as 2065 mills. Hence, if the divisor is an abstract number, the process of division can differ in no respect from ordinary division as shown in **36**, and the resulting quotient must express units of the same name as the dividend [Prin. 2, **34**].

ILLUSTRATIVE EXAMPLE

Divide \$593.95 by 47.

SOLUTION

```
                  4
47)$593.95($12.6~~3~~
   47
   ---
   123
    94
   ---
    299
    282
    ---
     175
     141
     ---
      34
```

EXPLANATION. Divide as in ordinary division. As the dividend expresses 59395 cents, one forty-seventh of the dividend, or 1264 must also express cents [Prin. 2, **34**]. Hence change the cents (1264) to dollars by cutting off two figures from the right of the quotient [**41**, *d*] obtaining \$12.64.

As there is no coinage in U. S. money below the cents' order, it is customary to stop the division after bringing down and dividing the cents' order of the dividend, and if the final remainder be one-half the divisor, or more, to increase the last figure of the quotient by 1. Hence, as 34, the final remainder is more than half the divisor (47), the last figure of the quotient (3) is increased by 1.

NOTE. If the dividend is expressed in dollars only, and it should not be exactly divisible by the divisor, change the dividend to cents [**41**, *a*] by annexing two ciphers, and continue the division until these annexed ciphers have been brought down and divided. The resulting quotient will then express cents.

CAUTION. If the division is not the final operation of a solution, but is to be followed by an operation in multiplication, it will be necessary to retain the exact remainder and express it as a fraction of a cent as in Ill. Ex., **36**; for any omitted inconsiderable fraction of a cent if multiplied by a considerable multiplier may cause an error in the final answer of many *integral* cents.

EXAMPLES FOR PRACTISE

Divide	Divide
1. \$6783.25 by 5	**5.** \$763 by 19
2. \$786.97 by 8	**6.** \$578 by 83
3. \$428.34 by 23	**7.** \$2783.42 by 327
4. \$562.78 by 53	**8.** \$8675 by 465

DIVISION OF UNITED STATES MONEY BY UNITED STATES MONEY

47. When both the divisor and dividend express United States money, and they are regarded as integers of the same name as their respective right-hand orders, they must first be changed to express units of the same name *if their right-hand* orders are dissimilar [Prin. 1, **34**]. This is accomplished by annexing the necessary number of ciphers to that term which has the fewer figures at the right of the point, so that the right-hand orders of both the divisor and the dividend shall be equidistant from their respective points.

ILLUSTRATIVE EXAMPLE

Divide \$6273 by \$18.25.

SOLUTION

\$18.25)\$6273.00(343 $\frac{1325}{1825}$
5475
7980
7300
6800
5475
1325

EXPLANATION. As the divisor expresses 1825 cents, reduce the dividend (\$6273) to cents by annexing two ciphers [Prin. 1, **34**]. Divide as in ordinary division, finding that 1825 cents are contained in 627300 cents 343 $\frac{1325}{1825}$ times.

EXAMPLES FOR PRACTISE

Divide	Divide
1. \$178.32 by \$.03	**6.** \$9258.63 by 18¢
2. \$2735.15 by \$2.83	**7.** \$62583 by 9¢
3. \$6825.42 by \$17.38	**8.** \$43897.38 by \$27
4. \$48578 by \$52.67	**9.** \$59687.42 by \$826
5. \$89264 by \$128.30	**10.** \$18273 by \$145

RELATION OF NUMBERS

48. (*a*) All arithmetical operations include only four elementary mechanical processes — addition, subtraction, multiplication, and division; and the intelligent solution of any problem, however complicated, depends upon a knowledge of the conditions to which each of these processes is invariably limited. (*b*) The only conceivable manner in which a given numerical expression of a quantity can be operated upon is by adding another quantity to it (increasing it by addition or by multiplication), or by taking a quantity from it (decreasing it by subtraction or by division). The arithmetical law of increase or decrease is uniform in application, and may be stated as follows:

(*c*) A given quantity is increased by *addition* if the quantity or quantities by which it is required to be augmented are *unequal* to the given quantity, or to each other; or (*d*) by *multiplication* if the quantity or quantities by which it is required to be augmented are *equal* to the given quantity and to each other. (*e*) The result of the increase is distinguishably called the *sum* if the augmentation has been by *unequal* quantities; or (*f*) the *product* if the augmentation has been by *equal* quantities. (*g*) Each *unequal numerical quantity* employed in obtaining a sum is distinguishably called a *component* of that sum; and (*h*) *one of the equal numerical quantities* employed in obtaining a product is called its *multiplicand*, or quantity to be repeated; and (*i*) *the number of such equal quantities* is called its *multiplier*, or times the equal quantity is to be repeated.

(*k*) A given quantity is decreased by recognizing it as a *total* of two or more quantities; and this total is distinguishably called a

minuend if it is the *total of unequal quantities;* or a *dividend* if it is the *total of equal quantities.*

(*m*) Though all increases can be effected by addition, the process of addition is usually limited to such increases as are made by combining *unequal* quantities [17], and the process of multiplication is more conveniently employed in combining *equal* quantities [25]. (*n*) While all decreases can be accomplished by subtraction, the process of subtraction is usually limited to the decrease of such quantities as express a total of *unequal* components [21], and the process of division is more conveniently employed in the decrease of quantities which express a total of *equal* components [33]. Hence,

PRINCIPLES. 1. *To find the total of two or more given unequal quantities, employ addition.* 2. *To find the total of two or more given equal quantities, employ multiplication.* 3. *To find one quantity of a given total of two or more unequal quantities, employ subtraction.* 4. *To find one quantity of a given total of two or more equal quantities, employ division.*

NOTE 1. It is thus seen that addition and subtraction are opposite processes pertaining to unequal quantities, and that multiplication and division are opposite processes pertaining to equal quantities; also, that addition and multiplication are different processes for obtaining a required total, the former limited to unequal and the latter to equal quantities; and that subtraction and division are different processes for diminishing a given total, the former by one or more unequal, and the latter by one or more equal quantities.

NOTE 2. It is also seen that the successive steps in the solution of any problem are: 1. Identifying its given numerical terms. 2. Ascertaining the relation of these terms to each other. 3. Perfecting that relation by performing the appropriate operation [Note, 7].

49. (*a*) The *total* of two or more unequal quantities is distinguishably called the *sum* if required, or (*b*) the *minuend* if given. (*c*) The total of two or more equal quantities is distinguishably called the *product* if *required*, or (*d*) the *dividend*, if *given.* (*e*) The several unequal quantities which constitute a total are called its *components* if the *total* (sum) *is required*, or (*f*) they are called its *subtrahend* and its *remainder* if the *total* (minuend) is given. (*g*) If a total of two or more equal quantities (product) *is required, one* of the several equal quantities which constitute

the required total is distinguishably called its *multiplicand factor*, and (*h*) the number of times the equal quantity is to be repeated is called its *multiplier factor*. (*i*) If a total of two or more equal quantities (dividend) is given, that one of its two factors just explained in *g* and *h*, which is given with it, is distinguishably called its *divisor;* and (*j*) its other factor [Note 1, 33] which will then be required, is called its *quotient*.

50. Addition and subtraction contrasted.

ILLUSTRATIVE EXAMPLES

1. J. A. Smith and S. H. Brown have formed a copartnership, Smith investing $5000 and Brown $7000. What is the capital of the firm?

2. The capital of the firm of Smith and Brown is $12000. If Smith's investment is $5000, how much is Brown's?

SOLUTIONS

$$\text{Components of a given minuend} \left\{ \begin{matrix} \$5000 \\ 7000 \end{matrix} \right\} \text{Components of a required sum}$$

$$\text{Minuend, if given} = \$12000 = \text{Sum, if required.}$$

EXPLANATION. The total of two or more unequal quantities is the *sum* if required as in Ex. 1, or the *minuend* if given as in Ex. 2 [*a* and *b*, 49]. Hence,

In Ex. 1, add the given unequal quantities, obtaining $12000 as the required sum or capital [Prin. 1, 48].

In Ex. 2, subtract the given unequal quantity ($5000), now called the subtrahend, from the given total of two unequal quantities ($12000), now called the minuend, to obtain the remaining unequal quantity ($7000), now called the remainder [Prin. 3, 48].

SUMMARY. 1. *If a total of two or more unequal quantities is required, add the several unequal quantities.* 2. *If a total of two unequal quantities is given, diminish it by its given unequal quantity to find its remaining unequal quantity.* 3. *If a total of more than two unequal quantities is given, diminish it by the sum of all its unequal quantities except one, and the remainder will be its one remaining unequal quantity.*

51. Multiplication and division contrasted.

ILLUSTRATIVE EXAMPLES

1. What is the cost of 12 pounds of butter at 35¢ per pound?

2. If 12 pounds of butter cost \$4.20, what is the price of one pound?

3. At 35¢ per pound, how many pounds of butter can be bought for \$4.20?

SOLUTIONS

$$\text{Factors of a given dividend} \left\{ \begin{matrix} \$\ .35 \\ 12 \end{matrix} \right\} \text{Factors of a required product.}$$

$$\text{Dividend, if given} = \$4.20 = \text{Product, if required.}$$

EXPLANATION. That factor of a product which expresses one of the considered equal quantities (35¢) is the multiplicand [Note 1, *a*, **25**]; and that factor which expresses the number of such equal quantities (12) is the multiplier [Note 1, *b*, **25**]; and the total of all the considered equal quantities (\$4.20) is the product if required [Note 1, *c*, **25**]; or the dividend if given [Note 1, **33**]. Hence,

In Ex. 1, multiply one of the equal quantities or multiplicand (35¢) by the number of such quantities or multiplier (12), obtaining the *required* total of all the equal quantities or product (\$4.20).

In Ex. 2, divide the *given* total of all the equal quantities or product (\$4.20), now called the dividend, by the number of such equal quantities or its given multiplier factor (12), now called the divisor, obtaining one of the required equal quantities or its multiplicand factor (35¢), now called the quotient [Note 1, **33**].

In Ex. 3, divide the *given* total of all the equal quantities or product (\$4.20), now called the dividend, by one of the equal quantities or its given multiplicand factor (35¢), now called the divisor, obtaining the required number of such equal quantities or multiplier factor (12), now called the quotient.

SUMMARY. 1. *If a product is required, multiply its two given factors.* 2. *If a product is given, divide it by its given factor, and the quotient will be its other factor.*

52. Addition and multiplication contrasted.

ILLUSTRATIVE EXAMPLES

1. A farm consists of three fields, the first containing 275 acres, the second 342 acres, and the third 378 acres. How many acres does the farm contain?

2. A farm consists of three fields, each containing 468 acres. How many acres does the farm contain?

SOLUTIONS

(1)	(2)
275	468
342	× 3
378	1404 acres
995 acres	

EXPLANATION. In each example, the same question is asked, but the proper process for Ex. 1 is addition, because the several components of the required total are unequal [Prin. 1, **48**]; and for Ex. 2 is multiplication, because the several components of the required total are equal. [Prin. 2, **48**].

SUMMARY. 1. *Employ addition to find the total of two or more unequal quantities.* 2. *Employ multiplication to find the total of two or more equal quantities.*

NOTE. The multiplier factor, denoting the number of equal quantities is peculiar to multiplication and division which relate exclusively to equal quantities. The multiplier factor cannot have its counterpart in addition or subtraction because in each of these processes the *number* of unequal quantities is altogether fortuitous, *depending upon the extent of the inequality* of the several components of a given or required total.

53. Subtraction and division contrasted.

ILLUSTRATIVE EXAMPLES

1. How many acres will remain after selling one field of 78 acres and another field of 65 acres from a farm of 216 acres?

2. How many fields of 72 acres each can I sell from a farm containing 186 acres, and how many acres will remain unsold?

SOLUTIONS

(1)		(2)
78	216	72)186(2 fields
65	143	144
143 sold	73 acres unsold	42 acres unsold

EXPLANATION. In Ex. 1, the remainder is obtained by adding the unequal subtrahends (78 + 65) to find the complete subtrahend (143) and subtracting it from the given total of all the unequal fields or minuend (216), obtaining 73 acres as the remaining unequal field [Summary, 3, **50**].

In Ex. 2, multiply the *equal* subtrahends (72 acres) by the obtained number of such subtrahends (2) or quotient [from the Latin, *quot*, how many] to find the complete subtrahend (144 acres); and subtract this product or total of equal subtrahends (144) from the dividend or total of equal subtrahends plus a possible remainder (186), obtaining 42 acres as the remainder [**48**, *k*, *m*, *n*].

SUMMARY. 1. *Employ subtraction to find the remainder, if any, after diminishing a given total* (minuend) *by its known unequal component* (subtrahend). 2. *Employ division to find how many times* (quotient) *a given total* (dividend) *can be diminished by a known equal component* (divisor), *and what will be the final remainder, if any.*

REVIEW EXAMPLES

54. **1.** What is the cost of 853 yards of cloth at $1.85 per yard?

2. If 42 barrels of flour cost $273, what will be the cost of 78 barrels of the same kind of flour?

3. What is the total cost of 28 pounds of sugar at 6 cents per pound, and 17 pounds of coffee at 23 cents per pound?

4. A man bought 278 bushels of wheat at $1.07 per bushel, 327 bushels of oats at 56 cents per bushel, and 129 bushels of rye at 95 cents per bushel. What was the total cost?

5. A man who had an estate of $52356 willed $25378 to his wife, $9678 to his son, $12368 to his daughter, and the remainder to an asylum. How much of the estate did the asylum receive at his death?

6. A dealer bought 285 tons of coal at $6.25 per ton, and sold it for $1995. What was his profit?

7. A speculator bought two adjoining tracts of land, the first containing 78 acres at $42 per acre and the second containing 94 acres at $35 per acre, expended $2537 in erecting a barn, $300 for other improvements, and then sold both tracts for $12300. What was his profit?

8. A merchant bought 435 barrels of flour at \$7.15 per barrel and sold it for \$3175.50. What was his gain per barrel?

9. If I buy a house for \$6500, pay \$3200 on the day of purchase and \$2758 on a subsequent day, how much shall I still owe?

10. If I buy 28 yards of muslin at 12 cents per yard, 30 yards of linen at 64 cents per yard, 15 yards of cloth at \$1.37 per yard, and give the seller a fifty-dollar bank note, how much change should I receive?

11. A dealer bought 75 barrels of potatoes and 863 bushels of corn for \$774.34. If the price of the potatoes was \$2.50 per barrel, what was the price per bushel of the corn?

12. I originally contracted a debt of \$3468.75. How much do I now owe, having paid \$1268.75 at one time and \$875.36 at another time?

13. A firm's sales were as follows: Monday, \$375.82; Tuesday, \$468.74; Wednesday, \$416.25; Thursday, \$396.75; Friday, \$298.16; and Saturday, \$547.60. What were its average daily sales?

EXPLANATION. To average or equalize two or more unequal quantities means to change them to the same number of *equal* quantities. Hence, the *sum* of these six unequal quantities when found should be regarded as the total of six equal quantities, or *product* [Note 1, *c*, **25**], and 6 should be regarded as the number of such equal quantities, or multiplier factor [Note 1, *b*, **25**]. Hence, divide the product by its multiplier factor to obtain its multiplicand factor, or one of the six equal daily sales.

14. There were five boys whose ages were 8, 10, 12, 14 and 16 years, respectively. What was the average age of these boys?

15. The temperature of a certain locality was 75° Fahrenheit for October, 64° for November, and 56° for December. What was the average temperature for these three months?

16. A grocer mixed 170 pounds coffee at 12 cents per pound, 105 pounds coffee at 15 cents per pound, and 84 pounds coffee at 18 cents per pound. What was the average price per pound of the mixture?

17. If it is estimated that the labor of 9 men is necessary

to complete a certain contract in 6 days, in how many days can one man complete it?

EXPLANATION. 6 days is a multiplicand, as it expresses the equal labor estimated to be performed by each man; 9 is a multiplier, as it expresses the considered number of equal men. Hence, multiply these two factors to find the product which will express the total days' labor of one man which is performed by all 9 of the men. Problems of this character are solved upon the supposition that all laborers possess equal strength, equal skill, and equal industry.

18. If 35 men can complete a given amount of work in 13 days, in how many days can one man, working alone, complete the same work?

19. If one man working alone can perform a certain task in 24 days, how many days will be required for 6 men to complete the same task?

20. If 50 men can do a certain job of work in 24 days, how many days will be required for 40 men to perform the same job?

21. A man was born in the year 1840 and died in the year 1911, how old was he at the time of his death?

EXPLANATION. 1840 years after the birth of Christ (one given unequal component of a given sum) + how many years (a required second unequal component of that given sum) = 1911 years after the birth of Christ (the given sum or minuend) [Note 1, *b*, **21**].

22. George Washington died in the year 1799, aged 67 years. What was the year of his birth?

23. The government of the United States was established in 1789. How many years had intervened between that event and the commencement of the Civil War in 1861?

24. William H. Taft was born in the year 1857, and was inaugurated President of the United States in the year 1909. How old was he at the time of his inauguration?

25. A, B, C and D formed a copartnership with a joint capital of $55000. If B invested twice as much as A; C, three times as much as A; and D, as much as B and C together; how much did each partner invest?

EXPLANATION. Twice (or 2 times) is the given multiplier of A's investment to produce B's investment; 3 is the given multiplier of A's investment to produce C's investment; and 2 + 3, or 5, is the indirectly given multiplier of A's investment to produce D's investment. Therefore, 1 time A's investment or A's investment + 2 times A's investment, or B's investment + 3 times A's investment or C's investment + 5 times A's investment or D's investment, that is, 11 times A's investment, or all the partners' investments, must equal the joint capital of the firm ($55000) or joint product.

Hence, divide the joint product ($55000) by its joint multiplier (11) to find the common multiplicand or A's investment ($5000). Next multiply the obtained common multiplicand ($5000) by the respective multipliers for B's investment (2), for C's investment (3) and for D's investment (5), to find their respective investment products.

NOTE. If the sum of two or more products is given, its multiplicand factor will be the primary multiplicand (from which all the other mentioned multiplicands have been successively derived); and its multiplier factor will be the sum of the expressed multipliers + 1, first changing the multipliers which have been expressed upon derivative multiplicands, to equivalent multipliers upon the primary multiplicand [Prin. 1, **18**].

26. A and B together have $260, B having three times as much as A. How much has each?

27. A, B, and C have $520, B having four times as much as A, and C twice as much as B. How much has each?

28. A man bought two houses for $6275, paying $275 more for one than for the other. How much did he pay for each?

EXPLANATION. $6275 is the given sum (minuend) of two equal quantities, and $275 an unequal quantity [Note 1, *b*, **21**]; that is, $6275 is $275 more than the sum of two equal quantities, each denoting the cost of the cheaper house. Hence, diminish the minuend ($6275) by its given unequal quantity ($275) to find the sum of its two remaining equal quantities ($6000). As $6000 is the total of two equal quantities, or product, divide it by its multiplier factor 2 (the number of equal quantities), obtaining its multiplicand factor, or the cost of the cheaper house ($3000), etc.

29. A and B formed a copartnership with a capital of $13168, A investing $568 more than B. How much did each invest?

30. The firm of A and B has a capital of $12387, A's investment being $2387 less than B's. What is each partner's investment?

31. A, B, and C together own property worth $17000, B's property being worth $1200 more than A's, and C's $400 less than B's. What is the worth of the property of each?

32. A man bequeathed $37025 to his wife, son, and daughter; his son receiving $625 less than his wife, and his daughter $925 more than his son. How much did each receive?

33. Two ships leave a certain harbor at the same time; one going east at the rate of 350 miles per day, and the other going west at the rate of 275 miles per day. How far apart will they be at the end of the ninth day?

34. At an election for mayor of a certain city, the total number of votes cast for three candidates was 52300. The first candidate received 34500 votes, and the second 1828 votes less than the third. How many votes did the third candidate receive?

35. A man made five partial payments on a debt of $1565.78, and then found that he still owed $237.62. If his first payment was $128.37, his second $75.40, his third $216.15, and his fourth $375.28, what was the amount of his fifth payment?

36. Two steamers start from the same place at the same time to go in the same direction. Four days after starting the faster steamer was 140 miles ahead of the slower. If the faster steamer's average speed was 375 miles per day, how far was the slower steamer from the starting point?

37. Two pedestrians lived 75 miles apart, and started from their respective homes at the same time to approach each other, one walking at the rate of four miles per hour, and the other at the rate of three miles per hour. How far apart were they after walking eight hours?

38. A farmer exchanged a tract of land containing 275 acres, and valued at $45 per acre for another tract containing 495 acres. What was the estimated value per acre of the other tract?

39. A certain library contains 43 book cases, each case containing 12 shelves, and each shelf 78 books. How many books does the library contain?

40. A merchant examined the contents of his cash drawer

and found that it contained $48. Of this sum he found $30 to be in notes, and the remainder consisted of an equal number of 50-cent coins, 25-cent coins, dimes and nickels. How many coins did he find in the cash drawer?

FACTORING

55. Factoring is that process by which a number is separated into two or more integers [Note 1, *e*, 3] which, when multiplied, will produce that number.

NOTE 1. Each of the integers into which a number is separated as above described, is called a *factor* of that number.

NOTE 2. When a number cannot be separated into integral factors other than itself or 1, it is called a *prime number;* and when it can be so separated, it is called a *composite number.*

NOTE 3. The factors of a number are also distinguished as *prime factors* when they are not separable into minor integral factors; and as *composite factors* when they are so separable.

56. An exact divisor of a number is one which will divide it without a remainder.

NOTE. Reversely, one number is said to be divisible by another when it contains that other a certain number of times without a remainder.

57. Any number is divisible by itself and 1; and it will be divisible:

(*a*) By 2, if it ends with a cipher, or with a figure which is divisible by 2.

Thus, 480 and 7818 are divisible by 2; the former because it ends with a cipher, the latter because it ends with 8 which is divisible by 2.

(*b*) By 3, if the sum of all its figures is divisible by 3.

Thus, 4791 is divisible by 3 because $4 + 7 + 9 + 1$, or 21, is divisible by 3.

(*c*) By 4, if it ends with two ciphers, or with two figures which express a number that is divisible by 4.

Thus, 34700 and 72324 are divisible by 4; the former because it ends with two ciphers, the latter because it ends with two figures (24) which express a number that is divisible by 4.

(*d*) By 5, if it ends either with a cipher or with 5.

Thus, 720 and 1465 are divisible by 5; the former because it ends with a cipher, and the latter because it ends with 5.

(*e*) By 6, if it ends with a figure which is divisible by 2 (one factor of 6), and the sum of all its figures is divisible by 3 (the other factor of 6).

Thus, 1368 is divisible by 6 because it ends with a figure (8) which is divisible by 2; and $1 + 3 + 6 + 8$, or 18, is divisible by 3.

(*f*) By 8, if it ends with three ciphers, or with three figures which express a number that is divisible by 8.

Thus, 276000 and 18376 are divisible by 8; the former because it ends with three ciphers, and the latter because it ends with three figures (376) which express a number that is divisible by 8.

(*g*) By 9, if the sum of its figures is divisible by 9.

Thus, 32472 is divisible by 9 because $3 + 2 + 4 + 7 + 2$, or 18, is divisible by 9.

(*h*) By 10, if it ends with one or more ciphers.

Thus, 6730 is divisible by 10 because it ends with a cipher.

(*i*) By 12, if it ends with two figures which express a number that is divisible by 4 (one factor of 12), and the sum of all its figures is divisible by 3 (the other factor of 12).

Thus, 71928 is divisible by 12 because it ends with two figures (28) which express a number that is divisible by 4, and $7 + 1 + 9 + 2 + 8$, or 27, is divisible by 3.

ILLUSTRATIVE EXAMPLE

58. What are the prime factors of 630?

SOLUTION

$$\begin{array}{r} 5)\overline{630} \\ 2)\overline{126} \\ 3)\overline{63} \\ 3)\overline{21} \\ \overline{7} \end{array}$$

EXPLANATION. As 630 ends with a cipher, divide it by its prime factor 5 [**57**, *d*], obtaining 126 as the unfactored part of 630. As 126 ends with 6 which is divisible by 2, the entire unfactored part is so divisible [**57**, *a*], obtaining 63 as the remaining unfactored part of the original number. As the sum of the figures of 63 ($6 + 3$), or 9, is divisible by 3, the entire unfactored part (63) is so divisible, obtaining 21 as the remaining unfactored part [**57**, *b*]. As the sum of the figures of 21 ($2 + 1$), or 3, is divisible by 3, the entire unfactored part (21) is so divisible, obtaining 7, a prime number. Therefore, the several prime divisors and

the final prime quotient 7, that is, 5, 2, 3, 3, 7, are the required prime factors of 630.

PROOF. The continued product of all its prime factors should equal the factored number. Thus $5 \times 2 \times 3 \times 3 \times 7$ should equal 630.

RULE. 1. *Divide the given composite number by any prime number which will exactly divide it.* 2. *Divide the resulting quotient by any prime number which will exactly divide it; and so continue until a quotient is obtained which is a prime number.* 3. *The several divisors and the final quotient will be the prime factors of the given composite number.*

EXAMPLES FOR PRACTISE

Find the prime factors of

1. 54	**4.** 360	**7.** 1260	**10.** 27000	**13.** 16632
2. 36	**5.** 252	**8.** 5292	**11.** 17640	**14.** 45000
3. 84	**6.** 945	**9.** 3375	**12.** 28512	**15.** 16380

LEAST COMMON DIVIDEND

59. A dividend of a number is one that contains it an exact number of times.

Thus, 63 is a dividend of 7 because 63 contains 7 exactly nine times.

60. A common dividend of two or more numbers is one that contains each of them an exact number of times.

Thus, 24 is a common dividend of 3, 6 and 8, because it contains 3 exactly eight times, 6 exactly four times, and 8 exactly three times.

61. The **least common dividend** of two or more numbers is the *least* number which contains each of them an exact number of times.

62. General principles. 1. *A dividend of a number must include all its prime factors, and may include other factors.* 2. *A common dividend of two or more numbers must include all the prime factors of each of those numbers, and may include other factors.* 3. *The least common dividend of two or more numbers must include all the prime factors of those numbers,* and no other factor.

ILLUSTRATIVE EXAMPLE

63. Find the least common dividend of 12, 18, and 20.

SOLUTION

$$\begin{array}{r} 2)12 - 18 - 20 \\ \hline 2)6 - 9 - 10 \\ \hline 3)3 - 9 - 5 \\ \hline 1 - 3 - 5 \end{array}$$

$$2 \times 2 \times 3 \times 3 \times 5 = 180$$

EXPLANATION. 2 is a common prime factor of each of the given numbers, therefore, the least common dividend must contain the factor 2 *at least once.*

Of the unfactored part of 12 (6), of 18 (9) and of 20 (10), 2 is a common prime factor of 6 and 10. Therefore the least common dividend must contain the factor 2 *at least twice* to contain 12 and 20 which include this factor *twice.*

Of the yet unfactored part of 12 (3), of 18 (9) and of 20 (5), 3 is a common prime factor of 3 and 9. Therefore the least common dividend must contain the factor 3 *at least once* in order to contain 12 and 18 which include this factor *at least once.*

The least common dividend must therefore not only contain the prime factors already found (2, 2, 3), but also the yet unfactored part of 18 (3) or it cannot contain 18, and the yet unfactored part of 20 (5) or it cannot contain 20. Therefore the *least* common dividend of 12, 18 and 20 must contain these *necessary* factors (2, 2, 3, 3 and 5) and *no other factor*, and the continued product of these factors is 180.

NOTE 1. When no two of the given numbers have a common prime factor, the least common dividend of those numbers must be their *continued product.*

NOTE 2. When it is seen that *all* of the given numbers have a common composite factor [Notes 2 and 3, **55**], it will save unnecessary labor to divide at once by the composite factor.

NOTE 3. When one of the given numbers of which it is required to find the least common dividend is found to be an exact divisor of another of the given numbers, it may be omitted from the solution, as all of its factors must then be included in that other number.

EXAMPLES FOR PRACTISE

Find the least common dividend of

1. 15, 18, 24, and 36
2. 21, 49, 56, and 63
3. 18, 27, 45, and 72
4. 24, 36, 48, and 64
5. 27, 36, 54, and 81
6. 30, 45, 75, and 90
7. 360, 450, 540, and 630
8. 150, 500, 600, and 900
9. 132, 165, 231, and 297
10. 5, 15, 25, 30, 45 [Note 3]
11. 12, 36, 48, 72, and 96
12. 2, 3, 7, 11, 13 [Note 1]

GREATEST COMMON DIVISOR

64. A common divisor of two or more numbers is an exact divisor of each of them.

Thus, 7 is a common divisor of 21, 35 and 56, as 7 is an exact divisor, or common factor, of each of them.

65. The **greatest common divisor** of two or more numbers is the *greatest* exact divisor of each of them.

Thus, while 2 is a common divisor of 24, 40 and 56, it is not the *greatest* common divisor; and while 4 is a *greater* common divisor of these numbers than 2, it is also not the *greatest* common divisor. The greatest divisor common to these numbers is 8, because 8 is the *greatest factor* common to all of them.

66. General principles. 1. *A common divisor of two or more numbers must include in itself only such prime factors as are common to those numbers.*

Thus, 6 is a common divisor of 30 and 42, because *all* the prime factors of 6 (3×2) are found both in 30 ($\mathbf{3} \times \mathbf{2} \times 5$) and in 42 ($\mathbf{3} \times \mathbf{2} \times 7$).

2. *The greatest common divisor of two or more numbers must include all the prime factors which are common to those numbers, and no other factor.*

Thus, 9 is the greatest common divisor of 45 and 63 because *all* the prime factors which are common to 45 ($\mathbf{3} \times \mathbf{3} \times 5$) and to 63 ($\mathbf{3} \times \mathbf{3} \times 7$) are also found in 9 ($3 \times 3$). A greater composite number than 9 cannot be a common divisor of 45 and 63 because it will include a factor which is not common to 45 and 63; and a less number than 9, say 3, which contains one factor common to 45 and 63 (3×1), and therefore a common divisor, would not contain *all* the common factors of those numbers and therefore could not be their *greatest* common divisor.

3. *The greatest common divisor of two numbers must also be a divisor* (a) *of their sum,* (b) *of their difference,* (c) *of any number of times their sum,* (d) *or of any number of times their difference.*

4. *The greatest common divisor of two numbers must also be* (e) *a divisor of any number of times each of them; or* (f) *of any number of times one of them and a different number of times the other; or* (g) *of the difference between the greater number and any number of times the less number that can be subtracted from the greater number.*

5. *The greatest common divisor of more than two numbers cannot be greater than the greatest common divisor of the two which express the fewest units.*

ILLUSTRATIVE EXAMPLE

67. Find the greatest common divisor of 531 and 767.

SOLUTION

531)767(1
531
236)531(2
472
59)236(4
236

EXPLANATION. The greatest divisor of 531 could not also be the greatest divisor of 767 except it be also a divisor of their difference which is 236 [Prin. 3, *b*, **66**].

Therefore, the G. C. D. of 531 and 767 must also be the G. C. D. of 236, 531 and 767; and this G. C. D. cannot be greater than the G. C. D. of 236 and 531 [Prin. 5, **66**]. Now the greatest divisor of 236 could not also be the greatest divisor of 531 except it be also a divisor of the difference between 531 and twice 236, or 472, which is 59 [Prin. 4, *g*, **66**].

Therefore, the G. C. D. of 236, 531 and 767 must also be the G. C. D. of 59, 236, 531 and 767, and this G. C. D. cannot be greater than the G. C. D. of 59 and 236 [Prin. 5, **66**]. As 59 will divide itself and as it is also found to be an exact divisor of 236, 59 is the G. C. D. of 59 and 236, and as 236 is the difference between 531 and 767, 59 must also be the G. C. D. of 531 and 767 [Prin. 3, *b*, **66**].

RULE. 1. *Divide the greater number by the less; and if there be no remainder, the less number will be the greatest common divisor.*

2. *Divide the last divisor by the remainder if there should be a remainder, and if the division be exact, that remainder will be the greatest common divisor.*

3. *If the second division be inexact, continue to divide the last divisor by the last remainder until there shall be no remainder. The last divisor will be the greatest common divisor.*

NOTE 1. If, at any time, 1 is obtained as a remainder, it will demonstrate that the numbers are prime to each other.

NOTE 2. To find the G. C. D. of more than two numbers, first find the G. C. D. of two of them, preferably of the two which express the fewest units; then find the G. C. D. of this obtained G. C. D. and one of the remaining numbers; and thus continue until all the given numbers have been considered. The last obtained G. C. D. will be the G. C. D. of all the numbers.

Note 3. If the numbers are not inconveniently great, their greatest common divisor may also be obtained by finding the continued product of *such prime factors as* are common to *all* of them.

EXAMPLES FOR PRACTISE

Find the greatest common divisor of

1. 185 and 259	5. 539 and 637	9. 16767 and 29889
2. 315 and 405	6. 10625 and 14375	10. 14406 and 26411
3. 204 and 612	7. 1344 and 4416	11. 12096, 19008, 22464
4. 875 and 1375	8. 1215 and 2673	12. 704, 448, 880

CANCELATION

68. Cancelation is the process of diminishing the mechanical labor of division by casting out factors which are common to both divisor and dividend.

Thus, in dividing 168 by 56, if it be noticed that 8 is a common factor of both divisor and dividend, the labor of division will be diminished by mentally dividing one-eighth of 168, or 21, by one-eighth of 56, or 7, obtaining 3 as the quotient [Prin. 5, **34**].

69. General principles. 1. *To cancel any factor of a number has the same effect as dividing that number by the canceled factor.*

2. *To cancel the same factor, or the same combination of factors, or equivalent combinations of factors, from both divisor and dividend, does not affect the value of the quotient* [Prin. 5, **34**].

ILLUSTRATIVE EXAMPLE

The factors of a dividend are 210, 16 and 18, and of its divisor are 27, 8 and 7. What is the quotient?

SOLUTION

$$\frac{\overset{10}{\cancel{210}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{18}}}{\underset{\cancel{8}}{\cancel{27}} \times \cancel{8} \times \cancel{7}} = 40$$

EXPLANATION. Instead of finding the continued product of 210, 16 and 18, which is 60480, and dividing this result by the continued product of $27 \times 8 \times 7$, which is 1512, obtaining 40, the mechanical labor of the solution can be materially diminished by cancelation, or continued division, [**37**], as follows:

Place the factors of the dividend above a horizontal line and the factors of the divisor below that line. Then cancel the common factor 9 from 27 in the divisor and from 18 in the dividend, writing underneath 27 its uncanceled factor 3, and above 18 its uncanceled factor 2. Then cancel the common factor 8 from 8 in the divisor and from 16 in the dividend, and over 16 place its uncanceled factor 2. Then cancel the common *factor 7 from 7 in* the divisor and from 210 in the dividend, and

over 210 place its uncanceled factor 30. Then cancel the yet uncanceled factor 3 from 3 in the divisor and from 30 in the dividend, and place over 30 its uncanceled factor 10. The product of the uncanceled factors in the dividend ($10 \times 2 \times 2 = 40$) will be the required quotient.

NOTE 1. If, after the cancelation has been completed, any uncanceled factors are still found in the divisor, the product of the uncanceled factors of the dividend should be divided by the product of the uncanceled factors of the divisor, to obtain the quotient.

NOTE 2. If the product of two or more factors on one side of the horizontal line is seen to be equal to a single factor on the other side of that line, they should be canceled in one operation.

NOTE 3. If 1 is obtained as a result of cancelation, it should be expressed in the dividend, but it may be omitted from the divisor.

EXAMPLES FOR PRACTISE

1. Divide $73 \times 27 \times 16$ by 18×4.
2. Divide $145 \times 36 \times 12$ by $30 \times 18 \times 4$.
3. Divide $648 \times 75 \times 32$ by $54 \times 45 \times 8$.
4. Divide $180 \times 63 \times 24$ by $48 \times 18 \times 9$.
5. How many bushels of wheat worth \$1.12 per bushel should a farmer deliver at a mill to receive 3 barrels of flour worth \$5.60 per barrel?
6. A speculator offered a tract of land containing 375 acres in exchange for 250 acres of land worth \$75 per acre. At what price per acre did the speculator value his land?

COMMON FRACTIONS

70. INTRODUCTION. Hitherto, all the operations which have been described and illustrated have been performed upon integral units [Note 1, *e*, 3]. It is frequently necessary, however, to operate upon a *part* of an integral unit, as upon the half of one integral unit ($\frac{1}{2}$), the third of one integral unit ($\frac{1}{3}$), etc.; or upon a *part* of a collection of integral units, as upon a third of two integral units ($\frac{2}{3}$), an eighth of five integral units ($\frac{5}{8}$), etc. The general principles already learned are unlimited in their application to all numbers, whether integral or fractional.

Any perplexity attending the study of this new *fractional form of numerical expression* will disappear if every fraction be regarded as an unexecuted division of which its numerator is the dividend, its denominator is the divisor, and the value of the entire fractional expression *is the* quotient. Thus regarded, the general

principles relating to division [34], so far as they apply to dividends, will also apply to numerators [72]; so far as they apply to divisors, will also apply to denominators [73]; and so far as they apply to quotients, will also apply to the entire fractional expression [71]. All reductions of fractions [82], therefore, are applications of the general principles of 34.

71. **A fraction** expresses the *result* of dividing one or more units into a given number of equal parts.

72. The **numerator** is that term of a fraction which is written *above* a horizontal line, and which denotes *how many* of the equal parts are expressed.

Thus, $\$\frac{5}{8}$ expresses 5 fractional units, the value of each of which is one-eighth of a dollar. Num(b)er-ator means *that which numbers.*

73. The **denominator** is that term of a fraction which is written *below* a horizontal line, and which denotes the *value* of each fractional unit expressed by the numerator.

Thus, in the fraction $\$\frac{4}{5}$, the numerator (4) denotes the *number* of fractional units; and the denominator (5) names the *value* of each fractional unit as being one-fifth of a dollar. Denominator means *that which denominates* or *names.*

74. The **terms** of a fraction are its numerator and denominator, considered separately.

Thus, in $\frac{7}{8}$, the numerator 7 is one term and the denominator 8 is the other. Considered jointly, they constitute the fraction $\frac{7}{8}$.

75. **A common fraction** is one which expresses the division of one or more units into *any* number of equal parts other than such as are represented by 1 followed by one or more ciphers.

Thus, $\frac{2}{3}$, $\frac{5}{16}$, $\frac{9}{800}$ are common fractions; but $\frac{8}{10}$, $\frac{7}{100}$, $\frac{19}{1000}$ are *not.*

76. **A proper fraction** is one in which the numerator is less than the denominator.

A proper fraction is so-called because a fraction properly means *part* of a whole; and if the numerator of a fraction should exceed its denominator, it would express *more than* a whole and could not *properly* be called a fraction, though for convenience it may be so indicated.

77. **An improper fraction** is one in which the numerator either equals or exceeds its denominator.

Thus, $\frac{1}{2}$, $\frac{3}{4}$ are *proper fractions* because the value of each is less than 1 integral unit; and $\frac{4}{4}$, $\frac{5}{4}$ are *improper fractions*, the former because its value is exactly equal to 1 integral unit, and the latter because its value is greater than 1 integral unit.

NOTE. Any integer may be expressed in the form of an improper fraction by writing 1 under it for a denominator. Thus, $5 = \frac{5}{1}$; $19 = \frac{19}{1}$.

78. A mixed number is an expression which is composed partly of an integer and partly of a fraction.

Thus, $35\frac{1}{2}$ is suggestively called a mixed number because it is composed partly of an integer (35) and partly of a fraction ($\frac{1}{2}$), *mixed* or united in one expression ($35\frac{1}{2}$).

79. The **value** of a fraction is the value expressed by consummating the unexecuted division of its numerator by its denominator.

Hence, the nearer a numerator of a *proper fraction* approximates to its denominator, the greater its value; because any increase in a dividend (numerator), the divisor (denominator) remaining the same, must necessarily increase the value of the quotient. Prin. 4, **34**, relating to an increase of the dividend (numerator) by multiplication must, though in a different manner, apply also to an increase by addition.

80. (*a*) The **unit of a fraction** is *one of the integral units* of which the fraction expresses a part; and (*b*) a **fractional unit** is *one of the equal parts* into which the unit of the fraction is divided.

Thus, in the expression $\$\frac{3}{8}$, *one* dollar is the unit of the fraction; and *one* eighth-of-a-dollar is the fractional unit.

81. General principles. 1. *To multiply or to divide the numerator by any number is equivalent to multiplying or to dividing the fraction by that number* [Prin. 4, **34**].

2. *To multiply the denominator by any number is equivalent to dividing the fraction by that number* [Prin. 6, **34**].

3. *To divide the denominator by any number is equivalent to multiplying the fraction by that number* [Prin. 7, **34**].

4. *To multiply or to divide both the numerator and the denominator by the same number will produce no change in the value of the fraction* [Prin. 5, **34**].

SUMMARY. (*a*) Any change in the numerator by multiplication or division, the denominator remaining the same, will produce a *similar* change in

the value of the fraction; (*b*) Any change in the denominator by multiplication or division, the numerator remaining the same, will produce an *opposite* change in the value of the fraction; (*c*) A *similar change* in both numerator and denominator, by multiplication or division, will produce *no change* in the value of the fraction.

REDUCTION OF FRACTIONS TO HIGHER TERMS

82. Reduction of fractions is the process of changing their *form* (appearance) without producing any change in their *value.*

Thus, if it is desired to change a fraction in *low* terms (possessing a *less* numerator and denominator) to an equivalent fraction in higher terms (possessing a *greater* numerator and denominator), the *name* of the process is *reduction*, and of the act, *to reduce.*

ILLUSTRATIVE EXAMPLE

Reduce $\frac{6}{7}$ to 126ths.

SOLUTION

$126 \div 7 = 18$

$\frac{6 \times 18}{7 \times 18} = \frac{108}{126}$

EXPLANATION. The required denominator (126) is 18 times the given denominator 7 ($126 \div 7 = 18$); therefore, the proper numerator for the required denominator must be 18 times the given numerator 6 ($6 \times 18 = 108$), to maintain a value equivalent to that of $\frac{6}{7}$ [Prin. 4, **81**].

RULE. *Multiply both the numerator and the denominator by such a number as will change the given denominator to the required denominator.*

NOTE 1. To find the common multiplier of both terms to produce a required higher denominator, *divide the required denominator* (the greater) *by the given denominator* (the less).

NOTE 2. To reduce an integer to a fraction of a required denominator, first change the integer to the form of a fraction by writing the denominator 1 under it, or by *conceiving it to be so written.* Thus, $5 = \frac{5}{1}$; $19 = \frac{19}{1}$; etc.

EXAMPLES FOR PRACTISE

Reduce	Reduce	Reduce
1. $\frac{3}{5}$ to 25ths	**5.** $\frac{2}{3}$ to 216ths	**9.** $\frac{1}{8}$ to 256ths
2. $\frac{5}{8}$ to 56ths	**6.** $\frac{3}{8}$ to 512ths	**10.** $\frac{3}{16}$ to 64ths
3. $\frac{3}{4}$ to 32nds	**7.** $\frac{1}{4}$ to 16ths	**11.** 28 to 4ths
4. $\frac{1}{2}$ to 64ths	**8.** $\frac{7}{8}$ to 128ths	**12.** 176 to 16ths

REDUCTION OF FRACTIONS TO LOWER TERMS

ILLUSTRATIVE EXAMPLE

83. Reduce $\frac{85}{544}$ to 32nds.

SOLUTION

$544 \div 32 = 17$

$\frac{85}{544} \div 17 = \frac{5}{32}$

EXPLANATION. First divide the given denominator (544) by the required denominator (32) to find the divisor of the given denominator (544) which will produce the required denominator (32), obtaining 17.

If the given denominator must be divided by 17 to find the required denominator, the given numerator (85) must also be divided by 17 to find the corresponding numerator (5) of the required denominator [Prin. 4, 81].

RULE. *Divide both the numerator and the denominator by such a common divisor as will change the given denominator to the required denominator.*

NOTE. To find the common divisor of both terms of a fraction which will produce an equivalent fraction of the required lower denominator, *divide the given denominator* (the greater) *by the required denominator* (the less).

EXAMPLES FOR PRACTISE

Reduce	Reduce	Reduce
1. $\frac{255}{408}$ to 8ths	**4.** $\frac{153}{288}$ to 32nds	**7.** $\frac{143}{704}$ to 64ths
2. $\frac{69}{368}$ to 16ths	**5.** $\frac{482}{964}$ to halves	**8.** $\frac{152}{8}$ to an integer (firsts)
3. $\frac{48}{172}$ to 4ths	**6.** $\frac{174}{232}$ to 4ths	**9.** $\frac{6888}{8868}$ to an integer

REDUCTION OF FRACTIONS TO LOWEST TERMS

84. A fraction is said to be reduced to lowest terms when its numerator and denominator are prime to each other.

ILLUSTRATIVE EXAMPLES

Reduce (1) $\frac{168}{384}$ and (2) $\frac{949}{2336}$ to lowest terms.

SOLUTIONS

(1)

$\frac{168}{384} \div 8 = \frac{21}{48}$

$\frac{21}{48} \div 3 = \frac{7}{16}$ Ans.

(2)

G. C. D. of 949 and 2336 = 73

$\frac{949}{2336} \div 73 = \frac{13}{32}$ Ans.

EXPLANATIONS. (1) Divide both terms of $\frac{168}{384}$ by any observed common factor, say 8, obtaining $\frac{21}{48}$ as an equivalent fraction in *lower* terms [Prin. 4, **81**]. It will now be further observed that 3 is a common factor of both terms of $\frac{21}{48}$. Hence, divide both terms of $\frac{21}{48}$ by 3, obtaining $\frac{7}{16}$ as an equivalent fraction in *still lower* terms. As the terms of $\frac{7}{16}$ are prime to each other, they are incapable of further reduction, and therefore $\frac{7}{16}$ is in its *lowest* possible terms.

(2) If the factor or factors common to both terms are not easily discoverable, as in $\frac{949}{2336}$, it will be more convenient to divide both numerator and denominator by their greatest common divisor [**67**].

EXAMPLES FOR PRACTISE

Reduce the following fractions to lowest terms:

1. $\frac{16}{32}$	**5.** $\frac{192}{320}$	**9.** $\frac{2880}{9216}$	**13.** $\frac{32805}{52488}$
2. $\frac{18}{24}$	**6.** $\frac{135}{216}$	**10.** $\frac{2112}{6144}$	**14.** $\frac{6912}{24576}$
3. $\frac{56}{64}$	**7.** $\frac{375}{625}$	**11.** $\frac{1872}{2304}$	**15.** $\frac{14336}{16384}$
4. $\frac{27}{72}$	**8.** $\frac{216}{288}$	**12.** $\frac{4864}{8192}$	**16.** $\frac{41472}{55296}$

REDUCTION OF FRACTIONS TO APPROXIMATE LOWEST TERMS

85. INTRODUCTION. To approximate means to approach *very near* without exactly reaching. Approximations are frequently necessary in business calculations to secure serviceable results. The market reports of our daily papers, contain no fractional quotations other than halves, quarters, eighths, and sometimes, though rarely, sixteenths. In exceptional occupations which require other denominators than the above, any obtained fractional results are approximated to such exceptional denominators. Hence, if the *final result* of a calculation contains a fraction with an impractical denominator, it is customary to reduce it to a practical denominator by approximation.

ILLUSTRATIVE EXAMPLES

86. Reduce (1) $\frac{21}{56}$ to 8ths; (2) $\frac{413}{548}$ to 4ths.

FIRST SOLUTION

$21 \times 8 = 168$

$168 \div 56 = 3$, exactly

Hence, $\frac{21}{56} = \frac{3}{8}$, exactly

FIRST EXPLANATION. If the division of its numerator by its denominator expresses the value of any given fraction [**79**], then to divide 8 times the numerator of $\frac{21}{56}$ ($21 \times 8 = 168$) by the denominator of $\frac{21}{56}$ ($168 \div 56$) must express a result (exactly 3) which is exactly 8 times the value of $\frac{21}{56}$ [Prin. 1, **81**]; and this result (3) divided by 8, or $\frac{3}{8}$, must be the *exact* value of $\frac{21}{56}$ expressed in eighths.

SECOND SOLUTION

$413 \times 4 = 1652$

$1652 \div 548 = 3$, approximately

Hence, $\frac{413}{548} = \frac{3}{4}$, approximately

SECOND EXPLANATION. To divide 4 times the numerator of $\frac{413}{548}$ ($413 \times 4 = 1652$) by the denominator of $\frac{413}{548}$ ($1652 \div 548$) will produce a result (3, approximately) which must be approximately 4 times the value of $\frac{413}{548}$ [Prin. 1, 81]. Hence, this result (3) divided by 4, or $\frac{3}{4}$, must be the approximate value of $\frac{413}{548}$ when expressed in fourths.

RULE. *Multiply the numerator of the given fraction by the required denominator, and divide the resulting product by the denominator of the given fraction, disregarding any remainder from the division if less than half the divisor, or increasing the quotient by* 1 *if the remainder is half the divisor, or more. The quotient will be the exact numerator if the division be exact, or the approximate numerator if the division be inexact. Under the quotient place the required denominator. If the resulting fraction is not in lowest terms, mentally apply Solution* 1, **84**.

NOTE. Intermediate fractions should *never* be approximated as the least departure from strict accuracy near the beginning of a continued operation may terminate in a considerable error when the final result is reached.

EXAMPLES FOR PRACTISE

Reduce the following to lowest terms:

1. $\frac{203}{584}$, not higher than 8ths
2. $\frac{107}{212}$, not higher than 4ths
3. $\frac{223}{256}$, not higher than 16ths
4. $\frac{185}{656}$, not higher than 8ths
5. $\frac{2141}{5704}$, not higher than 16ths
6. $\frac{1288}{1708}$, not higher than 8ths
7. $\frac{8870}{8464}$, not higher than 32nds
8. $\frac{4281}{5655}$, not higher than 4ths.

REDUCTION OF INTEGRAL OR MIXED NUMBERS TO IMPROPER FRACTIONS

ILLUSTRATIVE EXAMPLE

87. Reduce $37\frac{5}{8}$ to an improper fraction.

SOLUTION

$37 \times 8 = 296$ eighths

$296 + 5 = 301$ eighths

Hence, $37\frac{5}{8} = \frac{301}{8}$

EXPLANATION. 8 eighths = 1, therefore, 37 times 8 eighths, or 296 eighths must equal 37 times 1 or 37; and $37\frac{5}{8}$ must equal 5 eighths more than 296 eighths or 301 eighths.

PRINCIPLES. 1. *One integral unit is expressed as a fraction with a given denominator by making the numerator the same as the given denominator.*

2. *Two or more integral units are expressed as a fraction with a given denominator by making the numerator as many times the given denominator as the integer is times* 1.

RULE. *Multiply the integer by the denominator of the fraction, add the numerator to the product, and the result will be the numerator of the required improper fraction; under which write the given denominator.*

NOTE. When no particular denominator is required, an integer is reduced to the form of an improper fraction by simply writing 1 underneath it as a denominator [Note, **77**].

EXAMPLES FOR PRACTISE

Reduce the following to improper fractions:

1. 9 to 16ths	**5.** $28\frac{1}{2}$	**9.** $463\frac{1}{32}$	**13.** $354\frac{97}{128}$
2. 75 to 8ths	**6.** $53\frac{1}{4}$	**10.** $89\frac{13}{25}$	**14.** $2186\frac{13}{32}$
3. $6\frac{7}{8}$	**7.** $176\frac{3}{16}$	**11.** $756\frac{3}{8}$	**15.** $854\frac{5}{16}$
4. $12\frac{3}{4}$	**8.** $253\frac{9}{64}$	**12.** $817\frac{5}{8}$	**16.** $612\frac{4}{5}$

REDUCTION OF IMPROPER FRACTIONS TO INTEGRAL OR MIXED NUMBERS

ILLUSTRATIVE EXAMPLE

88. Reduce $\frac{173}{8}$ to an integer or mixed number.

SOLUTION

$173 \div 8 = 21\frac{5}{8}$

EXPLANATION. As 8 eighths equal one integral unit [Prin. 1, **87**], there must be as many integral units in 173 eighths as 8 eighths are contained times in 173 eighths; that is, 21 integral units and 5 eighths.

NOTE. The fractional part of the obtained mixed number should be reduced to lowest terms, if not already so.

EXAMPLES FOR PRACTISE

Reduce to integers or to mixed numbers:

1. $\frac{32}{2}$	**4.** $\frac{350}{25}$	**7.** $\frac{429}{64}$	**10.** $\frac{7831}{16}$	**13.** $\frac{16275}{128}$
2. $\frac{128}{4}$	**5.** $\frac{723}{16}$	**8.** $\frac{1731}{16}$	**11.** $\frac{4198}{5}$	**14.** $\frac{27357}{32}$
3. $\frac{64}{8}$	**6.** $\frac{857}{32}$	**9.** $\frac{32566}{8}$	**12.** $\frac{8263}{125}$	**15.** $\frac{53573}{64}$

COMMON DENOMINATOR

89. A common denominator of two or more fractions is any *one* denominator to which each of them can be separately reduced.

90. The **least common denominator** of two or more fractions is the *least* denominator to which each of them can be separately reduced.

NOTE. A fraction can be reduced to any higher denominator which is a dividend of its own denominator [Note 1, **82**]. A *common denominator* of two or more fractions must therefore be any *common dividend* [**60**] of their several denominators; and their *least common denominator* must be the *least* common dividend [**61**] of their several denominators.

REDUCTION OF FRACTIONS TO THEIR LEAST COMMON DENOMINATOR

ILLUSTRATIVE EXAMPLE

91. Reduce $\frac{5}{6}$, $\frac{3}{8}$, and $\frac{7}{12}$ to their least common denominator.

SOLUTION

L. C. D. of 6, 8, 12 = 24

$\frac{5}{6} = (24 \div 6) \times 5 = \frac{20}{24}$

$\frac{3}{8} = (24 \div 8) \times 3 = \frac{9}{24}$

$\frac{7}{12} = (24 \div 12) \times 7 = \frac{14}{24}$

EXPLANATION. First find the least common dividend of the several denominators, as shown in Ill. Ex., **63**, obtaining 24 as the least common denominator.

Then reduce each fraction to the obtained least common denominator (24) as shown in Ill. Ex., **82**.

MENTAL PROCESS. The required least common denominator cannot obviously be less than the greatest of the given denominators (12); and it must also contain all the factors of the remaining denominators 6 and 8 which are not found in 12. That is, 12 contains the denominator 6 in $\frac{5}{6}$ and it contains the factor 4 of the denominator 8 in $\frac{3}{8}$. Hence, incorporate with 12 the remaining factor 2 of the denominator 8 in $\frac{3}{8}$, and 12×2, or 24 is the required least common denominator.

RULE. 1. *Find the least common dividend of the denominators of the given fractions as shown in Ill. Ex.,* **63**, *and this will be the required least common denominator.*

2. *Reduce each of the given fractions to this obtained least common denominator as shown in Ill. Ex.,* **82**.

NOTE 1. If the denominators of the several fractions have no common factor, they are said to be prime to each other. The continued product of all the denominators will then be their least common denominator [Note 1, 63].

NOTE 2. If any given denominator is seen to be an exact divisor of another given denominator, it need not be considered in finding the least common denominator [Note 3, 63].

NOTE 3. Before attempting to find their least common denominator, the several fractions should be reduced to lowest terms, if not already so.

EXAMPLES FOR PRACTISE

Reduce to equivalent fractions of the least common denominator:

1. $\frac{5}{6}, \frac{3}{8}$
2. $\frac{1}{2}, \frac{2}{3}, \frac{7}{8}$
3. $\frac{9}{35}, \frac{11}{12}, \frac{4}{5}$
4. $\frac{3}{4}, \frac{5}{6}, \frac{13}{16}$
5. $\frac{1}{4}, \frac{2}{3}, \frac{5}{12}$
6. $\frac{3}{5}, \frac{1}{6}, \frac{2}{9}$
7. $\frac{2}{5}, \frac{9}{10}, \frac{11}{15}$
8. $\frac{5}{8}, \frac{7}{12}, \frac{9}{16}$
9. $\frac{3}{16}, \frac{13}{24}, \frac{7}{36}$
10. $7, \frac{3}{5}, \frac{2}{3}$
11. $\frac{29}{64}, 4, \frac{7}{24}$
12. $\frac{2}{3}, \frac{3}{4}, \frac{4}{5}$
13. $\frac{5}{8}, \frac{2}{3}, \frac{7}{12}, \frac{11}{24}$
14. $\frac{1}{2}, \frac{3}{4}, \frac{5}{8}, \frac{11}{12}$
15. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$
16. $\frac{7}{8}, 9, 5, \frac{11}{12}$

ADDITION OF FRACTIONS

92. Addition of fractions is the process of finding the sum of two or more unequal fractions.

PRINCIPLES. 1. *Only like numbers can be added* [Prin. 1, **18**]. *Therefore the several numbers of fractional units* (the several numerators, **72**), *must express like fractional parts* (similar denominators).

2. *Numerators express the number of fractional units* [**72**] *and denominators simply denote the names of those fractional units* [**73**]. *The sum of two or more fractions must therefore be the sum of their respective numerators.*

3. *The sum expresses a quantity of the same name as the numbers added* [Prin. 2, **18**]. *The sum of the several numerators must therefore express similar fractional units* (must have the same denominator) *as the several fractions added.*

TO FIND THE SUM OF TWO OR MORE FRACTIONS

ILLUSTRATIVE EXAMPLES

1. Find the sum of $\frac{1}{3}$, $\frac{3}{8}$, and $\frac{7}{12}$.

SOLUTION

L. C. D. of 3, 8, 12 = 24

$\frac{1}{3} = (24 \div 3) \times 1 = 8$ (24ths)

$\frac{3}{8} = (24 \div 8) \times 3 = 9$ (24ths)

$\frac{7}{12} = (24 \div 12) \times 7 = \frac{14}{31}$ (24ths)

$\frac{31}{24} = 1\frac{7}{24}$, Ans.

EXPLANATION. First reduce the fractions to a common denominator as shown in Ill. Ex., **91**. Then add the resulting numerators ($8 + 9 + 14 = 31$), and under the obtained sum (31) place the common denominator (24). As the result ($\frac{31}{24}$) is an improper fraction, reduce it to a mixed number as shown in Ill. Ex. **88**.

2. Find the sum of $25\frac{3}{4}$, $216\frac{5}{12}$ and $73\frac{11}{16}$.

SOLUTION

L. C. D. of 4, 12, 16 = 48

$25\frac{3}{4} = (48 \div 4) \times 3 = 36$ (48ths)

$216\frac{5}{12} = (48 \div 12) \times 5 = 20$ (48ths)

$73\frac{11}{16} = (48 \div 16) \times 11 = \frac{33}{89}$ (48ths)

$\frac{89}{48} = 1\frac{41}{48}$

1

$315\frac{41}{48}$, Ans.

EXPLANATION. First add the fractions as if there were no integers (as shown in preceding Ill. Ex. 1), obtaining $\frac{89}{48}$, or $1\frac{41}{48}$. Write the fractional part of the obtained mixed number ($\frac{41}{48}$) under the column of fractions; and carry the integral part of the obtained mixed number (1) to the units' column of the integers. Then add the integers (1, the carried figure, + 73 + 216 + 25), obtaining $315\frac{41}{48}$ as the complete sum.

EXAMPLES FOR PRACTISE

Find the sum of

1. $\frac{5}{6} + \frac{3}{4}$
2. $\frac{7}{8} + 1\frac{1}{2}$
3. $\frac{2}{3} + \frac{3}{4} + \frac{1}{6}$
4. $\frac{3}{8} + \frac{4}{5} + \frac{1}{2}$
5. $\frac{1}{2} + \frac{2}{3} + \frac{7}{12}$
6. $\frac{3}{4} + \frac{5}{8} + \frac{5}{12}$
7. $\frac{3}{16} + 1\frac{1}{2} + \frac{2}{3} + \frac{1}{8}$
8. $\frac{3}{4} + \frac{1}{3} + \frac{2}{8} + 1\frac{1}{2}$
9. $\frac{5}{8} + 17\frac{7}{32} + \frac{5}{8} + \frac{7}{16}$
10. $\frac{7}{8} + \frac{7}{9} + \frac{7}{10} + \frac{7}{12}$
11. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5}$
12. $\frac{3}{4} + \frac{5}{8} + \frac{9}{16} + 1\frac{2}{32}$
13. $8\frac{3}{4} + 16\frac{4}{8} + 27\frac{5}{8} + 46\frac{2}{3}$
14. $231\frac{7}{8} + 69\frac{5}{12} + 171\frac{3}{4} + 29\frac{1}{3}$
15. $73\frac{4}{5} + 27\frac{1}{3} + 62\frac{5}{6} + 91\frac{3}{10}$
16. $49\frac{2}{3} + 56\frac{5}{8} + 16\frac{1}{6} + 39\frac{7}{12}$
17. $628\frac{3}{16} + 9\frac{1}{12} + 28\frac{5}{9} + 63\frac{2}{3}$
18. $19\frac{1}{2} + 17\frac{2}{3} + 26\frac{3}{4} + 34\frac{1}{8}$

NOTE 1. It is usual, in the dry-goods business, to express all fractions of a yard in *fourths*. The denominator, being thus understood, is not expressed; and the numerator is written, like an exponent, at the right of the integer.

19. Add 47^{1} yards, 51^{2} yards, 39^{1} yards, 48^{3} yards, 45^{2} yards, 46 yards.

20. Add 38^{3} yards, 46^{2} yards, 51 yards, 42^{1} yards, 38^{2} yards, 47 yards, 39^{3} yards.

NOTE 2. It is customary in the grocery business to express all fractions of a pound in sixteenths, or ounces, and to omit writing the denominator understood.

21. Add 12^{9} lbs., 35^{3} lbs., 26 lbs., 18^{8} lbs., 23^{11} lbs., 41^{1} lbs., 23^{15} lbs.

22. Add 273^{5} lbs., 286^{12} lbs., 563^{8} lbs., 725^{3} lbs., 586^{13} lbs., 75^{11} lbs.

NOTE 3. In the grain business fractions of a bushel are similarly expressed in 60ths (pounds) for wheat, in 56ths for corn, in 32nds for oats, etc.

23. Add the following weights in wheat: 758^{37} bu., 645^{52} bu., 698^{43} bu.

24. How much corn in 687^{39} bu., 587^{23} bu., 934^{18} bu., 896^{42} bu.?

25. How much oats in 2183^{12} bu., 968^{27} bu., 1472^{30} bu., 5683^{19} bu.?

SUBTRACTION OF FRACTIONS

93. Subtraction of fractions is the process of finding the difference between two unequal fractions.

PRINCIPLES. 1. *The difference can be found only between like numbers* [Prin. 1, **22**]. *Therefore, the number of fractional units* (the numerator) *of the subtrahend must be of the same name* (must have the same denominator) *as the number of fractional units* (the numerator) *of the minuend.*

2. *Numerators express the number of fractional units* [**72**] *and denominators merely denote the names of those fractional units* [**73**]. *The difference between two fractions must therefore express the difference between their respective numerators.*

3. *The difference between like numbers must also be like* [Prin. 2, 22]. *Therefore, the difference between two numbers of fractional units* (two numerators) *must express fractional units of the same name* (the same denominator) *as those of the subtrahend and minuend.*

TO FIND THE DIFFERENCE BETWEEN TWO FRACTIONS

ILLUSTRATIVE EXAMPLE

94. Subtract $\frac{3}{18}$ from $2\frac{7}{12}$.

SOLUTION

L.C.D. of 18 and 12 = 36

$2\frac{7}{12} = \frac{31}{12}$ (36 ÷ 12) × 31 = 93(36ths)

$\frac{3}{18}$ (36 ÷ 18) × 13 = 26(36ths)

$\frac{67}{36} = 1\frac{31}{36}$, Ans.

EXPLANATION. First reduce the given mixed number ($2\frac{7}{12}$) to an improper fraction ($\frac{31}{12}$) as shown in Ill. Ex., **87**. Then reduce to equivalent fractions of the least common denominator as shown in Ill. Ex., **91**, obtaining $\frac{93}{36}$ and $\frac{26}{36}$.

Next subtract the less numerator (26) from the greater numerator (93), obtaining 67 as the difference between the two numerators, under which write the common denominator (36ths) to denote the name of this difference. As the result ($\frac{67}{36}$) is an improper fraction, reduce to a mixed number as shown in Ill. Ex., **88**, obtaining $1\frac{31}{36}$.

NOTE. If the mixed number contains *few* integral units, reduce it to the form of an improper fraction before performing the subtraction; but if it contains *many* integral units, apply the method shown in **95**.

EXAMPLES FOR PRACTISE

Find the difference between

1. $\frac{3}{8}$ and $\frac{3}{16}$
2. $\frac{2}{3}$ and $\frac{3}{4}$
3. $\frac{4}{9}$ and $\frac{1}{12}$
4. $\frac{3}{4}$ and $\frac{5}{6}$
5. $\frac{1}{6}$ and $\frac{7}{8}$
6. $\frac{1}{2}$ and $\frac{2}{3}$
7. $\frac{5}{12}$ and $\frac{1}{16}$
8. $\frac{5}{9}$ and $\frac{5}{8}$
9. $\frac{3}{5}$ and $\frac{7}{9}$
10. $\frac{11}{15}$ and $\frac{23}{25}$
11. $\frac{3}{8}$ and 1
12. $\frac{7}{8}$ and $1\frac{1}{4}$
13. $2\frac{1}{3}$ and $3\frac{1}{4}$
14. $1\frac{5}{8}$ and $2\frac{1}{12}$
15. $2\frac{7}{16}$ and $3\frac{5}{32}$
16. $2\frac{3}{4}$ and $3\frac{1}{8}$

SUBTRACTION OF MIXED NUMBERS

ILLUSTRATIVE EXAMPLES

95. 1. Subtract 75 from $87\frac{3}{4}$.

SOLUTION

$87\frac{3}{4}$	3(4ths)
75	0(4ths)
$12\frac{3}{4}$	3(4ths)

EXPLANATION. First find the fractional part of the remainder by subtracting *no* fourths in the subtrahend from 3 fourths in the minuend, obtaining $\frac{3}{4}$.

Next find the integral part of the remainder by subtracting the integral part of the subtrahend (75) from the integral part of the minuend (87), obtaining 12, thus making the complete remainder $12\frac{3}{4}$.

2. Subtract $216\frac{5}{6}$ from 325.

SOLUTION

4	
$32\not{5}$	6(6ths)
$216\frac{5}{6}$	5(6ths)
$108\frac{1}{6}$	1(6th)

EXPLANATION. As 5 sixths in the subtrahend cannot be subtracted from *no* sixths in the minuend, "borrow" 1 integral unit (= 6 sixths) from the integral part of the minuend, 325. Then subtract 5 sixths of the subtrahend from the borrowed 6 sixths of the minuend, obtaining 1 sixth as the fractional part of the remainder. Next subtract the integral part of the subtrahend (216) from the integral part of the minuend (324, that is, 325 less the borrowed 1), obtaining 108 as the integral part of the remainder, thus making the complete remainder $108\frac{1}{6}$.

3. Subtract $176\frac{3}{4}$ from $358\frac{5}{6}$.

SOLUTION

L.C.D. of 4 and 6 = 12

$358\frac{5}{6}$	$(12 \div 6) \times 5 =$	10(12ths)
$176\frac{3}{4}$	$(12 \div 4) \times 3 =$	9(12ths)
$182\frac{1}{12}$, Ans.		1(12th)

EXPLANATION. First reduce the fractional parts of the subtrahend and minuend to a common denominator (12ths). Then subtract 9 twelfths in the subtrahend from 10 twelfths in the minuend, obtaining 1 twelfth as the fractional part of the remainder, to which prefix the difference between the integers (182), obtaining $182\frac{1}{12}$ as the complete remainder.

4. Subtract $249\frac{11}{12}$ from $368\frac{3}{8}$.

SOLUTION

L.C.D. of 12 and 8 = 24

$368\frac{3}{8} = 367\frac{11}{8}$	$(24 \div 8) \times 11 =$	33	(24ths)
$249\frac{11}{12}$	$(24 \div 12) \times 11 =$	22	(24ths)
$118\frac{11}{24}$, Ans.		11	(24ths)

EXPLANATION. First reduce the fractions to a common denominator (24ths), obtaining 9 twenty-fourths in the minuend and 22 twenty-fourths in the subtrahend.

As 22 twenty-fourths in the subtrahend cannot be taken from 9 twenty-fourths in the minuend, borrow 1 integral unit from the integral part of the minuend (368). Add the borrowed integral unit, which equals 24 twenty-fourths to the 9 twenty-fourths, obtaining $367\frac{9}{24}$ as the equivalent minuend. Subtract 22 twenty-fourths in the subtrahend from 33 twenty-fourths in the minuend, obtaining 11 twenty-fourths as the fractional part of the remainder.

Lastly, subtract the integral part of the subtrahend (249) from the integral part of the minuend (367, that is, 368 less the borrowed unit) obtaining 118 as the integral part of the remainder, thus making the complete remainder $118\frac{11}{24}$.

EXAMPLES FOR PRACTISE

Find the difference between

1. 617 and $928\frac{4}{7}$
2. $248\frac{2}{3}$ and 562
3. 325 and $715\frac{5}{8}$
4. $183\frac{4}{5}$ and 327
5. $29\frac{3}{8}$ and $86\frac{2}{3}$
6. $42\frac{2}{5}$ and $59\frac{7}{8}$
7. $72\frac{1}{4}$ and $92\frac{11}{12}$
8. $23\frac{3}{16}$ and $58\frac{2}{3}$
9. $52\frac{7}{8}$ and $71\frac{3}{5}$
10. $34\frac{7}{9}$ and $63\frac{5}{12}$
11. $28\frac{11}{16}$ and $57\frac{5}{12}$
12. $65\frac{2}{3}$ and $82\frac{1}{4}$
13. $24\frac{1}{3}$ and $46\frac{7}{8}$
14. $35\frac{9}{32}$ and $64\frac{3}{24}$
15. $48\frac{3}{5}$ and $71\frac{4}{15}$
16. $56\frac{1}{2}$ and $84\frac{5}{6}$

MULTIPLICATION OF FRACTIONS

96. Multiplication of fractions is the process of finding the product if one or both factors are fractions.

NOTE. If the multiplicand denotes the value of *one* (entire) considered equal quantity [Note 1, *a*, **25**], and the multiplier denotes the number of such considered quantities [Note 1, *b*, **25**], and the product denotes the value of *all* such considered quantities [Note 1, *c*, **25**], it must follow that if the multiplier is 1, or an improper fraction equal to 1, the product must exactly equal the multiplicand; that if the multiplier is a mixed number or an improper fraction, that is, if the multiplier is greater than 1 and therefore denotes that the multiplicand is required to be taken more than one time, the product must be greater than the multiplicand; and if the multiplier is a proper fraction, or less than 1, it will denote that the multiplicand is required to be taken less than one time, and, therefore, the product must be less than the multiplicand. Hence,

PRINCIPLE. *Multiplication signifies increase when the multiplier is greater than* 1, *and decrease when the multiplier is less than 1.*

TO MULTIPLY ONE FRACTION BY ANOTHER FRACTION

ILLUSTRATIVE EXAMPLES

97. Multiply (1) $\frac{3}{4}$ by $\frac{8}{9}$; (2) 16 by $\frac{3}{8}$; (3) $\frac{5}{9}$ by 12; (4) $2\frac{1}{2}$ by $\frac{8}{25}$.

SOLUTION

(1)

$$\frac{3}{4} \times \frac{8}{9} = \frac{24}{36} = \frac{2}{3}$$

Or,

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{9}}} = \frac{2}{3}$$

FIRST OPERATION. 8 times 3 fourths equal 24 fourths; therefore one-ninth of 8 times 3 fourths, that is, $\frac{8}{9}$ times $\frac{3}{4}$, must equal $\frac{1}{9}$ of $\frac{24}{4}$, or $\frac{24}{36}$ [Prin. 3, 26] which in lowest terms [84] equal $\frac{2}{3}$.

SECOND OPERATION. As the numerators 3 and 8 have been shown in the first operation to be factors of the numerator of the answer, and therefore the factors of the dividend of an unexecuted division [79], and the denominators 4 and 9 have been there shown to be factors of the denominator of the answer, and, therefore, factors of the divisor of the same unexecuted division [79], and any factor common to both divisor and dividend may be canceled as shown in Ill. Ex., 69, it will be found more convenient first to cancel all factors common to *opposite terms* as in the second operation; and then multiply the uncanceled factors of the numerators to obtain the numerator of the product, and the uncanceled factors of the denominators to find the denominator of the product.

SOLUTIONS

(2)

$$\overset{2}{\cancel{16}} \times \frac{3}{\underset{1}{\cancel{8}}} = 6$$

(3)

$$\frac{5}{\underset{3}{\cancel{9}}} \times \overset{4}{\cancel{12}} = \frac{20}{3} = 6\frac{2}{3}$$

(4)

$$\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}} \times \frac{\overset{4}{\cancel{8}}}{\underset{1}{\cancel{25}}} = \frac{4}{5}$$

The explanation of Ex. 1 will also apply to Exs. 2 and 3, as will become evident by conceiving the denominator 1 to be written under each integral factor [Note, 77].

RULE. *Multiply the numerators of the fractions to obtain the numerator of the product, and multiply their denominators to obtain the denominator of the product, first canceling all factors which are common to opposite terms.*

NOTE 1. Before multiplication, mixed numbers which express few integral units may be reduced to improper fractions [87]; and simple integers should be regarded as numerators with the denominator 1 understood [Note, 77].

NOTE 2. If one of the factors is a simple integer of many figures, it will usually be found more convenient to multiply the integer by the numerator of the fraction, and to divide the result by the denominator.

NOTE 3. When the word "of" immediately follows a fraction, it signifies *times;* thus, $\frac{2}{3}$ of $\frac{1}{4}$ means $\frac{2}{3}$ times $\frac{1}{4}$; and $\frac{2}{3}$ of a fortune means $\frac{2}{3}$ times a fortune, *a fortune* being called the *verbal* multiplicand to distinguish it from the ordinary *numerical* multiplicand.

EXAMPLES FOR PRACTISE

Multiply	Multiply	Multiply
1. $\frac{5}{6}$ by $\frac{8}{10}$	**6.** $\frac{9}{16}$ by $5\frac{1}{3}$	**11.** 673 by $\frac{3}{4}$
2. $\frac{2}{3}$ by $\frac{5}{8}$	**7.** $7\frac{1}{3}$ by $\frac{6}{9}$	**12.** $\frac{2}{3}$ by 276
3. $\frac{5}{9}$ by $\frac{8}{20}$	**8.** $2\frac{6}{7}$ by $\frac{21}{25}$	**13.** 783 by $2\frac{1}{4}$
4. $\frac{8}{16}$ by $\frac{4}{9}$	**9.** $2\frac{1}{2}$ by $\frac{3}{5}$	**14.** Find $\frac{5}{8}$ of $\frac{4}{15}$
5. $\frac{6}{7}$ by $3\frac{1}{9}$	**10.** $6\frac{2}{3}$ by $5\frac{1}{4}$	**15.** Find $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{8}{15}$

MULTIPLICATION OF INTEGRAL AND MIXED NUMBERS

ILLUSTRATIVE EXAMPLES

98. Multiply (1) $426\frac{2}{3}$ by 8; and (2) 675 by $29\frac{3}{4}$.

SOLUTIONS

(1)

$426\frac{2}{3}$
$\underline{8}$
$3413\frac{1}{3}$

$(\frac{2}{3} \times 8 = \frac{16}{3} = 5\frac{1}{3})$

(2)

675
$29\frac{3}{4}$
4)2025
$506\frac{1}{4}$
6075
1350
$20081\frac{1}{4}$

EXPLANATION. (1) 8 times 2 thirds equal 16 thirds, or $5\frac{1}{3}$. Write $\frac{1}{3}$ as the fractional part of the product, and carry the integral part (5) to the integral part of the product (426×8), obtaining $3413\frac{1}{3}$ as the complete product.

(2) First multiply 675 by the fractional part of the multiplier ($\frac{1}{4}$), as directed in Note 2, **97**, obtaining $506\frac{1}{4}$. Then multiply 675 by the integral part of the multiplier (29), and add the several partial products, obtaining $20081\frac{1}{4}$ as the complete product.

EXAMPLES FOR PRACTISE

Multiply	Multiply	Multiply
1. $678\frac{3}{4}$ by 7	**5.** 816 by $8\frac{3}{4}$	**9.** 629 by $18\frac{6}{8}$
2. $295\frac{5}{8}$ by 9	**6.** 295 by $6\frac{2}{3}$	**10.** $571\frac{2}{3}$ by 35
3. $427\frac{2}{3}$ by 8	**7.** 378 by $52\frac{7}{8}$	**11.** $826\frac{7}{8}$ by 42
4. *572 by* $3\frac{5}{8}$	**8.** 483 by $25\frac{3}{4}$	**12.** $382\frac{4}{8}$ by 85

MULTIPLICATION OF MIXED NUMBERS BY MIXED NUMBERS

ILLUSTRATIVE EXAMPLE

99. Multiply $79\frac{3}{8}$ by $27\frac{2}{3}$.

SOLUTION

$$79\frac{3}{8}$$
$$27\frac{2}{3}$$

$\frac{3}{8} \times \frac{2}{3} = \frac{1}{4} = 6$(24ths)
$79 \times \frac{2}{3} = 52\frac{2}{3} = 16$(24ths)
$\frac{3}{8} \times 27 = 10\frac{1}{8} = 3$(24ths)
$79 \times 7 = 553 \quad \frac{25}{24} = 1\frac{1}{24}$
$79 \times 2 = 158$
1

$2196\frac{1}{24}$, Ans.

EXPLANATION. The total product must equal the product of each part of the multiplicand separately multiplied by each part of the multiplier [Prin. 2, **26**]. Therefore, multiply the fractional part of the multiplicand ($\frac{3}{8}$) by the fractional part of the multiplier ($\frac{2}{3}$), obtaining $\frac{1}{4}$. Next multiply the integral part of the multiplicand (79) by the fractional part of the multiplier ($\frac{2}{3}$), obtaining $52\frac{2}{3}$. Now that *all* of the multiplicand has been multiplied by the fractional part of the multiplier, separately multiply the fractional and integral parts of the multiplicand ($\frac{3}{8}$ and 79) by the integral part of the multiplier (27), and add the several partial products obtaining $2196\frac{1}{24}$ as the complete product of *all the orders of the multiplicand* multiplied by *all the orders of the multiplier.*

RULE. 1. *Multiply the fraction of the multiplicand by the fraction of the multiplier.* 2. *Multiply the integer of the multiplicand by the fraction of the multiplier.* 3. *Multiply the fraction of the multiplicand by the integer of the multiplier.* 4. *Multiply the integer of the multiplicand by the integer of the multiplier.* 5. *Add the several partial products.*

EXAMPLES FOR PRACTISE

Multiply	Multiply	Multiply
1. $56\frac{2}{3}$ by $36\frac{5}{8}$	**6.** $286\frac{5}{7}$ by $28\frac{3}{5}$	**11.** $5684\frac{3}{8}$ by $28\frac{5}{16}$
2. $42\frac{3}{4}$ by $64\frac{1}{2}$	**7.** $672\frac{1}{8}$ by $40\frac{4}{5}$	**12.** $9275\frac{7}{9}$ by $93\frac{1}{4}$
3. $63\frac{7}{8}$ by $75\frac{2}{3}$	**8.** $548\frac{5}{8}$ by $36\frac{1}{4}$	**13.** $3246\frac{1}{2}$ by $74\frac{3}{5}$
4. $37\frac{9}{16}$ by $23\frac{4}{5}$	**9.** $715\frac{4}{9}$ by $12\frac{2}{3}$	**14.** $4183\frac{7}{8}$ by $300\frac{3}{4}$
5. $51\frac{3}{8}$ by $18\frac{1}{3}$	**10.** $368\frac{5}{8}$ by $74\frac{3}{4}$	**15.** $958\frac{1}{12}$ by $673\frac{5}{9}$

DIVISION OF FRACTIONS

100. Division of fractions is the process of dividing when the divisor or dividend, or both of them, are fractions or mixed numbers.

NOTE. If division is the opposite process to multiplication [Note 1, 48], Principle, 96, taken reversely, will apply to division; that is,

PRINCIPLES. 1. *If the divisor is greater than 1, the quotient will be less than the dividend.* 2. *If the divisor is less than 1, the quotient will be greater than the dividend.*

DIVISION OF FRACTIONS BY INTEGERS

ILLUSTRATIVE EXAMPLES

101. Divide (1) $\frac{8}{9}$ by 4; (2) $\frac{5}{8}$ by 3; (3) $\frac{4}{9}$ by 6; (4) $3\frac{1}{2}$ by 21.

SOLUTIONS

(1)

$$\frac{8}{9} \div 4 = \frac{2}{9}, \text{ Ans.}$$

(2)

$$\frac{5}{8} \times 3 = \frac{5}{24}, \text{ Ans.}$$

(3)

$$\frac{\overset{2}{\cancel{4}}}{9} \times \frac{1}{\cancel{6}_{3}} = \frac{2}{27}, \text{ Ans.}$$

(4)

$$\frac{\overset{1}{\cancel{7}}}{2} \times \frac{1}{\cancel{21}_{3}} = \frac{1}{6}, \text{ Ans.}$$

EXPLANATION. (1) 8 ninths divided by 4 equal 2 ninths [Prin. 1, 81]. For, if $\frac{8}{9}$ expresses the quotient of an unexecuted division [79] of which the numerator (8) is the dividend and the denominator (9) is the divisor, then to divide the dividend (8) by 4 is equivalent to dividing the quotient (the fraction) by 4 [Prin. 4, 34].

(2) As the numerator (5) is not divisible by the divisor (3), as in the preceding operation, use Prin. 2, 81, and multiply the denominator (8) by the divisor (3), obtaining $\frac{5}{24}$ as the quotient. For, to multiply the divisor of an unexecuted division by 3 is equivalent to dividing the quotient (the fraction) by 3 [Prin. 6, 34].

(3) As the numerator (4) is not divisible by the entire divisor (6) but is divisible by one of its factors (2), divide the numerator (4) by the factor 2, as in Ill. Ex. 1, and multiply the denominator (9) by the remaining factor 3 ($2 \times 3 = 6$), as in Ill. Ex. 2, obtaining $\frac{2}{27}$ [Summary, 1, 37].

(4) First reduce the small mixed number ($3\frac{1}{2}$) to an improper fraction ($\frac{7}{2}$). As the numerator of $\frac{7}{2}$ is not divisible by the entire divisor (21) but is divisible by one of its factors (7), divide the numerator (7) by the factor 7 of the divisor ($21 = 7 \times 3$), obtaining 1 as the new numerator and $\frac{1}{2}$ as the quotient, then, in succession, [Summary 1, 37] divide $\frac{1}{2}$ by the remaining factor of the divisor (3) as already shown in the second Ill. Ex., obtaining $\frac{1}{6}$ as the final quotient.

RULE. 1. *Divide the numerator of the fractional dividend by the divisor, and let the denominator remain unchanged.* Or,

2. *Multiply the denominator of the fractional dividend by the divisor, and let the numerator remain unchanged.* Or,

3. *Divide the numerator of the fractional dividend by the greatest factor of the divisor which will exactly divide the numerator, and multiply the denominator of the fractional dividend by the remaining factor of the divisor.*

NOTE. If the dividend is a mixed number which contains *few* integral units, it should be reduced to an improper fraction.

EXAMPLES FOR PRACTISE

Divide	Divide	Divide
1. $\frac{5}{8}$ by 3	**6.** $\frac{9}{16}$ by 6	**11.** $4\frac{3}{5}$ by 7
2. $\frac{3}{4}$ by 6	**7.** $5\frac{1}{4}$ by 14	**12.** $6\frac{2}{3}$ by 5
3. $\frac{1}{3}\frac{5}{2}$ by 5	**8.** $6\frac{3}{5}$ by 8	**13.** $\frac{5}{16}$ by 20
4. $\frac{8}{9}$ by 16	**9.** $2\frac{1}{4}$ by 9	**14.** $\frac{7}{8}$ by 28
5. $\frac{4}{9}$ by 20	**10.** $\frac{8}{7}$ by 8	**15.** $8\frac{1}{6}$ by 14

DIVISIONS OF INTEGERS BY FRACTIONS

102. Division by any number, integral or fractional, is the *reverse* of multiplication by the same number [Note 1, **48**]; that is, 8 divided by 2 is the opposite process to 8 multiplied by 2; for in the former 8 is made *one-half* as great, and in the latter, 8 is inversely made *twice* as great. Therefore, any division by a fractional divisor must be the *inverse* process of multiplication by that same fractional quantity. Now, in multiplying 6 by $\frac{2}{3}$, it has already been seen that 6 is multiplied by the numerator (2) and divided by the denominator (3). Hence, when performing the reverse operation of dividing 6 by $\frac{2}{3}$, the *reverse process* should be followed and 6 should be multiplied by the denominator (3) and divided by the numerator (2); that is, in multiplying 6 by $\frac{2}{3}$, the statement is $6 \times \frac{2}{3}$ [Ill. Ex. 2, **97**]; but in dividing 6 by $\frac{2}{3}$, the statement should be $6 \times \frac{3}{2}$, or the former process with the terms of the divisor reversed. Hence,

SUMMARY. 1. *Division by a fractional divisor is the opposite process to multiplication by a fractional multiplier.*

2. *Division by any fractional divisor is the equivalent of multiplication by that same fractional divisor with its terms inverted.*

ILLUSTRATIVE EXAMPLES

Divide (1) 8 by $\frac{4}{5}$; (2) 15 by $3\frac{1}{8}$.

SOLUTIONS

(1)

$$8 \div \frac{4}{5} =$$

$$\overset{2}{\cancel{8}} \times \frac{5}{\cancel{4}} = 10$$

$$1$$

(2)

$$15 \div \frac{25}{8} =$$

$$\overset{3}{\cancel{15}} \times \frac{8}{\cancel{25}} = \frac{24}{5} = 4\frac{4}{5}$$

$$5$$

EXPLANATION. (1) If the dividend (8) be divided by the numerator of the divisor (4), that is, by five times the true divisor ($\frac{4}{5}$), the resulting quotient (2) will be one-fifth of the true quotient [Prin. 6, **34**]. Therefore, five times 2 or 10, must be the true quotient; and 5 times $\frac{1}{4}$ of 8 or $\frac{5}{4}$ of 8, that is, 8 multiplied by $\frac{5}{4}$ with its terms inverted ($8 \times \frac{5}{4}$) must be the proper process for dividing 8 by $\frac{4}{5}$.

(2) The quotient of 15 divided by 25 eighths ($3\frac{1}{8}$) is the same as the quotient of one-fifth of 15, or 3, divided by one-fifth of 25 eighths, or $\frac{8}{5}$ [Prin. 5, **34**]; and $3 \div \frac{8}{5}$, as shown in the first solution, is equal to $3 \times \frac{5}{8}$, or $\frac{24}{5} = 4\frac{4}{5}$.

RULE. *Multiply the dividend by the fractional divisor with its terms inverted.*

EXAMPLES FOR PRACTISE

Divide	Divide	Divide
1. 6 by $\frac{3}{4}$	**5.** 15 by $3\frac{5}{7}$	**9.** 15 by $2\frac{1}{2}$
2. 15 by $\frac{5}{8}$	**6.** 2 by $\frac{6}{7}$	**10.** 2967 by $\frac{4}{7}$
3. 24 by $\frac{8}{9}$	**7.** 19 by $\frac{3}{7}$	**11.** 6183 by $3\frac{1}{2}$
4. 8 by $\frac{4}{5}$	**8.** 28 by $\frac{7}{8}$	**12.** 7458 by $8\frac{1}{4}$

DIVISION OF FRACTIONS BY FRACTIONS

ILLUSTRATIVE EXAMPLE

103. Divide $\frac{8}{9}$ by $\frac{2}{3}$.

SOLUTION

$$\frac{8}{9} \div \frac{2}{3} =$$

$$\frac{\overset{4}{\cancel{8}}}{9} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}} = \frac{4}{3} = 1\frac{1}{3}$$

$$3$$

EXPLANATION. If 8 ninths be divided by 2, which is three times the true divisor ($\frac{2}{3}$), the quotient ($\frac{4}{9}$) will be one-third of the true quotient [Prin. 6, **34**]. Therefore 3 times $\frac{8}{9}$ divided by 2, that is, three times one-half of $\frac{8}{9}$, or $\frac{8}{9} \times \frac{1}{2}$ must be the true quotient, and this is equivalent to multiplying the dividend by the divisor with its terms inverted.

RULE. *Multiply the dividend by the fractional divisor with its terms inverted.*

EXAMPLES FOR PRACTISE

Divide	Divide	Divide
1. $\frac{5}{8}$ by $\frac{3}{4}$	**5.** $3\frac{3}{4}$ by $\frac{5}{8}$	**9.** $\frac{8}{9}$ by $\frac{3}{4}$
2. $\frac{7}{9}$ by $\frac{2}{3}$	**6.** $8\frac{1}{2}$ by $\frac{3}{4}$	**10.** $3\frac{1}{2}$ by $4\frac{2}{3}$
3. $\frac{4}{5}$ by $\frac{2}{9}$	**7.** $\frac{5}{16}$ by $2\frac{5}{8}$	**11.** $1\frac{1}{4}$ by $2\frac{1}{2}$
4. $2\frac{1}{2}$ by $\frac{3}{5}$	**8.** $\frac{7}{12}$ by $4\frac{2}{3}$	**12.** $5\frac{7}{8}$ by $3\frac{4}{9}$

DIVISION OF MIXED NUMBERS BY INTEGERS

ILLUSTRATIVE EXAMPLE

104. Divide $919\frac{7}{8}$ by 27.

SOLUTION

$$27)919\tfrac{7}{8}(34\tfrac{5}{72}$$
$$\underline{81}$$
$$109$$
$$\underline{108}$$
$$1\tfrac{7}{8} = \tfrac{15}{8}$$

$$\frac{\overset{5}{\cancel{15}}}{8} \times \frac{1}{\underset{9}{\cancel{27}}} = \frac{5}{72}$$

EXPLANATION. First divide the integral part of the dividend (919) by the divisor (27), obtaining 34 as the integral part of the quotient.

Then bring down the fractional part of dividend ($\frac{7}{8}$) to the remainder (1), obtaining $1\frac{7}{8}$ or $\frac{15}{8}$; and divide this as yet undivided part of the dividend ($\frac{15}{8}$) by the divisor (27), obtaining $\frac{5}{72}$ as the fractional part of the quotient, thus making the complete quotient $34\frac{5}{72}$.

If desired, $\frac{5}{72}$ may be reduced to any desired denominator as shown in Ill. Ex., **86**.

RULE. *Separately divide the integral and the fractional parts of the dividend by the divisor.* Or, proceed as shown in **105**.

EXAMPLES FOR PRACTISE

Divide	Divide	Divide
1. $8375\frac{2}{3}$ by 15	**4.** $6251\frac{5}{9}$ by 45	**7.** $3265\frac{1}{8}$ by 372
2. $5268\frac{5}{6}$ by 36	**5.** $9286\frac{5}{16}$ by 84	**8.** $58293\frac{5}{12}$ by 173
3. $6739\frac{3}{4}$ by 28	**6.** $2480\frac{1}{32}$ by 16	**9.** $6875\frac{1}{8}$ by 540

DIVISION OF INTEGERS BY MIXED NUMBERS

105. Divide 896 by $27\frac{5}{8}$.

SOLUTION

$$
\begin{array}{r}
27\frac{5}{8})896 \\
\underline{8 \quad 8} \\
221)7168(32\frac{96}{221} \\
\underline{663} \\
538 \\
\underline{442} \\
96
\end{array}
$$

$96 \div 221 = \frac{96}{221}$

EXPLANATION. To multiply both divisor and dividend by the same number does not affect the value of the quotient [Prin. 5, **34**]. Therefore, multiply both divisor ($27\frac{5}{8}$) and dividend (896) by the denominator of the fractional part of the divisor (8), obtaining 221 eighths as the equivalent divisor and 7168 eighths as the equivalent dividend. Divide as with integers [**36**]. If necessary, $\frac{96}{221}$ can be reduced to an approximate fraction of any desired denominator by **86**.

RULE. *Multiply both divisor and dividend by the denominator of the terminal fraction, and proceed with the results as in integral division.*

EXAMPLES FOR PRACTISE

Divide	Divide	Divide
1. 6783 by $41\frac{3}{5}$	**4.** 8729 by $328\frac{2}{3}$	**7.** 28973 by $269\frac{1}{8}$
2. 5286 by $37\frac{5}{8}$	**5.** 7123 by $416\frac{5}{7}$	**8.** 35286 by $713\frac{4}{7}$
3. 2857 by $25\frac{3}{4}$	**6.** 9285 by $876\frac{3}{8}$	**9.** 41627 by $697\frac{1}{2}$

DIVISION OF MIXED NUMBERS BY MIXED NUMBERS

ILLUSTRATIVE EXAMPLE

106. Divide $684\frac{2}{3}$ by $47\frac{1}{2}$.

SOLUTION

$$
\begin{array}{r}
47\frac{1}{2})684\frac{2}{3} \\
\underline{6 \quad 6} \\
285)4108(14\frac{118}{285} \\
\underline{285} \\
1258 \\
\underline{1140} \\
118
\end{array}
$$

$118 \div 285 = \frac{118}{285}$

EXPLANATION. Multiply both divisor and dividend by the least common denominator of their terminal fractions (L. C. D. of 2 and 3 = 6), obtaining 285 sixths as the equivalent divisor, and 4108 sixths as the equivalent dividend [Prin. 5, **34**]. Divide as with integers [**36**].

If required, $\frac{118}{285}$ can be reduced to an approximate fraction of any desired denominator by **86**.

EXAMPLES FOR PRACTISE

Divide	Divide	Divide
1. $695\frac{7}{8}$ by $27\frac{5}{8}$	**4.** $6395\frac{3}{8}$ by $34\frac{3}{4}$	**7.** $3486\frac{5}{8}$ by $537\frac{1}{6}$
2. $438\frac{1}{2}$ by $18\frac{4}{5}$	**5.** $5284\frac{3}{16}$ by $65\frac{1}{3}$	**8.** $2758\frac{7}{9}$ by $238\frac{4}{5}$
3. $741\frac{3}{4}$ by $53\frac{2}{3}$	**6.** $9172\frac{3}{5}$ by $24\frac{1}{4}$	**9.** $8175\frac{3}{4}$ by $196\frac{5}{7}$

Approximating the fractional part of the quotient to the specified denominator, divide

10. $475\frac{2}{3}$ by $73\frac{4}{5}$, to 16ths	**13.** $7283\frac{4}{5}$ by $196\frac{3}{8}$, to 4ths
11. $9183\frac{5}{6}$ by $29\frac{1}{2}$, to 8ths	**14.** $5976\frac{7}{8}$ by $39\frac{1}{2}$, to 64ths
12. $6198\frac{3}{4}$ by $16\frac{2}{3}$, to 32nds	**15.** $2743\frac{1}{3}$ by $216\frac{3}{5}$, to halves

RELATION OF COMMON FRACTIONS

107. INTRODUCTION. The principles which govern the relation of integers [**48** to **53**, inclusive] are unlimited in their application to all numerical expressions, whether integral or fractional. It only remains to explain the fractional form which the multiplier now assumes, and to harmonize its definition [Note, **96**] with that of an integral multiplier [Note 1, *b*, **25**]. A *number* is the expression of a quantity by the use of figures [**3**], and may mean "how much" as well as "how many." Hence, $\frac{3}{4}$ of a pound is as much a number of pounds as 1 pound is, or as 2 or more pounds are. The only difference between them is *in the form of expression;* $\frac{3}{4}$ (3 fourths) expressing a fractional *number of pounds*, and the others expressing integral *numbers of pounds*. Hence,

PRINCIPLES. 1. *The multiplicand denotes the value of one considered equal quantity, whether integral or fractional.* 2. *The multiplier denotes the integral or fractional number of considered equal quantities.* 3. *The product denotes the value of all the integral or fractional number of considered equal quantities.*

NOTE. The student is cautioned against confounding the expression "fractional number of units," with "number of fractional units." In the numerical expression $\frac{3}{4}$, $\frac{3}{4}$ is the fractional number of units, and the numerator 3 is the number of fractional units, or fourths.

ILLUSTRATIVE EXAMPLES

1. Find the cost of $\frac{3}{4}$ of a yard of silk at 80 cents per yard.

2. If $\frac{3}{4}$ of a yard of silk cost 60 cents, what is the price per yard?

3. How much silk at 80 cents per yard can be bought for 60 cents?

SOLUTION

Factors of dividend { 80¢ = cost of one entire yard = multiplicand; $\frac{3}{4}$ = fractional number of yards = multiplier } Factors of product

Dividend, if given = 60¢ = cost of all the fractional number of yds. = product, if required

EXPLANATION. In Ex. 1, as 80¢ is the price of one (entire) yard, it is a multiplicand; as $\frac{3}{4}$ expresses the fractional number of yards, it is a multiplier. Hence, multiply these two factors, obtaining 60¢ as the cost of all the fractional number of yards, or product.

In Exs. 2 and 3, the cost of all the fractional number of yards, or product (60¢), is given. Hence, in Ex. 2, divide the given product (60¢) by its given multiplier factor (the fractional number of yards, $\frac{3}{4}$) to obtain its required multiplicand factor (the price of one entire yard, 80¢); and in Ex. 3, divide the given product (the cost of all the fractional number of yards, 60¢) by its given multiplicand factor (the price of one entire yard, 80¢) to obtain its required multiplier factor (the fractional number of yards, $\frac{3}{4}$).

4. If a man can walk $3\frac{1}{3}$ miles in one hour, what part of an hour will he require to walk $2\frac{1}{2}$ miles?

EXPLANATION. $3\frac{1}{3}$ miles is a multiplicand because it is the distance walked in *one* entire hour [Prin. 1] and $2\frac{1}{2}$ miles is a product because it is the distance walked in all the required fractional number of hours [Prin. 3]. Therefore, divide the given product ($\frac{5}{2}$) by its given multiplicand factor ($\frac{10}{3}$), to obtain its required multiplier factor ($\frac{5}{2} \times \frac{3}{10} = \frac{3}{4}$), or the fractional number of hours.

EXAMPLES FOR PRACTISE

108. 1. What is the cost of $\frac{5}{8}$ of a gallon of syrup bought at $\$\frac{3}{4}$ per gallon?

2. At $\$\frac{3}{8}$ per yard, how many yards of goods can be bought for $\$2\frac{1}{4}$?

3. If a horse is fed $\frac{3}{8}$ of a peck of corn twice every day, how much corn will be necessary to last $4\frac{1}{2}$ days?

4. How much butter at $\$\frac{3}{8}$ per pound can be bought for $3?

5. If the speed of a steamboat is $15\frac{3}{4}$ miles per hour, in what time can it go a distance of $9\frac{1}{3}$ miles?

6. A gardener wishes to divide $2\frac{1}{4}$ acres of land into 16 equal flower beds. How much land will each bed contain?

7. If a man can complete $\frac{3}{7}$ of a task in $\frac{5}{12}$ of an hour, in what time can he perform the entire task? [Explanation of Ex. 17, 54].

8. If 8 men can complete a piece of work in $\frac{3}{5}$ of a working day, how many working days will be required for one man to complete it?

9. If 28 men can do a certain piece of work in $3\frac{1}{4}$ working days, how many working days will be necessary for 7 men to do the same piece of work?

10. If there are 146 days in $\frac{2}{5}$ of a year, how many days are in $\frac{1}{7}\frac{9}{3}$ of a year?

11. The distance between two villages is $18\frac{3}{4}$ miles. If a traveler has walked $\frac{5}{8}$ of the distance, how many miles has he yet to walk?

12. What is the total cost of $\frac{3}{4}$ of a pound of cheese at $\$\frac{1}{2}$ per pound, $\frac{5}{16}$ of a pound of butter at $\$\frac{3}{8}$ per pound, and $9\frac{2}{3}$ gallons of syrup at $\$\frac{4}{5}$ per gallon?

13. If it cost $\$\frac{3}{16}$ to buy $\frac{2}{3}$ of a yard of goods, how many yards of the same goods can be bought for $\$6\frac{3}{5}$?

14. An estate was bequeathed to a mother and her five children, the mother to receive one-third, and the remainder to be equally divided among the children. What part of the estate did each child receive?

15. From a barrel of cider containing $49\frac{3}{4}$ gallons, there were drawn $7\frac{5}{8}$ gallons at one time, $16\frac{3}{4}$ gallons at another time, $5\frac{3}{8}$ gallons at a third drawing, and $12\frac{1}{2}$ gallons at a fourth. How many gallons remained in the barrel?

16. $2\frac{3}{4}$ times $17\frac{3}{8}$ yards of goods are how much less than $5\frac{1}{2}$ times $28\frac{1}{3}$ yards of the same goods?

COMPARISON OF QUANTITIES

109. INTRODUCTION. (*a*) In a comparison of quantities whether integral or fractional, *any one* of them may be arbitrarily assumed as the standard, as the author of the problem may direct. It is therefore necessary that the problem should *indicate* which of the considered quantities is to be accepted as the *standard* of comparison. The conventional form of language for arbitrarily indicating the standard is the expression "*of*" [Note 3, 97] placed immediately *after* the fractional multiplier and immediately *before the standard* of comparison, or multiplicand. Thus,

in the completed comparison, \$2400 are $\frac{2}{3}$ of \$3600, "\$3600" is the indicated standard of comparison or multiplicand (because it is made to follow "$\frac{2}{3}$ *of*"), and expresses the value of *one* considered quantity [Prin. 1, **107**]; $\frac{2}{3}$ is the indicated multiplier (because it immediately precedes "of \$3600"), and expresses the fractional number of times the considered standard which is expressed by the quantity compared with the standard [Prin. 2, **107**]; and \$2400 is a product because it is the quantity which is placed in comparison with the standard by denoting the number of times ($\frac{2}{3}$) the standard (\$3600) which is expressed by \$2400, the quantity compared with the standard. That is, "\$2400 are $\frac{2}{3}$ of \$3600," simply means \$2400 are $\frac{2}{3}$ times \$3600.

(*b*) If, however, the standard of comparison is to be indicated with the view of increasing it, the phrase "more than" should immediately follow the fractional multiplier and precede the assumed standard of comparison. Thus, in the completed comparison "\$840 are $\frac{2}{5}$ more than \$600," \$600 is the indicated standard of comparison, or multiplicand (because it is thus pointed out by being made to follow "$\frac{2}{5}$ more than"), and expresses the value of one considered equal quantity; $\frac{2}{5}$ is the indicated multiplier (because it immediately precedes "more than \$600"), and expresses the fractional number of times the standard which the quantity compared with the standard is *more than* that standard; and \$840 is the quantity compared with the standard, or a product, because it expresses the value (\$840) of all the considered fractional number of times the standard ($\frac{2}{5}$ more than *once* or $\frac{5}{5}$ of the standard, that is, $\frac{1}{5}$ of \$600). Expressed in more familiar terms, "\$840 are $\frac{2}{5}$ more than \$600" means, "\$840 are $\frac{2}{5}$ of (times) \$600, added to \$600."

(*c*) The standard of comparison may be indicated in a way to show that it is to be decreased, in which case the indicator "less than" should follow immediately after the fractional multiplier and immediately precede the indicated standard of comparison, or multiplicand. Thus, in the completed comparison "\$750 are $\frac{3}{8}$ less than \$1200, the standard of comparison, or multiplicand, is \$1200; $\frac{3}{8}$ less than once ($\frac{8}{8}$) the standard, or $\frac{5}{8}$ times the standard, is the multiplier; and \$750 is the quantity compared with the standard or product. In other words, the problem means, \$750 are $\frac{3}{8}$ of (times) \$1200, subtracted from \$1200.

PRINCIPLES. 1. *The multiplicand is the value of one indicated quantity which is assumed as a standard with which some other quantity is to be compared.* 2. *The multiplier is the fractional part of the standard* (the fractional number of times the standard)

which is expressed by the quantity compared with the standard. 3. *The product is the quantity compared with the standard* (the value of all the fractional number of times the standard which is expressed by the multiplier).

NOTE 1. A problem in comparison is one in which two of the three terms of a perfected comparison are given, and it is required to find the omitted third term.

NOTE 2. In problems in which the numerical standard of comparison is the omitted third term, the *verbal standard* will have to be substituted after the indicators "of," "more than," or "less than." Thus, in the problem: "If 200 acres are $\frac{2}{5}$ of A's farm, how many acres does A's farm contain," "A's farm" is the indicated *verbal standard* of comparison, or verbal multiplicand which is required to be expressed numerically; 200 acres is the quantity compared with the standard (A's farm), or the product; and $\frac{2}{5}$ is the multiplier or fractional number of times A's farm expressed by the product (200 acres).

ILLUSTRATIVE EXAMPLES

1. What part of \$200 are \$80?

SOLUTION

$$80 \div 200 = \frac{80}{200} = \frac{2}{5}$$

EXPLANATION. \$200 is the indicated standard of comparison, or multiplicand; \$80 is the quantity compared with the indicated standard, or product. Hence, divide the given product (\$80) by its given multiplicand factor (\$200), expressing the unexecuted division as directed in Introduction, *a*, **70**, obtaining $\frac{2}{5}$ as the required multiplier factor, or fractional number of times (fractional part of) the standard (\$200) which is expressed by \$80.

2. $\frac{3}{4}$ of how much wheat equal $\frac{1}{6}$ of a bushel?

SOLUTION

$$\frac{1}{6} \div \frac{3}{4} = \frac{1}{\cancel{6}_{2}} \times \frac{\overset{3}{\cancel{9}}}{4} = \frac{3}{8} \text{ bu.}$$

EXPLANATION. "How much wheat" is the indicated *verbal* standard, or multiplicand, of which it is required to find the equivalent numerical expression [Note 2, above]. $\frac{3}{4}$ is the multiplier, because it denotes the fractional number of times the standard (how much wheat) which is expressed by the given product ($\frac{1}{6}$ of a bushel). Hence, divide the given product ($\frac{1}{6}$ bu.) by its given multiplier factor ($\frac{3}{4}$), obtaining $\frac{2}{9}$ bu. as the required multiplicand factor, or standard $\frac{3}{4}$ of which equal $\frac{1}{6}$ bu.

3. 280 men are $\frac{3}{4}$ more than how many men?

SOLUTION

$\frac{3}{4}$ more than $\frac{4}{4} = \frac{7}{4}$

$$280 \div \frac{7}{4} = \overset{40}{\cancel{280}} \times \frac{4}{\cancel{7}} = 160 \text{ men}$$
$$1$$

EXPLANATION. "How many men" is the indicated verbal standard of comparison, or multiplicand, of which it is required to find the equivalent numerical expression. 280 men is a product, because it is the quantity compared with the required standard. $\frac{3}{4}$ more than once the required standard, that is, $\frac{3}{4}$ more than $\frac{4}{4}$ of the standard, or $\frac{7}{4}$ of the standard, is the multiplier, because it denotes the fractional number of times the standard, or multiplicand, which is expressed by the product (280 men) or quantity compared with the standard. Hence, divide the given product (280 men) by its indirectly given multiplier factor ($\frac{7}{4}$), obtaining 160 men as the required multiplicand factor, or standard $\frac{4}{4}$ of which ($\frac{7}{4}$ times which) equal 280 men.

EXAMPLES FOR PRACTISE

110. 1. I paid \$7200 for a farm and afterwards sold it for $\frac{7}{8}$ of what I paid for it. How much did I receive for the farm?

2. If $\frac{2}{3}$ of a bushel of wheat weigh 40 pounds, what is the weight of an entire bushel?

3. What part of \$320 are \$180?

4. A dealer sold $\frac{6}{7}$ of his stock of merchandise for \$4824. At how much was the entire stock valued?

5. A man bequeathed $\frac{3}{16}$ of an estate worth \$12352 to a daughter. How much did she receive?

6. 48 bushels of corn were sold from a bin containing 56 bushels. What part of the contents of the bin was sold?

7. In a certain school were enrolled 246 boys and 318 girls. What part of the total enrolment were the girls?

8. The capital of a firm of two partners was \$5600, of which one partner invested \$2100. What part of the capital did the other partner invest?

9. A grazier sold $\frac{5}{8}$ of a flock of sheep and had 114 sheep remaining. How many sheep did he sell?

10. A man's salary is \$90 per month and his expenses \$60 per month. What part of his salary does he save?

11. A is worth \$13000, or $\frac{2}{3}$ more than B. How much is B *worth*?

12. A man paid $\frac{8}{9}$ of his indebtedness and then found that he still owed \$7310. How much did he originally owe?

REVIEW OF FRACTIONS

111. 1. What is the sum of $137\frac{5}{8}$, $463\frac{2}{3}$, $285\frac{1}{6}$, $348\frac{7}{12}$, $825\frac{11}{18}$, $567\frac{3}{4}$, and $362\frac{1}{2}$?

2. A man engaged to plow a field containing $78\frac{4}{5}$ acres. How many acres remained unplowed after completing $\frac{2}{3}$ of his engagement?

3. What is the cost of $58\frac{5}{8}$ yards of goods at $37\frac{1}{4}$ cents per yard?

4. A dealer sold $16\frac{1}{4}$ yards from a piece containing $38\frac{5}{12}$ yards. What part of the piece of goods did he sell?

5. The distance between two harbors is $847\frac{11}{16}$ miles. After a ship has sailed $\frac{3}{4}$ of this distance, how many miles remain to complete the voyage?

6. What is the total cost of $6\frac{7}{8}$ pounds beef at $18\frac{3}{4}$ cents per pound, $3\frac{1}{4}$ pounds mutton at $13\frac{5}{8}$ cents per pound, and $\frac{7}{8}$ of a pound of lard at $11\frac{1}{2}$ cents per pound?

7. A farm consists of 6 fields containing respectively $68\frac{3}{4}$, $57\frac{1}{8}$, $65\frac{19}{160}$, $42\frac{19}{32}$, $52\frac{2}{3}$, and $63\frac{13}{16}$ acres, a vegetable garden containing $\frac{11}{12}$ of an acre, an orchard of $1\frac{5}{8}$ acres, and a lawn of $2\frac{5}{6}$ acres. If it was bought at the average rate of \$36 per acre, how much did the farm cost?

8. If a ship completes $\frac{8}{16}$ of its voyage on the first day, $\frac{5}{12}$ on the second, $\frac{1}{4}$ on the third day, and is then 1300 miles from its destination, what is the total distance which it has already sailed?

9. If a railroad train moves with the average speed of $37\frac{1}{2}$ miles per hour, in how many hours can it traverse $718\frac{3}{4}$ miles?

10. What is the cost of $685\frac{7}{8}$ tons coal at \$5.75 per ton?

11. $\frac{7}{24}$ of a day is $\frac{3}{4}$ of what part of a day?

12. 345 gallons are what part of 460 gallons?

13. A grammar and speller together cost 92 cents, and the speller cost $\frac{1}{3}$ as much as the grammar. What was the cost of each?

14. A can complete a job of work in 8 days, and B in 10 days. In what time *can they* complete the task by working together?

EXPLANATION. If A can complete the entire job in 8 days, he can perform $\frac{1}{8}$ of the job in 1 day; and if B can complete the entire job in 10 days, he can perform $\frac{1}{10}$ of the job in 1 day. Therefore, by working together, A and B can perform $\frac{1}{8}$ of the job plus $\frac{1}{10}$ of the job, or $\frac{9}{40}$ of the job in 1 day. That is, $\frac{9}{40}$ of the job is the multiplicand, because it expresses the value of one equal day's labor performed by both men [Prin. 1, **107**]; $\frac{4}{4}\frac{9}{0}$ of the job is the product understood, because it expresses the value of all the equal days' labor performed by both men. Hence, divide the product understood ($\frac{4}{4}\frac{9}{0}$) by its given multiplicand factor ($\frac{9}{40}$), obtaining $4\frac{4}{9}$ days as the required multiplier factor, or number of times one day's labor of both men ($\frac{9}{40}$ of the job) which will equal the total labor of both men ($\frac{4}{4}\frac{9}{0}$ of the job), or the entire job.

15. A can do a piece of work in 5 days and B in 7 days. In how many days can they do the piece of work by uniting their labor?

16. A can perform a certain task in 6 days, B in 9 days, and C in 12 days. In what time can the task be completed if they work together?

17. A and B, working together, can finish a piece of work in 12 days. If A working alone can do it in 20 days, in what time can B do it without any help?

18. The greater of two numbers is $347\frac{5}{9}$ and the less $265\frac{4}{9}$. What is the difference between them?

19. The greater of two fractions is $\frac{5}{6}$, and their difference $\frac{3}{16}$. What is the less fraction?

20. The less of two numbers is $758\frac{2}{3}$, and the difference between them is $125\frac{5}{6}$. What is the greater number?

21. The total of six weighings is $936\frac{5}{8}$ pounds. If five of the weighings are respectively $168\frac{1}{4}$, $271\frac{7}{8}$, $82\frac{3}{16}$, $125\frac{3}{8}$, and $76\frac{1}{2}$ pounds, what is the sixth weighing?

22. If coffee loses $\frac{1}{16}$ of its weight in roasting, how much green coffee will be required to produce 375 pounds of roasted coffee?

23. A water tank has two outlets of different diameters. If only the smaller outlet is opened, the tank will be emptied in 15 minutes; and if both are opened, it will be emptied in 6 minutes. In what time will the tank be emptied if only the larger outlet be opened?

24. A merchant withdrew $\frac{1}{3}\frac{5}{2}$ of his deposit from his bank and afterwards had $510 remaining on deposit. How much did the *merchant withdraw*?

25. If a barrel containing 28 gallons of vinegar is only $\frac{2}{3}$ full, what part of the barrel will be filled if 7 gallons of vinegar are added?

26. The capital of a firm was \$24210. If the investment of one partner was $\frac{4}{5}$ of the investment of the other partner, what did each partner invest?

27. A man bought a house, paid $\frac{1}{3}\frac{9}{2}$ of the purchase money in cash and still owed \$5200. What was the cost of the house?

28. Multiply $275\frac{5}{8}$ by $38\frac{2}{3}$ and divide the product by $18\frac{3}{4}$.

29. Subtract the sum of $268\frac{3}{4}$, $425\frac{5}{8}$, and $362\frac{2}{3}$ from the sum of $349\frac{7}{8}$, $562\frac{1}{6}$, $416\frac{5}{12}$, and $183\frac{1}{4}$.

30. A man owed \$3500 and afterwards paid \$1800 at one time and \$300 at another time. What part of his original debt did he still owe?

31. A invested $\frac{1}{3}$ of the capital of a firm, B $\frac{1}{4}$, C $\frac{1}{5}$, and D the remainder. If D's investment was \$2782, what was A's, B's, and C's?

32. A man bought a house for \$8500, paid $\frac{1}{4}$ of the purchase money at one time, and $\frac{2}{3}$ of the remainder at another time. How much remained unpaid?

DECIMAL FRACTIONS

112. INTRODUCTION. The new form of fraction now to be considered differs from that of common fractions, already studied, in this one respect, that the denominators of decimals are not usually expressed, whereas in common fractions, by reason of the varying scale employed, the denominators require expression, except in a certain class of commercial fractions [Notes 1, 2, 3, **92**]. The distinguishing characteristic of decimal fractions is the uniform scale of *ten* employed in expressing the equal parts into which the unit of the fraction [**80**] is divided, instead of the arbitrary, inconstant scale by which the unit of common fractions may be divided into *any* number of equal parts. Reading and writing decimals are simply extensions on a *descending* scale of the manner of reading and writing integers. Taking the unit as the initial order, the first figure on the left of it is the *tens'* order, and the first figure on the right of it is the *tenths'* order; the second figure on the left of the units' order is the *hundreds'* order, and the second figure on its right is the *hundredths'* order, etc.

While the methods already shown for operating upon common

fractions are also applicable to decimal fractions, if the latter are made to assume the form of common fractions by expressing their denominators, the much more simple and convenient methods for reduction of integers, and for adding, subtracting, multiplying, and dividing integers, can be extended to decimal fractions by reason of the same decimal system of notation which is employed for expressing both.

113. A decimal fraction expresses one or more of the decimal parts of an integral unit.

Thus, any fraction the denominator of which is 10, or the product of 10 taken twice as a factor ($10 \times 10 = 100$), or the continued product of 10 taken any number of times as a factor ($10 \times 10 \times 10 = 1000$), is called a decimal fraction, or simply a *decimal.*

114. A decimal point is a dot (.) placed at the left of a decimal to indicate its denominator, and if a mixed decimal, to separate the fractional from the integral orders.

The denominator of a decimal is understood to be 10 taken as many times as a factor as the number of places which the right-hand order of its numerator is located to the right of the decimal point. Thus, in .5, the numerator 5 is located *one place* to the right of the decimal point and therefore the denominator is understood to be the factor 10 taken *once;* in .17 the right-hand order of the numerator (7) is located *two* places to the right of the decimal point, and the denominator is understood to be the factor 10 taken *two times;* that is, 10×10, or 100; etc.

115. The **decimal order** or **decimal value** of any figure of the numerator is its distance from the decimal point.

The *nearer* any order of the numerator is to the decimal point, the *greater* its value, because 10 will be a factor of its denominator *fewer* times, thus dividing the unit of the fraction into fewer equal parts, and correspondingly increasing the value of those parts; and the *more distant* any order of the numerator is from the decimal point, the *less* its value, because 10 will be a factor of its denominator a *greater number* of times, thus dividing the unit of the fraction into a *greater number* of equal parts, and correspondingly diminishing the value of those parts.

The names of the several decimal orders and their relative *values, are shown* in the following.

TABLE OF DECIMAL VALUES

Etc., etc.	Billions	Hundred-millions	Ten-millions	Millions	Hundred-thousands	Ten-thousands	Thousands	Hundreds	Tens	Units	. Decimal Point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths	Hundred-millionths	Billionths	Etc., etc.
	7	3	8	2	4	9	1	5	1	6		2	8	6	3	9	4	7	5	1	
	10th.	9th.	8th.	7th.	6th.	5th.	4th.	3d.	2d.	1st.		1st.	2d.	3d.	4th.	5th.	6th.	7th.	8th.	9th.	
	Integral Orders											*Decimal Orders*									

EXPLANATION. In considering the above TABLE, observe 1st, that the decimal point separates the decimal orders from the integral orders; 2nd, that, commencing at the decimal point, the integral orders are enumerated from right to left, and the decimal orders from left to right; 3rd, that the names of the several decimal orders correspond with those of the integral orders which are *equally distant from the units' order.*

116. A simple decimal is one which is composed entirely of decimal orders, having no integer at its left, and no common fraction at its right; as, .175.

117. A complex decimal is one which has a common fraction at the right of its lowest decimal order; as, .286⅔.

NOTE. A complex decimal is said to be *terminate* when the common fraction at its right can be expanded to an equivalent *simple* decimal **[116]** and to be *interminate* when it cannot be thus expanded.

118. A mixed decimal is one which contains integral orders at the left of its decimal point; as, 28.356.

119. The **unit of a decimal** is the whole unit of which the decimal expresses a part; and a **decimal unit** is one of the equal decimal parts into which the *unit of the decimal* has been divided.

Thus, in the expression .25 of a yard, *one* whole yard is the unit of the decimal, and *one* one-hundredth of a yard is the decimal unit.

120. PRINCIPLES. 1. *The denominator of a decimal is 1 followed by a cipher for every place that its right-hand order is distant from the decimal point.*

Thus, in .0375 the denominator is 1 followed by four 0's (10000), because its right-hand order (5) is four places to the right of the decimal point; and the decimal is read 375 ten-thousandths.

2. *Adding ciphers to, or taking them from the right of a decimal, makes no change in its value.*

To place 0's at the right of a decimal numerator is equivalent to multiplying both the numerator and the denominator by 10 for every 0 annexed [30]; and to omit 0's from the right of a decimal numerator is equivalent to dividing both the numerator and the denominator by 10 for every 0 omitted [38]; and therefore, in neither case, does it change the value of the fraction [Prin. 4, 81].

3. *Prefixing ciphers to a decimal numerator and moving the decimal point to the left of the prefixed ciphers, is equivalent to dividing the decimal by 10 for each cipher prefixed.*

To place 0's on the left of a decimal numerator does not affect the value of the numerator, but increases the denominator tenfold for each 0 so prefixed [Prin. 1], and therefore is equivalent to dividing the fraction by 10 for each prefixed cipher [Prin. 2, 81].

4. *Moving its decimal point one or more places to the right of its former position, multiplies the decimal by 10 for each place so removed.*

To move the decimal point to the right of its former position will bring the right-hand order of the decimal correspondingly nearer to the point, and thus diminish the number of 0's in the denominator [Prin. 1]; but to diminish the 0's in the denominator is equivalent to dividing the denominator by 10 for each 0 rejected, and therefore of correspondingly multiplying the decimal [Prin. 3, 81].

5. *Moving its decimal point one or more places to the left of its former position, divides the decimal by 10 for each place so removed.*

To move the decimal point to the left of its former position will throw the right-hand order of the decimal farther from the point, and thus increase the number of 0's in the denominator [Prin. 1]; but to increase the 0's in the denominator is equivalent to multiplying the denominator by 10 for each additional 0, and therefore of correspondingly dividing the decimal [Prin. 2, 81].

READING DECIMALS

121. Read the decimal .0289.

EXPLANATION. First read the decimal as if it were an integer (two hundred eighty-nine); then name its denominator (1 followed by four ciphers, or 10000, Prin. 1, **120**).

Read the following decimals:

1. .7	**7.** .375	**13.** .06345	**19.** 57.816372
2. .09	**8.** .0375	**14.** .00918	**20.** 219.0071832
3. .005	**9.** .58	**15.** .07016	**21.** 8.00031678
4. .35	**10.** .291	**16.** .00053	**22.** 2.006521873
5. .046	**11.** .8273	**17.** .21897	**23.** 15.000005281
6. .673	**12.** .625	**18.** .00725	**24.** 7.8123768542

WRITING DECIMALS

122. Express two thousand one hundred nine hundred thousandths by the use of figures.

EXPLANATION. The denominator (100000) contains five ciphers, and the numerator (2109) contains only four of the five places necessary for its right-hand order (9) to be removed from the decimal point to express such a denominator [Prin. 1, **120**]. Therefore, write the decimal point and one cipher for the one lacking decimal place (.0), and then annex the numerator, obtaining .02109, in which the right-hand order (9) is located *five* places to the right of the point, and is thus made to express a denominator of 1 followed by five 0's, or 100000.

NOTE. The number of ciphers, if any, to follow the decimal point (or to be prefixed to the numerator) will always be the number of ciphers in the denominator diminished by the number of figures necessary to express the numerator.

Express the following as numerical decimals:

1. Six tenths	**9.** 328 ten-thousandths
2. Three hundredths	**10.** 75 hundred-thousandths
3. Twenty-seven hundredths	**11.** 4768 millionths
4. Five thousandths	**12.** 27 ten-millionths
5. Forty-two thousandths	**13.** 92536 millionths
6. Nine hundredths	**14.** 527 hundred-thousandths
7. Sixty-three thousandths	**15.** 8 millionths
8. Eighteen ten-thousandths	**16.** 263 ten-millionths

17. Nineteen, and twenty-three thousandths
18. Forty-six, and two hundred fifty-three ten-thousandths
19. Ninety-five, and seventy-nine thousandths
20. Six, and one thousand three hundred four millionths
21. Two, and eight thousand ninety-six hundred thousandths
22. Three and four thousand seven millionths
23. Four hundred fifty-three, and twenty-eight thousand seven hundred forty-six hundred-millionths
24. Eight thousand three hundred sixty-two, and one hundred twenty-one thousand three hundred seventeen billionths
25. Nine, and eighty-seven million two hundred ninety-four thousand eight hundred seventy-one billionths

REDUCTION OF COMMON FRACTIONS TO DECIMALS

ILLUSTRATIVE EXAMPLE

123. Reduce $\frac{7}{16}$ to an equivalent decimal.

SOLUTION

$$
\begin{array}{r}
16)7.0000(.4375 \\
\underline{64} \\
60 \\
\underline{48} \\
120 \\
\underline{112} \\
80 \\
\underline{80}
\end{array}
$$

EXPLANATION. The value of any fraction is the quotient from dividing its numerator by its denominator [**79**]. Hence, if 10000 times the numerator 7 (or 70000) be divided by the denominator (16), the resulting quotient (4375) will be 10000 times the true quotient [Prin. 4, **34**]. Therefore cut off four places from the right of the quotient (.4375) which is equivalent to dividing it by 10000 [**38**], thus obtaining the true quotient.

If no numerator can be exactly divisible by its denominator except it contain all the factors of that denominator [Prin. 1, **62**], and the denominator (16) contains the factor 2 four times ($2 \times 2 \times 2 \times 2 = 16$), it is necessary for the numerator (7) to contain the factor 2 four times in order to be exactly divisible by the denominator (16). As annexing one cipher to the numerator is equivalent to multiplying it by 10 [**30**], and as 10 contains the factor 2 once ($2 \times 5 = 10$), it follows that the factor 2 is introduced *once* for every cipher annexed to the numerator, and therefore to annex four 0's to the numerator will introduce the factor 2 four times, and thus make it exactly divisible by the denominator.

RULE. *Annex ciphers to the numerator and divide by the denominator. Place a decimal point as many places from the right of the quotient as there have been ciphers annexed to the numerator.*

NOTE 1. The number of ciphers to be annexed to the numerator will be equal to the greatest number of factors 2 or 5 in the denominator. A common fraction in lowest terms cannot therefore be reduced to a *simple* decimal [116] if its denominator contains other prime factors than 2 or 5, because annexing 0's to the numerator cannot introduce such other factors without which exact division of the numerator by the denominator is impossible [Prin. 1, 62].

NOTE 2. Common fractions should be in lowest terms before they are reduced to decimals.

NOTE 3. Because they recur so frequently, learners should memorize the decimal equivalents of *halves, fourths,* and *eighths.* $\frac{1}{2} = .5$; $\frac{1}{4} = .25$; $\frac{3}{4} = .75$; $\frac{1}{8} = .125$; $\frac{3}{8} = .375$; $\frac{5}{8} = .625$; $\frac{7}{8} = .875$.

EXAMPLES FOR PRACTISE

Reduce the following to decimals:

1. $\frac{1}{2}$	**4.** $\frac{1}{4}$	**7.** $\frac{5}{16}$	**10.** $\frac{9}{64}$	**13.** $\frac{9}{250}$	**16.** $18\frac{3}{16}$
2. $\frac{3}{4}$	**5.** $\frac{7}{8}$	**8.** $\frac{9}{32}$	**11.** $\frac{11}{16}$	**14.** $\frac{17}{125}$	**17.** $465\frac{7}{32}$
3. $\frac{5}{8}$	**6.** $\frac{3}{8}$	**9.** $\frac{1}{8}$	**12.** $\frac{4}{5}$	**15.** $\frac{13}{64}$	**18.** $67\frac{127}{625}$

NOTE 4. Inconvenient long divisions may frequently be avoided by separating the common fraction into two factors, the decimal equivalent of one of which is already known [Note 3]. Thus, $\frac{19}{64} = \frac{1}{8}$ of $\frac{19}{8}$; and $\frac{19}{8} = 2\frac{3}{8}$ or 2.375; therefore $\frac{1}{8}$ of $\frac{19}{8}$ must equal $\frac{1}{8}$ of 2.375.

Reduce the following to decimals by factoring:

19. $\frac{23}{32}$	**22.** $\frac{11}{32}$	**25.** $\frac{35}{64}$	**28.** $63\frac{17}{32}$
20. $\frac{21}{64}$	**23.** $\frac{7}{64}$	**26.** $\frac{1}{32}$	**29.** $37\frac{3}{64}$
21. $\frac{15}{16}$	**24.** $\frac{13}{16}$	**27.** $58\frac{7}{16}$	**30.** $78\frac{1}{16}$

NOTE 5. It is usually sufficient for the requirements of business to extend a decimal result to a certain number of decimal places, ordinarily not more than three or four; that is, as many ciphers are annexed to the numerator as there are decimal places required in the result; and when the last quotient figure has been obtained, to increase it by 1 if the final remainder is at least one-half the divisor.

Reduce the following to decimals of the number of places indicated:

2 dec. pl.	3 dec. pl.	4 dec. pl.	5 dec. pl.	6 dec. pl.
31. $168\frac{7}{16}$	**34.** $91\frac{11}{16}$	**37.** $23\frac{15}{32}$	**40.** $8.26\frac{9}{16}$	**43.** $1.2764\frac{5}{16}$
32. $34\frac{7}{125}$	**35.** $26\frac{3}{32}$	**38.** $71\frac{49}{64}$	**41.** $7.348\frac{2}{3}$	**44.** $3.296\frac{3}{7}$
33. $72\frac{23}{32}$	**36.** $49\frac{1}{64}$	**39.** $37\frac{3}{16}$	**42.** $9.527\frac{3}{8}$	**45.** $2.8345\frac{3}{4}$

REDUCTION OF DECIMALS TO COMMON FRACTIONS

ILLUSTRATIVE EXAMPLES

124. Reduce $.57\frac{1}{7}$ and .1875 to common fractions.

FIRST SOLUTION

$$.57\tfrac{1}{7} = \frac{57\frac{1}{7} \times 7}{100 \times 7} = \frac{400}{700} = \frac{4}{7}$$

SECOND SOLUTION

$$.1875 = \frac{1875 \div 625}{10000 \div 625} = \frac{3}{16}$$

Or,

$$.1875 = \frac{1\frac{7}{8} \times 8}{10 \times 8} = \frac{15 \div 5}{80 \div 5} = \frac{3}{16}$$

FIRST EXPLANATION. Change the complex decimal to the form of a complex common fraction by expressing its decimal denominator 100 ($\frac{57\frac{1}{7}}{100}$). Then multiply both terms of the obtained common fraction by the denominator of the complex numerator (7) to obtain an equivalent simple common fraction $\frac{400}{700}$ [Prin. 4, **81**]. Finally, reduce to lowest terms [**84**], obtaining $\frac{4}{7}$.

SECOND EXPLANATION. Change the decimal to the form of a common fraction ($\frac{1875}{10000}$). Then reduce to lowest terms [**84**].

Or, applying Note 3, **123**, it is seen that the last three figures of the given decimal (875) equal $\frac{7}{8}$. Hence, .1875 equal $\frac{1\frac{7}{8}}{10}$. Then proceed as in the first solution.

NOTE. Business men do not usually take the unnecessary trouble to find the greatest common divisor as was done in the second solution; but as 5 and 2 are the only prime factors of a decimal denominator, they successively divide by 5 until that factor has been eliminated and then similarly by 2; or, to avoid many petty divisions by 2, they divide by 4 or by 8, if seen to be common to both terms.

RULE. *Omit the decimal point and prefixed ciphers from the given decimal; express the remaining figures in the form of a common fraction by writing the denominator understood; and reduce to lowest terms as in* **84.**

EXAMPLES FOR PRACTISE

Reduce the following to common fractions:

1. .75	**6.** .6875	**11.** .59375	**16.** 2.00875
2. .375	**7.** .9375	**12.** .00625	**17.** $19.66\frac{2}{3}$
3. .005	**8.** .0025	**13.** 5.0375	**18.** $6.333\frac{1}{3}$
4. .125	**9.** .15625	**14.** 17.0075	**19.** $9.714\frac{2}{7}$
5. .0875	**10.** .34375	**15.** 8.28125	**20.** $16.272\frac{8}{11}$

APPROXIMATION OF DECIMALS TO COMMON FRACTIONS

ILLUSTRATIVE EXAMPLE

125. Approximate .75173 to a common fraction with a denominator not greater than eighths.

SOLUTION

.75173
8
6.01384 = $\frac{6}{8}$ = $\frac{3}{4}$

EXPLANATION. The unit of the given decimal, expressed in eighths equals 8 eighths, therefore .75173 of the unit of the fraction must equal .75173 of (times) 8 eighths, or 6.01384 eighths. Hence, .75173 = $\frac{6}{8}$, approximately, or in lowest terms, $\frac{3}{4}$.

RULE. *Multiply the given decimal by the required denominator. Point off as many places in the product as are found in the given decimal. If the decimal part of the product equals one-half or more, increase the integral part of the product by 1. The integral part of the product will be the required numerator, under which write the given denominator.*

EXAMPLES FOR PRACTISE

Approximate to common fractions of the required denominator:

1. .6879 to 16ths
2. .4912 to 32nds
3. .7489 to 8ths
4. .6247 to 16ths
5. .7987 to 5ths
6. .2531 to 4ths
7. .5168 to 32nds
8. .6672 to 3ds
9. .8334 to 6ths

ADDITION OF DECIMALS

ILLUSTRATIVE EXAMPLES

126. 1. Add .375, 2.75, .0058, 15.5 and .0625.

SOLUTION

.375
2.75
.0058
15.5
.0625
18.6933

EXPLANATION. Only like orders can be added [Prin. 1, **18**]. Therefore arrange the decimals to be added so that their points shall fall in an upright column, thus causing orders which are at the same distance from the decimal point and which must consequently be like [**115**], to fall in the same column. Add the columns in the usual manner. Place the decimal point in the sum directly under the points of the numbers added, for the sum of any column must be of the same order as that of the column added [Prin. 2, **18**], and must therefore be located at the same distance from the decimal point as the column added, to be properly indicated.

2. Add $19.29\frac{3}{8}$, $.5\frac{2}{3}$, $3.06\frac{1}{16}$, and $25\frac{8}{32}$, to 4 decimal places.

SOLUTION

19.29375
.56667 —
3.06813 —
25.09375
48.02230
48.0223, Ans.

EXPLANATION. First expand the complex decimals [117] and mixed numbers [78] to *five* decimal places [Note 5, 123]. The expression to one decimal place more than the required number is intended to insure accuracy in the *four* places required in the answer by obtaining the necessary carrying figure (2) from the fifth decimal column. Then arrange, add, and point off, as in the preceding example.

NOTE. If the rejected figure at the right of any answer is 5 or more, the last retained figure should be increased by 1. That is, if, in the Ill. Ex., 48.02237 had been the obtained total, 48.0224 would have been a closer approximation to this exact result than 48.0223.

EXAMPLES FOR PRACTISE

Find the sum of

1. 37.41, 5.8967, .3562, 863.7, and 16.25.

2. 2.183, .713, .005, 43.81, 2.1875, and 32.025.

3. 682.5, 6.073, .058, 25.305, 97.1834, and 5.07168.

4. 29.38, 1.35867, 26.038, 534.7, 6.28734, and 59.0625.

5. .0635, .0072, .9, .85, 3.15825, .075, and .6.

6. $32.05\frac{1}{4}$, $7.08\frac{9}{16}$, $563\frac{28}{32}$, $16.07\frac{3}{4}$, and $6.9\frac{7}{8}$, as a simple decimal.

7. $.08\frac{11}{16}$, $.72\frac{3}{8}$, $.84\frac{3}{5}$, $46\frac{1}{16}$, $28.3\frac{3}{4}$, and $\frac{5}{32}$, as a simple decimal.

8. $.62\frac{1}{2}$, $\frac{7}{8}$, $42\frac{3}{4}$, $.07\frac{3}{16}$, $48\frac{1}{32}$, and $.25\frac{9}{125}$, as a simple decimal.

9. $52.6\frac{11}{32}$, $7.82\frac{1}{64}$, $.5\frac{2}{3}$, $.17\frac{3}{7}$, and $61.8\frac{6}{9}$, to 3 decimal places.

10. $7\frac{4}{9}$, $.5\frac{1}{3}$, $.68\frac{4}{125}$, $2.91\frac{1}{32}$, $.78\frac{3}{8}$, and $\frac{1}{6}\frac{3}{4}$, to 4 decimal places.

11. $.516\frac{2}{3}$, $6.8\frac{4}{9}$, $12.343\frac{1}{16}$, $.6\frac{3}{4}$, and $\frac{2}{7}$, to 5 decimal places.

12. Sixteen, and one hundred sixty-four ten thousandths; two hundred twenty-eight, and three thousand five hundred seventy-two millionths; forty-seven, and twenty-one ten-thousandths; eight hundred thirty-seven, and fifteen thousand one hundred ninety-three millionths.

13. Four hundred nineteen, and six thousand one hundred fifty-six hundred-thousandths; twenty-three, and eighteen thousand nine hundred seventy-four ten-millionths; eight, and forty-two thousand eight hundred fifty-one millionths; one hundred thirty-five thousand six hundred eighty-seven, and sixteen hundred-thousandths.

14. Twenty-three millionths; seven, and one thousand four hundred thirty-two hundred-thousandths; fifty-nine, and thirteen thousand seven hundred twenty-eight ten-millionths; eight thousand three hundred fifty-eight ten-thousandths; seventy-five, and eighty-six thousandths; nine thousand two hundred, and seven thousand three hundred nineteen millionths.

SUBTRACTION OF DECIMALS

ILLUSTRATIVE EXAMPLE

127. Subtract $68.25\frac{5}{2}$ from $183.6\frac{5}{8}$, to 3 decimal places.

SOLUTION

183.6833 +
68.2516 −
115.4317
115.432 −, Ans.

EXPLANATION. First expand each decimal to four decimal places, or to one more place than is required, as explained in Ill. Ex. 2, **126**; and so arrange them that the decimal point of the subtrahend shall fall directly underneath that of the minuend, thus causing similar orders to fall in the same column [Prin. 1, **22**]. Subtract in the usual manner. Place the decimal point in the result directly under those of the minuend and subtrahend, because the difference between like orders must also be like [Prin. 2, **22**], and must therefore be located at the same distance from the decimal point [**115**] to indicate this likeness.

EXAMPLES FOR PRACTISE

Find the difference between

1. 75.106 and 28.32
2. 837.25 and 256.3845
3. 72.68 and 34.2478
4. 39.658 and 4.8
5. 76.23 and .847
6. 43.007 and 18.26
7. 528.1586 and 271.03
8. 68 and .3129
9. 42.175 and 28
10. .75 and .21867

11. $53.28\frac{5}{8}$ and $27.2\frac{3}{16}$, expanded to simple decimals

12. $268.4\frac{1}{4}$ and $125.92\frac{3}{8}$, expanded to simple decimals

13. $9.68\frac{3}{2}\frac{7}{2}$ and $2.934\frac{7}{8}$, expanded to simple decimals

14. $67.2\frac{5}{7}$ and $39.16\frac{5}{8}$, to 3 decimal places

15. $95.13\frac{2}{3}$ and $27.9\frac{3}{8}\frac{3}{4}$, to 4 decimal places

16. $18\frac{1}{6}$ and $12\frac{1}{3}$, to 2 decimal places.

17. Subtract three hundred twenty-eight, and fifty-six ten-thousandths from five hundred sixty.

18. Subtract two thousand five hundred seventy-three hundred-*thousandths from* eight-tenths.

MULTIPLICATION OF DECIMALS

ILLUSTRATIVE EXAMPLE

128. Multiply 9.865 by 2.45.

SOLUTION

$$\begin{array}{r} 9.865 \\ 2.45 \\ \hline 49325 \\ 39460 \\ 19730 \\ \hline 24.16925 \end{array}$$

EXPLANATION. If the multiplicand (9.865) be expressed in the form of an equivalent improper common fraction ($\frac{9865}{1000}$); and if the multiplier (2.45) be similarly expressed ($\frac{245}{100}$); and their product obtained as common fractions by **97**, the numerator of the product would be the product of these numerators (9865 × 245, or 2416925), or the product of the several orders of the decimal multiplicand and decimal multiplier (9.865 × 2.45 = 2416925); and the denominator of the product would be the product of these denominators (1000 × 100 = 100000), that is, 1 followed by as many ciphers as are found in both factors, or five ciphers [**32**]; and as each cipher in the denominator requires a decimal place in the numerator to express it [Prin. 1, **120**], cut off five decimal places from the right of the product (24.16925) to indicate that the denominator is 1 followed by 5 ciphers, or 100000.

RULE. *So place one factor under the other that the right-hand order of the lower factor shall fall under the right-hand order of the upper factor. Multiply as in integral multiplication. Point off from the right of the product as many decimal places as are found in both factors.*

NOTE 1. In arranging the factors for multiplication, their decimal points need not fall in an upright column, as in addition and subtraction of decimals.

NOTE 2. If any obtained product has fewer figures than are required to be pointed off, *prefix* ciphers to supply the deficiency.

NOTE 3. To multiply a decimal by 10, 100, 1000, etc., move its decimal point as many places *to the right* as there are ciphers in the multiplier, *annexing* ciphers to the decimal if necessary.

NOTE 4. Complex decimals, if terminate [Note, **117**], should be expanded to simple decimals before multiplication; and if interminate, should be multiplied as shown in **98** or **99**.

EXAMPLES FOR PRACTISE

Multiply	Multiply	Multiply
1. 516.78 by 43	**4.** 2.086 by .034	**7.** .685 by .0058
2. 79.832 by 7.4	**5.** 56.8 by .0096	**8.** $7.6\frac{5}{8}$ by 5.2
3. *56.283 by .59*	**6.** 8.27 by 1.038	**9.** $6.28\frac{3}{16}$ by .76

Multiply	Multiply	Multiply
10. $75.1\frac{9}{32}$ by $.2\frac{3}{4}$	**14.** $2.41\frac{7}{25}$ by $1.6\frac{11}{16}$	**18.** $2.34\frac{1}{6}$ by $.16\frac{3}{7}$
11. $6.8\frac{4}{5}$ by 3.6	**15.** $2.89\frac{2}{3}$ by 3.5	**19.** .7183 by 100 [Note 3]
12. $2.96\frac{3}{8}$ by $5.12\frac{1}{4}$	**16.** $.684\frac{5}{6}$ by 2.63	**20.** .62 by 1000
13. $6.73\frac{3}{4}$ by $.913\frac{1}{2}$	**17.** $.715\frac{1}{3}$ by $3.8\frac{4}{9}$ [99]	**21.** .7183 by 10

DIVISION OF DECIMALS

ILLUSTRATIVE EXAMPLE

129. Divide 10.02677 by 2.873.

SOLUTION

```
2.873)10.02677(3.49
      8 619
      ------
      1 4077
      1 1492
      ------
        25857
        25857
        -----
```

EXPLANATION. If the dividend be expressed in the form of an equivalent improper common fraction ($\frac{1002677}{100000}$), and the divisor be similarly expressed ($\frac{2873}{1000}$), the quotient obtained as in common fractions [103] would be $\frac{1002677}{100000} \times \frac{1000}{2873}$, or $\frac{1002677}{2873 \times 100}$; that is, the quotient of the dividend regarded as an integer (1002677) divided by the divisor regarded as an integer (2873), obtaining 349, and this result (349) divided by 100 [38], or by 1 followed by as many ciphers as the number of decimal places in the dividend (five) exceed the number of decimal places in the divisor (three), obtaining 3.49.

DEDUCTIONS. 1. If the dividend is the product of the divisor and quotient (Note 2, **35**), then the dividend (10.02677) must contain as many decimal places as the divisor (2.873) and the quotient (3.49) together [Rule, **128**]. Therefore, if the divisor contains only *three* of the *five* decimal places found in the dividend, the quotient must contain the remaining two decimal places. Hence, *subtract the number of decimal places in the divisor from the number of decimal places in the dividend to find the number of decimal places to point off in the quotient.*

2. If the dividend contains as many decimal places as both the divisor and the quotient, it must contain at least as many as the divisor. Hence, *if, before division, any dividend is seen to contain fewer decimal places than its divisor, annex as many ciphers to such a dividend as will supply the deficiency.*

RULE. (1) *See that the dividend contains at least as many decimal places as the divisor.* (2) *Divide as with whole numbers.* (3) *From the right of the quotient point off as many decimal places as the number of decimal places in the dividend exceeds the number of decimal places in the divisor.*

NOTE 1. If the quotient does not contain as many figures as it must have decimal places, *prefix* ciphers to supply the deficiency.

NOTE 2. To divide a decimal by 10, 100, 1000, etc., move its decimal

point as many places to the left of its former position, as there are ciphers in the divisor, prefixing ciphers to the dividend if necessary.

NOTE 3. Complex decimals, if terminate [Note, 117], should be expanded to simple decimals before commencing the division; and if interminate, should be divided as shown in **105** and **106**.

NOTE 4. To obtain an approximate quotient of a given number of decimal places, use as many figures after the decimal point of the dividend as will equal the number of decimal places in the divisor *plus* the number of decimal places required in the quotient, omitting the remaining decimal places of the dividend if it has an excess, or annexing decimal ciphers to the dividend if it has a deficiency, of decimal places.

EXAMPLES FOR PRACTISE

Divide

1. 16.341 by 3
2. 138.1716 by 49.347
3. 32.50676 by 56.83
4. 2175.4754 by 842
5. 37 by .0008
6. .0366234 by .537
7. .00003 by .005
8. .000219936 by 68.73

Divide

9. 95 by .0005
10. .0002512 by .628
11. 9.72 by .00036
12. .001 by .000008
13. $153.8671\frac{7}{8}$ by $37.8\frac{3}{4}$ [Note 3]
14. .5224 by $.006\frac{2}{3}$
15. $16.471\frac{1}{9}$ by $3.7\frac{7}{9}$
16. $11.7196\frac{1}{2}$ by $.016\frac{1}{8}$

Applying Note 4, divide

17. 19.3 by 17 (to 4 dec. places)
18. 43.5782 by 3.4 (to 2 dec. places)
19. .268345 by 5.3 (to 3 dec. places)
20. 5.93472 by .26 (to 2 dec. places)
21. 617.35 by 48.3 (to 3 dec. places)
22. $.18\frac{5}{16}$ by .348 (to 3 dec. places)
23. $394.8\frac{2}{3}$ by .417 (to 2 dec. places)
24. $58.52\frac{3}{4}$ by .78 (to 3 dec. places)
25. $16.7\frac{3}{4}$ by $.18\frac{5}{8}$ (to 4 dec. places)
26. $7.5\frac{1}{6}$ by $13\frac{2}{3}$ (to 2 dec. places)

Applying Note 2, divide

27. 283.45 by 100
28. 4167.8 by 10
29. 52.384 by 1000
30. $3.4\frac{5}{7}$ by 100
31. $.6\frac{2}{3}$ by 10 (to 3 dec. pl.)
32. $1.7\frac{5}{8}$ by 100 (to 6 dec. pl.)
33. 38.74 by 400 (= 100 × 4)
34. 763.4 by 7000 (= 1000 × 7)

WHEN THE DIVISOR IS A SIMPLE DECIMAL OF MANY PLACES

130. It is frequently necessary in commercial calculations to divide by a simple decimal of many places. This will require much unnecessary labor if the methods of **129** are employed. The following contraction is absolutely correct, and will often save more than half the labor required by the ordinary processes.

ILLUSTRATIVE EXAMPLE

Divide \$5.86 by .621783 to 2 decimal places (cents).

SOLUTION

.6217$\not{8}\not{3}$)\$5.8600(\$9.42 +
5 5960
2640
2487
153
124
29

EXPLANATION. First see that the dividend contains no greater or less number than four decimal places, cutting out all beyond the fourth place if it has more, or annexing ciphers to supply the deficiency if it has fewer places.

Then divide by as many figures of the divisor (.621783) as this dividend of four decimal places (5.8600) will contain, that is, by .6217$\not{8}\not{3}$, canceling the remaining figures of the divisor as unnecessary to secure the correct quotient. Dividing this adapted dividend, it is found that 58600 ten-thousandths contain 6217 ten-thousandths 9 (integral) times with a remainder of 2640 ten-thousandths.

Then dropping one figure (7) from the right of the preceding divisor (621$\not{7}$), it is seen that the remainder from the preceding division (2640 ten-thousandths) contains 621 thousandths, 4 (tenths) times, with a remainder of 153 ten-thousandths.

Again dropping one figure (1) from the right of the preceding divisor (.62$\not{1}$), it is seen that the remainder from the preceding division (153 ten-thousandths) contains 62 hundredths, 2 (hundredths) times. As the quotient is required to two decimal places, or to hundredths, the division is now completed.

RULE. 1. *First see that the dividend contains exactly four decimal places, annexing ciphers if it has a deficiency, or rejecting all beyond the fourth place if it has an excess.*

2. *At the first partial division, use as many of the higher orders of the divisor as the modified dividend of four decimal places can contain, and reject the remaining orders.*

3. *At each successive partial division, reject one additional order from the right of the immediately preceding divisor. The last partial division should be by the number expressed by the first two figures after the decimal point of the divisor.*

4. *If, at any stage of the division, the remainder does not contain the divisor, write a cipher as the next quotient figure, reject one more order from the right of the divisor, and proceed as before.*

NOTE 1. When multiplying by the successive quotient figures, always include one or two figures next adjoining the proper divisor, to secure the correct carrying figure to the expressed product.

NOTE 2. If the final remainder is one-half or more of the final divisor, increase the final quotient figure by 1.

NOTE 3. The above method of division will also be found useful when a divisor is the integer 1 followed by many decimal places. In this case, see that the dividend contains exactly *three* decimal places, instead of the four places required when the divisor is a simple decimal. The final divisor should be the two highest orders as before, but these two orders (regarded as one quantity) will now express *tenths*.

EXAMPLES FOR PRACTISE

Carrying the quotients to 2 decimal places, divide

1. \$9.16 by .48736291
2. \$12.50 by .9376284
3. \$35.72 by .8675913
4. \$138.65 by .9163748
5. \$400 by .28683297
6. \$3.167894 by .7264583
7. \$7.25 by 1.3427862 [Note 3]
8. \$45.6 by 1.2783245
9. \$25 by 1.6824357
10. \$18.67 by 1.2354987

REVIEW OF DECIMALS

131. The relation of mixed decimals [**118**] to each other the same as that of integers [**48–53**]; and the relation of simple decimal quantities, the same as that of proper fractions [**107–109**].

1. What is the difference between .087 times 72.5634 and the sum of 8.73, 16.035, 12.4875, and 15.3?

2. The average yield per acre of a field of wheat was 32.1875 bushels. What was the total yield if the field contained 75.0625 acres?

3. What is the sum of $15.6\frac{5}{16}$, $38.24\frac{3}{32}$, $39\frac{5}{8}$, 265, $\frac{5}{8}$, and $2.34\frac{1}{4}$, to four decimal places?

4. How many 32nds are in .34378? [**125**]

5. What is the cost of 37 barrels of vinegar, averaging 39.0625 gallons per barrel at $16\frac{2}{3}$ cents per gallon?

6. The product of two decimals is 11392.5625 and one of its factors is 156.0625. What is its other factor?

7. What decimal part of a yard of cloth can be bought for \$.60, at \$.80 per yard?

8. How much corn in .75 of 850 bushels?

9. A and B own a tract of land, A owning .625 of the tract. What decimal part of the tract does B own?

10. A merchant sold .375 of his stock of flour. How many barrels of flour did he originally have in stock if the quantity sold was 162 barrels?

11. After paying .1875 of his debts, a merchant found that he still owes \$7800. What was his original indebtedness?

12. A man had 650 yards of silk, and at various times sold 481 yards. What decimal part of the silk did he sell?

13. A merchant had a certain sum deposited in a bank. If he withdrew .35 of his deposit to buy 140 barrels of flour at \$6 per barrel, how much has he still remaining in the bank?

14. A man bequeathed \$28560 to his wife, son, and daughter, leaving .475 of this sum to his wife, .315 of it to his daughter, and the remainder to his son. How much did each receive?

15. A man bought a house and lot for \$3500. If the lot cost .75 of what he paid for the house, how much did he pay for each?

16. A man lost .465 of his wealth in speculation, and then found that he was still worth \$8774. How much was he worth before the speculation?

17. \$119.48 is .45 more than how much money?

18. \$28.90 is .15 less than how much money?

19. A man has \$678.60, which is .45 more than his neighbor has. How much has his neighbor?

20. Since the census of 1900, a town has lost .18 of its population. If the census of 1910 showed a population of 4428, how many inhabitants did it have at the census of 1900?

21. A and B engage in partnership, A investing \$31188 and B investing \$36612. What decimal part of the capital did each *invest?*

22. A farmer sold 450 bushels of wheat to one man at 97 cents per bushel, and .36 less than that quantity to another man at 95 cents per bushel. How much did he receive from both sales?

23. A man's income is $2500 per annum and his expenses $1800 per annum. What decimal part of his income does he save?

24. A man paid $23.75 for a suit of clothes, $33.25 for an overcoat, $3.80 for a hat, and had .36 of his money remaining. How much money did he have at first?

25. A man's expenses are .78 of his income. How much does he save, if his expenses are $1404 per annum?

COMPOUND NUMBERS

132. INTRODUCTION. (*a*) Quantities have now been considered the various orders of which have been expressed upon a uniform, ascending scale of 10, commencing with the unit as the initial order, as with integers [**9–13** and **17–69**]. It has also been seen that this same convenient arrangement of orders can be extended *conversely* to fractional parts of the initial unit by expressing them upon a uniform descending scale of 10, as with decimal fractions [**112–131**].

(*b*) In contrast with the subdivisions of the initial unit into convenient *decimal* fractional parts, has also been placed the more inconvenient divisions of the integral unit upon an arbitrary changeable scale, as with common fractions [**70–111**], in the consideration of which increased difficulty was experienced in completing the relation of quantities thus expressed, by reason of the *varying* scale in which the subdivisions (denominators) of the mutually related terms were expressed.

(*c*) A *compound number*, now to be considered, is the expression of an integral quantity upon the same arbitrary varying scale, as that of common fractions. So that, as common fractions are when contrasted with decimal fractions, so are compound integral quantities when contrasted with simple integral quantities; the scale in passing from one denominate order to the next higher or lower order being as irregular as in passing from a common fraction of one denominator to another fraction of a higher or lower denominator, thus requiring the application of the same principles of reduction in the one as in the other.

(d) The law of increase and decrease is universal in its appli-

cation to *all* quantities, in whatever form they may be expressed. The processes for completing the relation of quantities are therefore the same for *compound* integers as for simple integers [**48–53**], and the same for common fractions [**107–109**] as for decimals, though the method for executing these processes may *seem* to differ with the *form* in which the related terms are expressed. Any seeming difference will quickly disappear if related compound numbers or common fractions are conceived to be reduced to equivalent expressions which have a common decimal scale.

133. A denominate number is the expression of a quantity in units of weight, of measure, or of value.

134. A simple denominate number expresses a denominate quantity in units of a *single* denomination or name.

Thus, 3 pounds (units of weight), 5 bushels (units of measure), and 6 dollars (units of value) are simple denominate numbers.

135. A compound denominate number, or, as it is usually abbreviated, a **compound number**, is the expression of *one* denominate quantity in two or more different denominations or subdivisional names.

Thus, "5 gallons 3 quarts 1 pint" is a compound number, because it expresses *one* fluid quantity in units of three different denominations.

MEASURES OF VALUE

136. United States money is the legal currency of the United States. *Scale:* 10 cents (¢) = 1 dime (d.); 10 dimes or 100 cents = 1 dollar ($); 10 dollars = 1 eagle (E).

For convenience, the denomination *mill* is frequently used to express tenths of a cent.

137. English or sterling money is the legal currency of Great Britain. *Scale:* 4 farthings (far.) = 1 penny (d.); 12 pence = 1 shilling (s.); 20 shillings = 1 pound (£).

138. The following Table exhibits the intrinsic value of the several units of currency of the principal nations of the world, estimated in United States money, as proclaimed by the Secretary of the United States Treasury on Jan. 1, 1912.

Country	Legal Standard	Monetary Unit	Value in Terms of U. S. Gold Dollar
Argentine Republic	Gold	Peso	$ .965
Austria-Hungary	Gold	Crown	.203
Belgium	Gold and silver	Franc	.193
Brazil	Gold	Milreis	.546
British America	Gold	Dollar	1.000
Chili	Gold	Peso	.365
Denmark	Gold	Crown	.268
Egypt	Gold	Pound (100 piasters)	4.943
France	Gold and silver	Franc	.193
German Empire	Gold	Mark	.238
Great Britain	Gold	Pound Sterling	4.8665
India (British)	Gold	Rupee	.324⅓
Italy	Gold and silver	Lira	.193
Japan	Gold	Yen	.498
Mexico	Gold	Peso	.498
Norway	Gold	Crown	.268
Panama	Gold	Balboa	1.000
Peru	Gold	Libra	4.8665
Philippine Islands	Gold	Peso	.500
Portugal	Gold	Milreis	1.080
Russia	Gold	Ruble	.515
Spain	Gold and silver	Peseta	.193
Sweden	Gold	Crown	.268
Switzerland	Gold	Franc	.193
Turkey	Gold	Piaster	.044

MEASURES OF WEIGHT

139. Avoirdupois weight is the usual weight of commerce. *Scale:* 16 ounces (oz.) = 1 pound (lb.); 100 pounds = 1 hundredweight (cwt.); 20 hundredweight = 1 ton (T.).

140. Troy weight is used by jewelers in weighing gold, silver, and other precious metals. *Scale:* 24 grains (gr.) = 1 pennyweight (pwt.); 20 pennyweights = 1 ounce (oz.); 12 ounces = 1 pound (℔).

141. Apothecaries' weight is used by apothecaries and physicians in weighing the non-liquid ingredients of prescriptions. *Scale:* 20 grains (gr.) = 1 scruple (℈); 3 scruples = 1 dram (ʒ); 8 drams = 1 ounce (℥); 12 ounces = 1 pound (℔).

142. Miscellaneous table of weights: 2240 pounds = 1 long ton; 100 pounds of grain or flour = 1 cental; 100 pounds of nails = 1 keg; 196 pounds of flour = 1 barrel; 200 pounds of salted beef, pork, or fish = 1 barrel.

NOTE 1. It is customary to allow the following weights to 1 bushel:

Barley .....48 lb.	Ear corn .....70 lb.	Potatoes .....60 lb.
Beans.....60 "	Corn meal.....50 "	Rye.....56 "
Buckwheat ....48 "	Cotton seed ...30 "	Timothy seed...45 "
Clover seed....60 "	Oats.....32 "	Wheat .....60 "
Shelled corn ...56 "	Peas.....60 "	Wheat bran ....20 "

NOTE 2. Denominations of the same name in the *Troy Scale* and *Apothecaries' Scale* have the same weight; but both of these differ in weight from denominations of the same name in the *Avoirdupois Scale*. Thus, there are 5760 Troy grains in 1 pound Troy, and 7000 Troy grains in 1 pound Avoirdupois; and there are 480 Troy grains in a Troy ounce (5760 gr. ÷ 12), but only 437½ Troy grains in 1 ounce Avoirdupois (7000 gr. ÷ 16).

MEASURES OF EXTENSION

143. Linear measure is used in measuring lengths, widths, heights, depths, and distances. *Scale:* 12 inches (in.) = 1 foot (ft.); 3 feet = 1 yard (yd.); 5½ yards = 1 rod (rd.); 40 rods = 1 furlong (fur.); 8 furlongs or 320 rods = 1 mile (mi.).

144. Surveyors' linear measure is used by surveyors in measuring lengths of roads, boundaries of land, etc. *Scale:* 7.92 inches (in.) = 1 link (l.); 100 links = 1 chain (ch.); 80 chains = 1 mile (mi.).

145. Circular measure is used in measuring angles and arcs, (parts of a circle); and by navigators in estimating latitude and longitude. *Scale:* 60 seconds (″) = 1 minute (′); 60 minutes = 1 degree (°) ; 360 degrees = 1 circle (C), or circumference of the earth.

MEASURES OF SURFACE

146. Square measure is used in measuring ordinary areas, such as the surfaces of boards, of painting, plastering, papering, paving, etc., and, unofficially, of land. *Scale:* 144 square inches (sq. in.) = 1 square *foot* (sq. ft.); 9 square feet = 1 square yard

(sq. yd.); $30\frac{1}{4}$ square yards = 1 square rod (sq. rd.); 160 square rods = 1 acre (A); 640 acres = 1 square mile (sq. mi.).

147. Surveyors' square measure is used by surveyors in measuring land. *Scale:* 10,000 square links (sq. l.) = 1 square chain (sq. ch.); 10 square chains = 1 acre (A); 640 acres = 1 square mile (sq. mi.).

MEASURES OF VOLUME

148. Cubic measure is used in measuring the contents or volume of bodies. *Scale:* 1728 cubic inches (cu. in.) = 1 cubic foot (cu. ft.); 27 cubic feet = 1 cubic yard (cu. yd.).

149. Wood measure is used in measuring cord wood. *Scale:* 16 cubic feet (cu. ft.) = 1 cord foot (cd. ft.); 8 cord feet or 128 cubic feet = 1 cord (cd.).

MEASURES OF CAPACITY

150. Liquid measure is used in measuring ordinary liquids. *Scale:* 4 gills (gi.) = 1 pint (pt.); 2 pints = 1 quart (qt.); 4 quarts = 1 gallon (gal.).

151. Apothecaries' fluid measure is used by apothecaries and physicians in measuring liquid medicines. *Scale:* 60 minims (℥) = 1 fluiddrachm (f3); 8 fluiddrachms = 1 fluidounce (f℥); 16 fluidounces = 1 pint (O).

152. Dry measure is used in measuring fruits, vegetables, and grains. *Scale:* 2 pints (pt.) = 1 quart (qt.); 8 quarts = 1 peck (pk.) 4 pecks = 1 bushel (bu.).

MEASURE OF TIME

153. *Scale:* 60 seconds (sec.) = 1 minute (min.); 60 minutes = 1 hour (hr.); 24 hours = 1 day (da.); 365 days = 1 common year (yr.); 366 days = 1 leap year; 100 years = 1 century (cent.).

NOTE 1. Leap years and common years are identified as follows:

1. *Every year which is not exactly divisible by 4, is a common year.*
2. *Every year which is exactly divisible by 4, is a leap year.*
3. *Centennial years* (years ending in two ciphers) *when divisible by 400, are leap years; and when not divisible by 400, are common years.*

NOTE 2. The order, name, and length in days of each month are:

Order	Name	Length	Order	Name	Length
First	January	31 days	Seventh	July	31 days
Second	February	28 or 29 da.	Eighth	August	31 "
Third	March	31 days	Ninth	September	30 "
Fourth	April	30 "	Tenth	October	31 "
Fifth	May	31 "	Eleventh	November	30 "
Sixth	June	30 "	Twelfth	December	31 "

OTHER MEASURES

154. In enumerating certain articles of merchandise, the following *Scale* is used: 12 things = 1 dozen (doz.); 12 dozen = 1 gross (gro.); 12 gross = 1 great gross (g. gro.); 6 things = 1 set; 20 things = 1 score.

155. In measuring paper, the following *Scale* is used: 24 sheets (sh.) = 1 quire (qu.); 20 quires = 1 ream (rm.); 2 reams = 1 bundle (bdl.); 5 bundles = 1 bale (bl.).

REDUCTION OF COMPOUND DENOMINATE NUMBERS TO SIMPLE DENOMINATE NUMBERS

ILLUSTRATIVE EXAMPLE

156. Reduce 15 da. 17 hr. 28 min. to seconds.

SOLUTION

15 da. 17 hr. 28 min.
× 24 (hrs. in 1 da.)
377 (15 × 24 + 17)
×60 (min. in 1 hr.)
22648 (377 × 60 + 28)
× 60 (sec. in 1 min.)
1358880 seconds.

EXPLANATION. Commence with the highest denomination (15 da.) and reduce it to the next lower denomination (hr.) as follows: If 1 day = 24 hr., 15 days will equal 15 times 24 hr., or 360 hr., and 15 days 17 hr. will equal 360 hr. + 17 hr., or 377 hr.

Next reduce 377 hr. to the next lower denomination of the scale (min.) as follows: If 1 hr. = 60 min., 377 hr. will equal 377 times 60 min., or 22620 min., and 377 hr. 28 min. will equal 22620 min. + 28 min., or 22648 min.

Lastly, reduce 22648 min. to the next lower denomination (sec.) as follows: If 1 min. = 60 sec., 22648 min. will equal 22648 times 60 sec., or 1358880 sec., the required answer.

RULE. *Multiply the highest given denomination by the number of units of the next given denomination* which equal one unit of that

highest denomination, and to the result add the given units of that next given denomination.

Reduce the obtained results in a similar manner to the next lower given denomination, and continue this process until all the given denominations have been considered and the required lower denomination has been obtained.

NOTE. It should be remembered that a multiplicand and multiplier are not so called because they have a certain *position* with respect to each other, but because they possess a certain characteristic relation to each other [Note 1, **25**]. Thus, in the Ill. Ex., 24, 60, and 60 are the successive multiplicands though placed for convenience in positions *usually* occupied by multipliers.

EXAMPLES FOR PRACTISE

Reduce

1. £39 18s. 7d. to pence
2. 73 T. 16 cwt. to pounds
3. 16 bu. 3 pk. 5 qt. to pints
4. $19 and 8 ¢ to mills
5. 3 yr. 216 da. 12 min. to seconds
6. 18 lb. 11 pwt. to grains
7. £196 16s. 2d. to pence
8. 28 rd. 2 yd. 1 ft. to inches
9. 25 cd. 3 cd. ft. to cubic feet
10. 4 A. 25 sq. rd. 16 sq. yd. to square feet
11. 27 mi. 18 rd. 2 yd. to inches
12. 5 T. 12 cwt. 16 lb. to ounces
13. 3 wk. 4 da. 17 hr. to seconds
14. 35 bu. 2 pk. 3 qt. to quarts
15. 65 gal. 3 qt. 1 pt. to gills
16. 73 A. 75 sq. rd. 12 sq. yd. to square inches
17. 25 cu. yd. 13 cu. ft. to cubic inches
18. How much was received for 3 bu. 2 pk. chestnuts if sold at 6 cents per quart?
19. What is the cost of 1 lb. 7 oz. 13 pwt. of jewelers' gold at $1.05 per pennyweight?
20. How much should be received for 37 gallons of cider if retailed at 5 cents per pint?

REDUCTION OF SIMPLE DENOMINATE NUMBERS TO COMPOUND DENOMINATE NUMBERS

ILLUSTRATIVE EXAMPLE

157. Reduce 3470 gills to the higher denominations.

SOLUTION

4)3470 gi.
2)867 + 2 gi.
4)433 + 1 pt.
108 gal. + 1 qt.

EXPLANATION. 4 gills of the given denomination (3470 gills) equal 1 pint, the next higher denomination; and 3470 gi. contain 4 gi. 867 times and 2 gi. remaining. Therefore 3470 gi. = 867 pt. and 2 gi.

2 pt. of the obtained denomination (867 pt.) equal 1 qt., the next higher denomination; and 867 pt. contain 2 pt. 433 times and 1 pt. remaining. Therefore, 867 pt. = 433 qt. and 1 pt.

4 qt. of the last obtained denomination (433 qt.) equal 1 gal., the next higher denomination; and 433 qt. contain 4 qt. 108 times and 1 qt. remaining. Therefore 433 qt. = 108 gal. and 1 qt.; and 3470 gi. = 108 gal. 1 qt. 1 pt. 2 gi.

RULE. *Reduce the given denomination to the next higher denomination. Continue this process in regular order from one obtained denomination to the next higher until the highest required denomination is reached.*

NOTE. (*a*) Each successive divisor should be of the same denomination as its dividend; (*b*) each successive remainder is of the same denomination as the dividend from which it was obtained, and (*c*) each successive quotient is of that denomination in the scale which equals the divisor that produced it.

EXAMPLES FOR PRACTISE

Reduce

1. 6173 pt. to bushels
2. 3567282 sec. to days
3. 273465 in. to miles
4. 827356 sq. ft. to acres
5. 23468 far. to £'s
6. 853764 oz., Avoir., to tons
7. 5295 sheets to reams
8. 7835 cu. ft. to cords
9. 2346 pwt. to lbs, Troy
10. 3496 cd. ft. to cords

Reduce

11. 7683d. to £'s
12. 53867 mills to dollars
13. 6287 gi. to gallons
14. 2863452 sec. to days
15. 627853 in. to miles.
16. 7456 qt. to bushels
17. 876345 ft. to miles
18. 917345 cu. in. to cu. yds.
19. 17832d. to £'s
20. 1378 gi. to gallons.

REDUCTION OF DENOMINATE COMMON FRACTIONS TO INTEGERS

ILLUSTRATIVE EXAMPLE

158. Reduce $\frac{5}{9}$ of a year to integers of the lower denominations.

SOLUTIONS

$\frac{5}{9}$ of 365 da. = 202 da.(and $\frac{7}{9}$ da.)

$\frac{7}{9}$ of 24 hr. = 18 hr. (and $\frac{6}{9}$ = $\frac{2}{3}$ hr.)

$\frac{2}{3}$ of 60 min. = 40 min., Hence,

$\frac{5}{9}$ yr. = 202 da. 18 hr. 40 min.

EXPLANATION. $\frac{5}{9}$ of a year are equivalent to $\frac{5}{9}$ of (times) 365 da. or 202$\frac{7}{9}$ da. Reserving the integral part of this result (202 da.) as the first term of the required answer, similarly reduce the fractional part ($\frac{7}{9}$ da.) to the next lower denomination (hr.) as follows:

$\frac{7}{9}$ of a day = $\frac{7}{9}$ of (times) 24 hr., or 18$\frac{2}{3}$ hr. Reserving the integral part of the result (18 hr.) as the second term of the required answer, reduce the fractional part ($\frac{2}{3}$ hr.) to the next lower denomination (min.), as follows:

$\frac{2}{3}$ of an hour equals $\frac{2}{3}$ of (times) 60 min., or 40 min. Therefore $\frac{5}{9}$ of a year equals 202 da. 18 hr. 40 min.

EXAMPLES FOR PRACTISE

Reduce to integers of the lower denominations.

1. £$\frac{7}{8}$
2. $\frac{3}{4}$ yr.
3. $\frac{3}{16}$ gal.
4. $1\frac{7}{2}$ bu.
5. $\frac{3}{8}$ yr.
6. $\frac{4}{5}$ mi.
7. $\frac{5}{6}$ yd.
8. $\frac{3}{16}$ T.
9. £$\frac{11}{16}$
10. $\frac{11}{12}$ yd.

REDUCTION OF DENOMINATE DECIMALS TO INTEGERS

ILLUSTRATIVE EXAMPLE

159. Reduce £.375 to integers of the lower denominations.

SOLUTION

£.375
20
s. 7.500
12
d. 6.0

£.375 = 7s. 6d.

EXPLANATION. .375 of a £ = .375 of (times) 20s., or 7.5s. Reserving the integral part of this result (7s.) as the first term of the required answer, reduce the fractional part (.5s) to the next lower denomination (d.) as follows:

.5s = .5 of (times) 12d., or 6d. Therefore £.375 = 7s.6d.

EXAMPLES FOR PRACTISE

Reduce to integers of the lower denominations.

1. .1875 gal.	**4.** .84375 gal.	**7.** .34375 mi.
2. .0375 yr.	**5.** .876 T.	**8.** £.52375
3. .546875 bu.	**6.** .375 lb., Troy	**9.** .921875 bu.

REDUCTION OF COMPOUND INTEGRAL NUMBERS TO FRACTIONS OF ANY REQUIRED HIGHER DENOMINATION

ILLUSTRATIVE EXAMPLES

160. 1. Reduce 2 pk. 3 qt. to a common fraction of a bushel.

SOLUTION

2 pk. 3 qt. = (8 qt. × 2) + 3, or 19 qt.

1 bu. = 4 pk., or 4 times 8 qt. = 32 qt.

Hence, 2 bu. 3 qt. = $\frac{19}{32}$ of a bushel

EXPLANATION. 1 pk. = 8 qt., therefore 2 pk. 3 qt. = 2 times 8 qt. (16 qt.) + 3 qt., or 19 qt.

1 bu. = 4 pk., and 4 pk. = 4 times 8 qt., or 32 qt.

If 32 qt. = 1 bu., 1 qt. must equal $\frac{1}{32}$ of a bu., and 2 pk. 3 qt., or 19 qt., must equal 19 times $\frac{1}{32}$ of a bu., or $\frac{19}{32}$ of a bu.

2. Reduce 17s. 6d. to a decimal of a £ sterling.

SOLUTION

12)6.0
.5s.

20)17.5
£.875

EXPLANATION. 12d. = 1s.; therefore 6d. must be such a decimal part of a shilling as 6d. are of 12d., that is, $\frac{6}{12}$ or .5 of a shilling [**123**]; and 17s. 6d. must equal 17.5 shillings.

20s. = £1; and 17.5s. must be such a decimal part of a £ as 17.5s. are of 20s., that is $\frac{17.5}{20}$ or .875 of a £.

Or, 17s. 6d. may be reduced to a common fraction of a £ as in the preceding solution, obtaining £$\frac{7}{8}$, and this result reduced to a decimal by **123**, obtaining £.875.

EXAMPLES FOR PRACTISE

What fraction of

1. A £ are 2s. 6d.
2. A dollar are 12¢ 5 mills.
3. A gallon are 2 qt. 1 pt.
4. A bushel are 1 pk. 4 qt.
5. A day are 4 hr. 48 min.

What decimal of

6. A gallon are 3 qt. 1 pt.
7. A £ are 12s. 6d.
8. A yard are 1 ft. 6 in.
9. A bushel are 1 pk. 7 qt. 1 pt. (to 4 places).
10. A day are 5 hr. 19 min. (to 3 places).

ADDITION OF COMPOUND NUMBERS

ILLUSTRATIVE EXAMPLE

161. Add 3 gal. 2 qt. 1 pt.; 2 gal. 1 pt.; and 3 qt. 1 pt.

SOLUTION

gal.	qt.	pt.
3	2	1
2	0	1
	3	1
6	2	1

EXPLANATION. So arrange the given compound numbers that their denominations which are similar shall fall in the same column [Prin. 1, **18**]. Then add the column of the lowest given denomination (pt.), obtaining 3 pt., or 1 qt. 1 pt. Write 1 pt. under the column of pints, and carry 1 qt. to the column of quarts and add, obtaining 6 qt., or 1 gal. 2 qt. Write 2 qt. under the column of quarts, and carry 1 gal. to the column of gallons and add, obtaining 6 gal.

EXAMPLES FOR PRACTISE

1

da.	hr.	min.	sec.
5	13	25	19
17	21	18	27
12	15	35	40
8	7	43	52

2

T.	cwt.	lb.	oz.
45	18	29	9
37	3	75	3
15	12	34	11
7	6	17	6

3

£	s.	d.	far.
15	16	8	3
7	10	6	1
24	3	10	2
18	12	9	3

4. Add 5 bu. 7 qt.; 3 bu. 2 pk.; 7 bu. 1 pt.; and 3 pk. 5 qt.

5. Add £35 7s. 6d.; £53 8d.; £65 2 far.; £45 15s.

6. Add 16 da. 18 hr. 5 sec.; 7 da. 3 hr. 48 min.; 19 da. 7 hr. 50 sec.; 12 da. 2 hr. 34 sec.; and 2 da. 23 hr.

7. Add 19 cu. yd. 16 cu. ft. 1217 cu. in.; 35 cu. yd. 24 cu. ft. 197 cu. in.; 2 cu. yd. 15 cu. ft. and 718 cu. in.; 13 cu. yd. 16 cu. ft.; and 25 cu. yd. 11 cu. ft. 1235 cu. in.

8. Add $\frac{1}{5}$ da.; $\frac{4}{15}$ da.; $1\frac{1}{3}$ da.; and $\frac{2}{3}$ da.; reduced to compound integers.

9. Add .875 cd. ft.; .25 cd.;. .125 cd.; reduced to compound *integers.*

SUBTRACTION OF COMPOUND NUMBERS

ILLUSTRATIVE EXAMPLE

162. Subtract £15 12s. 3 far. from £48 7s. 6d.

SOLUTION

£	s.	d.	far.
48	7	6	0
15	12	0	3
£32	15s.	5d.	1 far.

EXPLANATION. So place the several denominations of the subtrahend that they shall fall under similar denominations of the minuend; and commence the subtraction with the lowest denomination.

As 3 far. cannot be subtracted from 0 far. borrow 1d. (= 4 far.) from the next higher denomination of the minuend (6d.), and then subtract 3 far. from the borrowed 4 far., obtaining 1 far. as the first partial remainder.

Next subtract the next higher denomination of the subtrahend (0d.) from the similar denomination of the minuend (6d. less the borrowed 1d., or 5d.), obtaining 5d. as the second partial remainder.

As the next higher denomination of the subtrahend (12s.) cannot be subtracted from the similar denomination in the minuend (7s.), borrow £1 (= 20s.) from the £48 of the minuend, and then subtract 12s. from 7s. increased by the borrowed 20s., or 27s., obtaining 15s. as the third partial remainder.

Lastly, subtract the next higher denomination of the subtrahend (£15) from the similar denomination of the minuend (£48 less the borrowed £1, or £47), obtaining £32 as the final partial remainder; thus making the complete remainder £32 15s. 5d. 1 far.

EXAMPLES FOR PRACTISE

1. Subtract 5 bu. 1 pk. 1 pt. from 12 bu. 3 qt.
2. Subtract 19 da. 18 hr. 45 min. from 45 da. 12 hr.
3. Subtract 25 cd. 95 cu. ft. from 42 cd. 37 cu. ft.
4. Subtract 167 gal. 3 qt. 2 gi. from 258 gal. 2 qt. 1 pt.
5. Subtract £235 17s. 6d. from £452 12s. 9d.
6. Subtract 15 A. 90 sq. rd. 18 sq. yd. and 5 sq. ft. from 23 A. 73 sq. rd. 24 sq. ft.
7. A grocer sold 27 gal. 3 qt. 1 pt. from a barrel of molasses containing 39 gal. 1 qt. How much molasses remained in the barrel?
8. A merchant in London owed £285 2s 6d and subsequently made a payment of £97 12s 9d. How much of the debt remained unpaid?

COMPOUND SUBTRACTION OF DATES

ILLUSTRATIVE EXAMPLE

163. Find the interval between Oct. 25, 1854, and July 16, 1911.

SOLUTION

yr.	mo.	da.
1911	7	16
1854	10	25
56 yr.	8 mo.	21 da.

EXPLANATION. Take the later date (July 16, 1911) as the minuend because it expresses a *greater* period of time, denoting the 1911th year of the Christian era, the 7th month [Note 2, **153**], and the 16th day; underneath which write the earlier date which expresses a *less* period of time, denoting the 1854th year of the Christian era, the 10th month, and the 25th day. Subtract as shown in Ill. Ex. **162**.

NOTE 1. In obtaining the interval between two dates by compound subtraction, it is customary to consider *any* borrowed month as equal to 30 days.

NOTE 2. If *both* the subtrahend and minuend express the last days of their respective months, and the subtrahend month is the longer, as in Ex. 7 and 9, following, the difference between them will be 0; but if the subtrahend month is the shorter as in Ex. 10, following, the difference is found in the usual manner. The courts have decided when a male child is born on Feb. 29th of a leap year, that he has reached his majority on Feb. 28th of the 21st year thereafter; and when a note falls due on the 29th, 30th, or 31st of a month which contains less than that number of days, that it is payable on the last day of such a month. Thus, 3 months after Aug. 31 = Nov. 30, and conversely from Aug. 31 to Nov. 30 must equal 3 months.

EXAMPLES FOR PRACTISE

By compound subtraction find the interval from

1. Mch. 16, 1892 to Dec. 3, 1911.
2. Oct. 28, 1886 to May 4, 1912.
3. July 2, 1875 to Feb. 12, 1912.
4. Dec. 18, 1895 to Aug. 9, 1911.
5. Nov. 30, 1845 to Feb. 8, 1910.
6. Sept. 19, 1902 to July 31, 1910.
7. Aug. 31, 1897 to June 30, 1911. [Note 2]
8. May 31, 1875 to Jan. 31, 1909.
9. Feb. 29, 1892 to Feb. 28, 1910.
10. Feb. 28, 1887 to Feb. 29, 1912.

TO FIND THE EXACT NUMBER OF DAYS BETWEEN TWO DATES

ILLUSTRATIVE EXAMPLE

164. How many days after Oct. 13, 1911, is Apr. 24, 1912?

SOLUTION

18 da., remainder of October
30 da., in November
31 da., in December
31 da., in January
29 da., in February
31 da., in March
24 da., included in April
194 da., required interval

EXPLANATION. Subtract the initial date (Oct. 13) from the full number of days in that month (31), obtaining 18 days as the included remainder of Oct., under which place in regular order the number of days in each of the succeeding full months of the interval, and the included days of the final month (24); and add, obtaining 194 days.

EXAMPLES FOR PRACTISE

Find the exact number of days from

1. Jan. 16, 1911 to Nov. 12, 1911.
2. July 23, 1912 to Mch. 8, 1913.
3. Sept. 3, 1909 to May 31, 1910.
4. Feb. 16, 1910 to Oct. 6, 1910.
5. Dec. 31, 1911 to Aug. 12, 1912.
6. Nov. 16, 1910 to Feb. 3, 1911.
7. Mch. 26, 1911 to Dec. 20, 1911.
8. Apr. 30, 1909 to Jan. 14, 1910.
9. June 16, 1911 to Apr. 8, 1912.
10. May 19, 1910 to May 28, 1910.
11. Aug. 27, 1911 to Mch. 5, 1912.
12. Oct. 1, 1907 to July 17, 1908.

MULTIPLICATION OF COMPOUND NUMBERS

ILLUSTRATIVE EXAMPLE

165. Multiply 28 da. 16 hr. 34 min. by 6.

SOLUTION

da.	hr.	min.
28	16	34
		6
172 da	3 hr.	24 min.

EXPLANATION. Commencing with the lowest denomination (34 min.) successively multiply each denomination of the multiplicand by the multiplier, as follows:

6 times 34 min. = 204 min., or 3 hr. 24 min. Write 24 min. as a partial product,

and carry 3 hr. to 6 times 16 hr., or 96 hr., obtaining 99 hr., or 4 da. 3 hr. Write 3 hr. as a second partial product, and carry 4 da. to 6 times 28 da., or 168 da., obtaining 172 da. as a third and final partial product; thus making the complete product equal 172 da. 3 hr. 24 min.

EXAMPLES FOR PRACTISE

Multiply	Multiply
1. 5 gal. 2 qt. 1 pt. by 8	**7.** 17 mi. 48 rd. 4 yd. 2 ft. by 73
2. £24 3s. 5d. by 7	**8.** 53 yr. 7 mo. 23 da. by 85
3. 16 da. 21 hr. 15 min. by 9	**9.** 78 bu. 3 pk. 2 qt. 1 pt. by 147
4. 46 lb. 8 oz. 12 pwt. by 6	**10.** 47 T. 15 cwt. 73 lb. by 524
5. 53 bu. 2 pk. 5 qt. by 12	**11.** 16 da. 15 hr. 49 min. by 97
6. £273 16s. 7d. 3 far. by 97	**12.** 9 gal. 3qt. 1 pt. by 276

13. The distance between two towns is 42 miles. If a man walk from one towards the other for 8 hours at an average speed of 3 miles and 192 rods per hour, how far will he then be from the other town?

DIVISION OF COMPOUND NUMBERS BY ABSTRACT NUMBERS

ILLUSTRATIVE EXAMPLES

166. 1. Divide 35 lb. 10. oz. 18 pwt. by 8.

SOLUTION

8)35 lb. 10 oz. 18 pwt.

4 lb. 5 oz. 17 pwt. 6 gr.

EXPLANATION. Commencing with the highest denomination (35 lb.) successively divide each denomination of the dividend by the divisor as follows:

35 lb. ÷ 8 = 4 lb. and an undivided remainder of 3 lb. Write 4 lb. as a partial quotient; and carry the undivided remainder (3 lb. or 36 oz.) to the next lower denomination of the dividend (10 oz.), obtaining 46 oz. as the second partial dividend.

46 oz. ÷ 8 = 5 oz. and an undivided remainder of 6 oz. Write 5 oz. as the second part of the quotient; and carry the undivided remainder (6 oz. or 120 pwt.) to the next lower denomination of the dividend (18 pwt.), obtaining 138 pwt. as the third partial dividend.

138 pwt. ÷ 8 = 17 pwt. and an undivided remainder of 2 pwt. Write 17 pwt. as the third part of the quotient; and reduce the undivided remainder (2 pwt.) to the next lower denomination of the Troy scale (2 pwt. = 48 gr.).

48 gr. ÷ 8 = 6 gr. and no undivided remainder. Write 6 gr. as the fourth and final part of the quotient. These several partial quotients combined *make the complete* quotient 4 lb. 5 oz. 17 pwt. 6 gr.

2. Divide £249 10s. 5d. by 29.

SOLUTION

29)£249 10s. 5d.(£8
232
£17 remainder
× 20 (and add 10s.)
29)350s.(12s.
29
60
58
2s. remainder
× 12 (and add 5d.)
29)29 d.(1d.
29

Complete quotient, £8 12s. 1d.

EXPLANATION. When the divisor is over 12, and not a composite number, long division may be advantageously employed.

The processes in the present example are similar to those of the preceding example; but here they are written because they are too unwieldy to be carried mentally.

EXAMPLES FOR PRACTISE

Divide	Divide
1. 39 gal. 1 qt. 1 pt. by 5.	**6.** £189 9s. 4d. by 56 (= 8 × 7)
2. 78 yd. 2 ft. 8 in. by 8	**7.** 105 T. 4 cwt. 38 lb. by 27 (9×3)
3. £225 18s. 6d. by 7	**8.** 369 gal. 1 pt. by 47
4. 155 cd. 50 cu. ft. by 9	**9.** 277 yr. 4 mo. 24 da. by 73
5. 287 bu. 2 pk. 4 qt. by 12	**10.** 930° 39′ 55″ by 269

DIVISION OF COMPOUND NUMBERS BY COMPOUND NUMBERS

ILLUSTRATIVE EXAMPLE

167. Divide 907 yd. 2 ft. 6 in. by 34 yd. 2 ft. 9 in.

SOLUTION

907 yd. 2 ft. 6 in. = 32682 in.
34 yd. 2 ft. 9 in. = 1257 in.

1257)32682(26 times
2514
7542
7542

EXPLANATION. Reduce both compound numbers to simple denominate numbers of the lowest denomination expressed in either, obtaining 32682 in. as the equivalent simple dividend, and 1257 in. as the equivalent simple divisor. Divide in the usual manner.

EXAMPLES FOR PRACTISE

1. Divide £1000 8s. 4d. by £35 14s. 7d.
2. Divide 764 bu. by 5 bu. 3 pk. 7 qt.
3. Divide 1172 gal. 2 qt. 1 pt. by 19 gal. 3 qt. 1 pt.
4. Divide 53 da. 11 hr. 56 min. 54 sec. by 17 hr. 35 min. 18 sec.
5. Divide 183 ℔. 3 oz. 12 pwt. by 3 ℔. 9 oz. 16 pwt. 12 gr.

REVIEW OF COMPOUND NUMBERS

168. 1. From a field of wheat containing 78 A., was gathered an average crop of 28 bu. 1 pk. 3 qt. per acre. How much wheat did the field produce?

2. How many spoons weighing 1 oz. 17 pwt. can be made from 3 ℔. 8 oz. 8 pwt of silver?

3. A tenant rented a house on March 18, 1909, at $22.50 per month, and occupied it until February 18, 1912. What was the total rent paid during his tenancy?

4. A man contracted a debt on June 16, 1910, and discharged it on April 23, 1911. How many days did the debt remain unpaid?

5. A coal dealer bought 58240 lb. of coal at $5.20 per long ton and sold it at $6.10 per short ton. What was his total gain?

6. A customer bought a piece of goods at 2s. 6d. per yard, and paid £5 7s. 6d. How many yards did the piece contain?

7. A grocer bought 3 bu. 2 pk. 7 qt. of peanuts and retailed his purchase at 5¢ per pint. How much did he receive?

8. What is the difference between 15 lb. Avoir., and 25 ℔. Troy expressed in denominations of Troy Weight?

9. A charity organization divided 51 bu. 1 pk. 7 qt. of potatoes equally between 27 needy families. How much did each family receive?

10. A gardener divided $1\frac{3}{8}$ of an acre of ground into 12 equal vegetable plats. How much space did each plat occupy, expressed integrally?

11. An importer paid £253 5s 7d. for one invoice of merchandise, £75 18s. 6d. for a second invoice, £148 12s. 9d. for a third, £62 9s. 3d. for a fourth, and £197 17s. 3d. for a fifth. What was his total payment for the five invoices?

12. A grocer sold 18 gal. 3 qt. 1 pt. 2 gi. of vinegar from a

barrel containing 42 gal. 1 qt. How much vinegar remained in the barrel?

13. A pedestrian walked 7 hours per day at an average speed of 4 mi. 37 rd. 3 yd. per hour. What distance had he traveled at the end of the ninth day's walk?

14. How much will be received for a barrel of cider containing 42.375 gallons, if retailed at 3¢ per pint?

15. How many barrels will be needed to ship 79 bu. of potatoes, if the average capacity per barrel is 2 bu. 1 pk. 7 qt?

16. What is the value in United States money of 873.45 marks (Germany)? **17.** Of 728.75 crowns (Sweden)? **18.** Of £673 2s. 6d. (Great Britain)? **19.** Of 3278 lira (Italy)? **20.** Of 348.25 yen (Japan)? **21.** Of 758.35 francs (France)?

22. Reduce $675.80 to French francs. **23.** $528.36 to £'s sterling. **24.** $912.50 to Russian rubles. **25.** $248.75 to Spanish peseta.

THE METRIC SYSTEM

169. INTRODUCTION. The **metric system** is the expression of compound denominate numbers upon the *decimal scale;* and by its use the convenient processes incident to operations upon simple integral quantities and upon decimal fractions can be substituted in the place of the inconvenient methods necessary for operating upon compound integral quantities and upon common fractions which are expressed upon an arbitrary variable scale. The system derives its name from the *meter* (39.37079 inches long) which is the established standard not only for measuring distances, but also for determining the size of the other standards for weights, measures, surfaces, and volumes, by expressing them in decimal multiples or aliquots of the meter.

170. The higher denominations of the metric system are denoted by prefixing the Greek numerals *deka* (10 standard units), *hekto* (100 standard units), *kilo* (1000 standard units), and *myria* (10,000 standard units).

Thus, 1 dekameter means 10 meters; 5 hektometers, 500 meters; 7 kilometers, 7000 meters; and 9 myriameters, 90000 meters. The metric compound 5 hektometers 9 dekameters and 7 meters, is written 597 M.; the meters (7) occupying the units' order, the dekameters (9) the tens' order, the hektometers (5) the hundreds' order, in accordance with the signification of *their respective prefixes.*

171. The lower denominations of the metric system are denoted by prefixing the Latin numerals *deci* (one-tenth of one standard unit), *centi* (one-hundredth of one standard unit), *milli* (one-thousandth of one standard unit).

Thus, 1 decimeter means one-tenth of a meter; 3 centimeters mean 3 hundredths of a meter; 4 millimeters mean 4 thousandths of a meter. The metric compound 2 decimeters, 4 centimeters, 3 millimeters is written .243 M., in accordance with the signification of their respective prefixes; and is usually read 243 thousandths of a meter, or 243 millimeters.

172. The **meter** (= 39.37079 inches) is the metric standard for measuring lengths, widths, heights, depths and distances.

NOTE 1. The meter (pronounced mee'ter) is used by merchants in measuring dry goods, and by others in measuring *short* distances; the millimeter being used in measuring *minute* distances; and the kilometer in measuring *long* distances.

NOTE 2. The higher and lower denominations of the meter are expressed by the use of the prefixes mentioned in **170** and **171.**

NOTE 3. Reductions from the common denominations of length to the metric, or the reverse, may be made by the use of the following

EQUIVALENTS OF LINEAR MEASURE

(Common to Metric)	(Metric to Common)
1 inch = 2.54001 centimeters	1 centimeter = .3937 of an inch
1 foot = .304801 of a meter	1 decimeter = .32818 of a foot
1 yard = .914402 of a meter	1 meter = 1.093611 yards
1 rod = 5.029211 meters	1 dekameter = 1.98838 rods
1 mile = 1.60935 kilometers	1 kilometer = .62137 of a mile

173. The **gram** (= .03527 oz., Avoir., or 15.432 gr., Troy) is the metric standard for weights. It is the weight of a cubic volume of distilled water each edge of which is .01 of a meter.

NOTE 1. The higher and lower denominations of the *gram* are expressed by the use of the same prefixes with the same signification as those of the meter [**170, 171**]. Thus, 8 hektograms mean 800 grams; 4 milligrams mean 4 thousandths of a gram; etc.

NOTE 2. The *gram* is used to express the weight of minute quantities, as jewels, medicines, etc.; and the kilogram (usually abbreviated to kilo) to *express ordinary weights.*

EQUIVALENTS OF WEIGHT

(Common to Metric)	(Metric to Common)
1 oz., Avoir. = 28.3494 grams	1 gram = .03527 oz., Avoir.
1 oz., Troy = 31.104 grams	1 gram = .03215 oz., Troy
1 lb., Avoir. = .45359 of a kilo.	1 hektogram = 3.527 oz., Avoir.
1 lb., Troy = .37324 of a kilo.	1 kilogram = 2.20462 lb., Avoir.

174. The **liter** (= 1.0567 liquid quarts, or .908 of a dry quart) is the metric standard for capacities. It is the capacity of a cubic measure each edge of which is one-tenth of a meter. The *liter* is used to measure not only liquids, but also salt, grains, fruits, vegetables, etc. It is pronounced "lee'ter."

NOTE 1. The higher and lower denominations of the liter are denoted by the prefixes mentioned in **170, 171.**

NOTE 2. Of liquids, the dekaliter is used to express large quantities, the *liter* moderate quantities, and the *centiliter* or *milliliter* minute quantities. Of grains, fruits, vegetables, the *hektoliter*, like the bushel, is used to express large quantities (wholesale); and the *dekaliter* or *liter*, small quantities (retail).

EQUIVALENTS OF CAPACITY

(Common to Metric)	(Metric to Common)
1 liquid quart = .94636 of a liter	1 liter = 1.05668 liquid qts.
1 dry quart = 1.1012 liters	1 liter = .9081 of a dry qt.
1 liquid gallon = .37854 of a dekaliter	1 dekaliter = 2.6417 liquid gal.
1 peck = .881 of a dekaliter	1 dekaliter = 1.1351 pecks
1 bushel = .35239 of a hektoliter	1 hektoliter = 2.8375 bu.

175. The **ar** (= 119.6034 square yards) is the metric standard for measuring land. It is a square each side of which is 10 meters, and is therefore equal to 100 square meters. It is pronounced "air."

Scale: 100 centars = 1 ar; 100 ars = 1 hektar.

176. The **square meter** (= 1.196 square yards) is the metric standard for measuring surfaces other than those of land. It is a square each side of which is 1 meter.

Scale: 100 sq. centimeters = 1 sq. decimeter; 100 sq. decimeters = 1 sq. meter; 100 sq. meters = 1 sq. dekameter.

NOTE. As in metric square measures 100 units of any order make one unit of the next higher order, it is necessary to give each order, except the highest, *two places* in expressing either land surfaces [175] or ordinary surfaces [176]. Thus, 19 sq. dekameters 5 sq. meters 49 sq. decimeters 9 sq. centimeters should be written 1905.4909 M.; and 2 hektars 7 ars 5 centars of land are written 207.05 ars.

EQUIVALENTS OF SQUARE MEASURE

(Common to Metric)	(Metric to Common)
1 sq. inch = 6.4516 sq. centimeters	1 sq. centimeter = .155 of a sq. in.
1 sq. foot = .0929 of a sq. meter	1 sq. decimeter = .1076 of a sq. ft.
1 sq. yard = .8361 of a sq. meter	1 sq. meter = 1.1960 sq. yd.
1 sq. rod = 25.293 sq. meters	1 ar = 3.954 sq. rods
1 acre = 40.469 ars	1 hektar = 2.47104 acres
1 sq. mile = 259 hektars	1 sq. kilometer = .3861 of a sq. mi.

177. The **ster** (= .2759 of a cord) is the metric standard for measuring wood. It is a cube each edge of which is 1 meter. It is pronounced "stair."

Scale: 10 decisters = 1 ster; 10 sters = 1 dekaster.

178. The **cubic meter** is the metric standard for measuring solids other than wood. Like the *Ster* [177], it is a cube each edge of which is 1 meter.

Scale: 1000 cubic millimeters = 1 cubic centimeter; 1000 cubic centimeters = 1 cubic decimeter; 1000 cubic decimeters = 1 cubic meter.

NOTE. As in metric cubic measure 1000 units of any order make 1 unit of the next higher order, it is necessary to allot *three places* to each order except the highest in expressing solids other than wood. In wood measure the scale is 10, and therefore only *one place* is required to express each order.

EQUIVALENTS OF CUBIC MEASURE

(Common to Metric)	(Metric to Common)
1 cu. inch = 16.387 cu. centimeters	1 cu. centimeter = .061 of a cu. in.
1 cu. foot = 28.317 cu. decimeters	1 cu. decimeter = .0353 of a cu. ft.
1 cu. yard = .7645 of a cu. meter	1 cu. meter = 1.308 cu. yards
1 cord = 3.624 sters	1 ster = .2759 of a cord

179. Reductions from one metric denomination to another are effected by simply placing a decimal point at the right of the *required denomination*, removing the original decimal point, and

making the corresponding change in the abbreviation to that of the required denomination.

Thus, to reduce to a higher metric denomination, remove the decimal point one place to the left if the scale of reduction is 10, two places to the left if the scale is 100, or three places to the left if the scale is 1000, for each intermediate denomination until the required higher denomination is reached, prefixing 0's if necessary; and to reduce to a lower metric denomination, remove the decimal point one, two, or three places to the right, according to the scale used, for each intermediate denomination until the required lower denomination is reached, annexing 0's if necessary.

NOTE 1. When a metric quantity is expressed in figures, *intermediate* vacant orders must also be expressed by writing as many 0's for each intermediate vacant order as there are ciphers in its scale.

NOTE 2. Metric denominations are abbreviated by writing the initial letter of the prefix followed by the initial letter of the standard unit; and by commencing the abbreviation with a capital letter when it expresses a higher denomination than the standard unit [**170**], and with a "small" letter when it expresses a lower denomination than the standard unit [**171**].

180. Metric quantities are added, subtracted, multiplied, and divided as ordinary decimals.

NOTE. Before adding or subtracting, it is necessary that the several metric quantities should be reduced to the same denomination [**179**], that is, that the decimal point of each should be *at the right of the same denomination*, thus throwing like denominations in the same column [Prin. 1, **18**].

EXAMPLES FOR PRACTISE

Reduce	Reduce
1. 4613.48 Sq. M. to sq. dm.	**8.** 268 Km. to miles.
2. 346834.57 dg. to Dg.	**9.** 56 Dl. of vinegar to gal.
3. 46.8173 Hg. to dg.	**10.** 475 Hl. of wheat to bush.
4. 567.8 ds. to S.	**11.** 468 gal. wine to Dl.
5. 196538.24 ca. to Ha.	**12.** 15.2 lb. Troy, to dg.
6. 28.246 Km. to mm.	**13.** 75 yd. cloth to M.
7. 341728.3 cl. to Dl.	**14.** 347.48 sq. yd. to Sq. M.

15. Add 468.75 M., 3.567 Dm., and 6178.35 cm.

16. Subtract 46.8 Hl. from 58722.348 dl.

17. Find the cost of 9348.56 Kg. of sugar at 11 cents per kilo.

18. Find *the cost of* 735.48 M. cloth at 5.37 francs per M.

COMPUTATION OF SURFACES

181. A surface is the outside or visible part of a solid body, without reference to its depth.

NOTE. A *plane surface* is one which is perfectly flat, having no undulations.

182. A rectangular surface is a plane surface bounded by four straight sides, each perpendicular to its adjacent side.

183. A square is a rectangular surface bounded by four *equal* sides.

Thus, a square foot is a rectangular surface each side of which is one foot; a square yard is a rectangular surface each side of which is one yard; etc.

184. The **area** of a rectangular surface is the amount of space included within its four sides, expressed in square units of any convenient denomination.

Thus, to find the area of a rectangular surface 1 yd. 1 ft. long and 1 yd. wide, a convenient measuring unit is one square foot. The area can then be found as follows:

FIG. 1

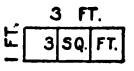


FIG. 2

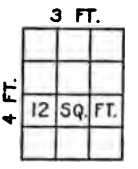


FIG. 3

EXPLANATION. Fig. 1 is the measuring unit, each side of which is 1 foot. Fig. 2 shows the number of these measuring squares in 1 row (3 sq. ft.). Fig. 3 shows the number of rows (4). Therefore, 4 times 3 sq. ft., or 12 sq. ft. is the area of a rectangular surface which is 1 yd. 1 ft. long, and 1 yd. wide.

THE LENGTH AND BREADTH BEING GIVEN TO FIND THE AREA

185. How many sq. yd. in a floor 18 ft. long and 14 ft. 6 in. wide?

SOLUTION

18 sq. ft. $\times$ $14\frac{1}{2}$ = 261 sq. ft.
261 $\div$ 9 = 29 sq. yd.

EXPLANATION. A floor 18 ft. long and only one foot wide contains 18 sq. ft.; therefore a floor 18 ft. long and $14\frac{1}{2}$ ft. wide must contain $14\frac{1}{2}$ times 18 sq. ft., or 261 sq. ft., which, when reduced to sq. yd. by 157, equal 29 sq. yd.

SUMMARY. (a) *The number of square measuring units in one row* (lying adjacent to one dimension) *is the multiplicand.* (b) *The number of rows* (as indicated by the number of the same square measuring units lying adjacent to the other dimension) *is the multiplier.* (c) *The area* (the number of square measuring units in all the rows) *is the product.*

NOTE. Before finding any required area, reduce the given dimensions to the same denomination if not already so expressed.

EXAMPLES FOR PRACTISE

Find the area of a surface which is

1. 32 ft. long and 24 ft. wide.
2. 56 yd. long and 18 ft. wide.
3. 7 yd. long and 2 ft. 6 in. wide.
4. 18 ft. 6 in. long and 12 ft. wide.
5. 34 yd. 2 ft. long and 18 yd. 1 ft. wide.
6. 35 rods long and 25 yd. 2 ft. wide.
7. 173 yd. 1 ft. long and 98 yd. 2 ft. wide.
8. 18 rd. 2 yd. long and 12 rd. 4 yd. 1 ft. wide.
9. 3 mi. 190 rd. long and 2 mi. 75 rd. wide.
10. 15 mi. long and 7 mi. 200 rd. wide.
11. 3 yd. 2 ft. 9 in. long and 2 yd. 1 ft. 6 in. wide.
12. 8 rd. 4 yd. long and 5 rd. 3 yd. 2 ft. wide.
13. How many square yards in a ceiling 28 ft. 9 in. long and 20 ft. 6 in. wide?
14. How many square yards in the walls of a room 30 ft. long, 20 ft. wide, and 8 ft. 6 in. high?
15. How many acres does a rectangular field contain which is 350 yd. 2 ft. long and 235 yd. 1 ft. 6 in. wide?
16. How many sq. ft. in the outer surface of a packing case which is 3 ft. 7 in. long, 2 ft. 9 in. wide, and 2 ft. 3 in. high?
17. How many sq. ft. of surface are in a blackboard 9 ft. 6 in. long and 4 ft. wide?
18. What is the cost of paving a cellar floor with cement at 35¢ per sq. yd., if the cellar is 30 ft. long and 18 ft. wide?
19. What is the cost of paving a sidewalk 18 ft. long and 12 ft. wide *at 25 cents per* square foot?

TO FIND ONE DIMENSION, WHEN THE AREA AND THE OTHER DIMENSION ARE GIVEN

ILLUSTRATIVE EXAMPLE

186. If a rectangular field contains 40 rods and is 22 yards wide, what is its length?

SOLUTION

40 rods = 1210 sq. yd., area

1210 ÷ 22 = 55 yd., length

EXPLANATION. The area (40 rods) is the given product [Summary, *c*, **185**]; and 22 (times the square yards adjoining the required width) is the given multiplier factor of that product [Summary, *b*, **185**]. Hence, complete the relation of these terms of multiplication by dividing the given product (40 rods, or 1210 sq. yd.) by its given multiplier factor (22), obtaining 55 sq. yd. (adjoining the required width) as its required multiplicand factor [Summary, *a*, **185**]. As only one *linear* yard of each of these 55 sq. yd. is in immediate contact with the required width, the width must be 55 linear yards.

NOTE. Before division, the given area should be made to express square units, and the given dimension linear units, of the *same* denomination as that of the required dimension.

EXAMPLES FOR PRACTISE

1. If the area of a floor is 35 sq. yd. 8 sq. ft., and its length is 6 yd. 1 ft., what is its breadth?

2. If a rectangular field contains 74 acres and is 880 yd. long, how many rods is its breadth?

3. If a blackboard contains 112 sq. ft. of surface and is 4 ft. wide, what is its length?

4. How high is a room 18 ft. long and 16 ft. wide, if its four walls contain 68 sq. yd.?

5. The area of a rectangular garden is 14 sq. rd. 8 sq. yd. 4 sq. ft. 72 sq. in., and its length 4 rd. 2 yd. What is its width?

CARPETING, PAVING, ROOFING

ILLUSTRATIVE EXAMPLE

187. Utilizing all the material, how many yards of carpeting 2 ft. 4 in. wide, are required to cover a floor 21 ft. long and 16 ft. *wide?*

SOLUTION

21 sq. ft. (in one row adjoining the length of the room) × 16 (the number of rows in the width of the room) = 336 sq. ft. (total floor area to be carpeted). 3 sq. ft. (in one row adjoining the length of 1 yd. of carpet) × 2⅓ (the number of such rows in the width of 1 yd. of carpet) = 7 sq. ft. (area of 1 yd. of carpet). Hence,

336 sq. ft. (area of floor) ÷ 7 sq. ft. (area of 1 yd. of carpet) = 48 times (1 yd. of carpet), or 48 yd. of carpet.

EXPLANATION. As many yards of carpet will be required as the obtained area of 1 yd. of the given carpet (7 sq. ft.) is contained times in the entire floor area to be carpeted (336 sq. ft.), or 48 times 1 yard.

SUMMARY. (a) *The floor area overlaid by one yard of the covering material is the multiplicand.* (b) *The total number of yards of the covering material is the multiplier.* (c) *The total floor area to be overlaid is the product.*

NOTE 1. To find the number of flagstones, bricks, etc., to overlay a given surface, divide the area of the surface to be overlaid by the *exposed* area of one unit of the overlaying material.

EXAMPLES FOR PRACTISE

Utilizing all the material, how many yards will carpet

1. A room 26 ft. long and 20 ft. wide; carpet 2 ft. 6 in. wide?
2. A room 19 ft. 3 in. long and 16 ft. wide; carpet 2 ft. 9 in. wide?
3. A room 27 ft. 8 in. long and 23 ft. 7 in. wide; carpet 1 yd. wide?
4. A room 37 ft. 4 in. long, 25 ft. 6 in. wide; carpet 2 ft. 4 in. wide?
5. What is the cost of laying matting 2 yd. wide on the floor of a room 24 ft. long and 18 ft. wide at 45¢ per yard?
6. How many bricks 8 in. long, 4 in. wide, and 2 in. thick, are necessary to cover a pavement 18 ft. long and 12 ft. wide, if the bricks are laid flatwise?
7. Laid edgewise, how many bricks are necessary to pave a stable entrance 14 ft. long and 12 ft. wide, if the bricks are 8 in. long, 4 in. wide, and 2 in. thick?

8. If placed endwise, how many bricks are needed to cover a sidewalk 32 ft. 6 in. long and 18 ft. 4 in. wide, if the bricks are 8 in. long, 4 in. wide, and 2 in. thick?

9. How many granite blocks 8 in. long and 8 in. wide are needed to cover a yard 18 ft. long and 15 ft. wide?

10. How many pieces of turf 1 ft. long and 9 in. wide are needed to sod a plat of ground 62 ft. 3 in. long and 43 ft. wide?

11. How many yards of carpet 2 ft. 6 in. wide are needed to cover the floor of a room 27 ft. long and 20 ft. wide, if the strips are laid crosswise?

SOLUTION

27 ft. $\div$ $2\frac{1}{2}$ ft. = $10\frac{4}{5}$ times 1 strip

or practically 11 strips.

20 ft. $\times$ 11 = 220 ft. = $73\frac{1}{3}$ yd.

EXPLANATION. As the strips are to be laid crosswise (*across* the length of the room), divide the length of the room (27 ft.) by the width of the carpet ($2\frac{1}{2}$ ft.) to find the number of required strips (practically 11 strips, the excess of the 11th strip, 6 in., being either cut off before laying or turned under while laying). As the length of each strip must be the same as the width of the room, multiply the length of each strip or multiplicand (20 ft.) by the number of strips or multiplier (11) to find the total number of yards of carpet needed (220 ft., or $73\frac{1}{3}$ yd.).

If there is no loss in matching figures, how many yards of carpet must be purchased to cover a floor

12. 26 ft. 3 in. long and 20 ft. 5 in. wide; carpet 1 yd. wide, laid crosswise?

13. 21 ft. 9 in. long and 18 ft. 8 in. wide; carpet 2 ft. 4 in. wide, laid lengthwise?

14. 18 ft. 6 in. long and 16 ft. 9 in. wide; carpet 2 ft. 6 in. wide, laid crosswise?

NOTE 2. In matching figures, an allowance should be made for the wastage caused by adding to the length of each following strip as much length of carpet as must be cut off to enable it to match the adjoining edge of the first strip. Thus, in Ex. 11, if 18 in. had been allowed for wastage in matching figures, the first strip would have been 20 ft. long and the 10 remaining strips 20 ft. + 18 in. long; and the length of carpet needed would have been 20 ft. + ($21\frac{1}{2}$ ft. $\times$ 10), or 235 ft., or $78\frac{1}{3}$ yd.

Allowing 10 inches per strip for matching figures, how many yards of carpet will be needed to cover a floor

15. 30 ft. 6 in. long and 26 ft. 3 in. wide; carpet 1 yd. wide, laid lengthwise?

16. 25 ft. 3 in. long and 18 ft. 6 in. wide; carpet 2 ft. 4 in. wide, laid crosswise?

COMPUTATION OF LUMBER

188. Lumber measure is used in measuring timber after it has been sawed into boards or planks, scantling, joists, and other building material.

189. The **board foot** is the standard by which lumber is measured. A *board foot* is 1 ft. long, 1 ft. wide and 1 in. thick; or 1 sq. ft. of the face of a board which is 1 in. thick.

Thus, a board 16 ft. long, 2 ft. wide, and 1 in. thick, is estimated to contain 16×2, or 32 board feet.

NOTE 1. If a board is more than 1 in. thick, the number of square feet in its face should be proportionately increased in estimating the number of board feet it contains. Thus, the number of board feet in a plank 12 ft. long, 2 ft. wide, and $1\frac{1}{4}$ in. thick = $1\frac{1}{4}$ times 12×2, or 30 bd. ft.; if $1\frac{1}{2}$ in. thick = $1\frac{1}{2}$ times 12×2, or 36 bd. ft.; if 2 in. thick = 2 times 12×2, or 48 bd. ft.

NOTE 2. Hewn timber is sold by board measure, or by cubic measure, and sometimes by the "running foot." Even when sold by the running foot, its width and thickness are the governing factors in determining the price per running foot.

NOTE 3. The average width of a *tapering* board is ascertained by measuring midway between its two ends, or by taking half the sum of the widths of its two ends.

NOTE 4. In measuring two or more boards of uniform length and thickness, it is more convenient to multiply in one operation the uniform length by the *total width* of all the boards to obtain the *total board* feet.

ILLUSTRATIVE EXAMPLE

How many bd. ft. in a plank 18 ft. long, 6 in. wide, and $1\frac{1}{2}$ in. thick?

SOLUTION

18 bd. ft. (if 1 ft. wide and 1 in. thick) $\times \frac{1}{2}$ (of 1 ft. wide) = 9 bd. ft. 9 bd. ft. (if 1 in. thick) $\times 1\frac{1}{2}$ (actual thickness in inches) = $13\frac{1}{2}$ bd. ft.

EXPLANATION. A plank 18 ft. long, 1 ft. wide, and 1 in. thick contains 18 bd. ft.; and if only $\frac{1}{2}$ of a foot (6 in.) wide, contains $\frac{1}{2}$ of 18 bd. ft., or 9 bd. ft.; and if $1\frac{1}{2}$ in. thick, contains $1\frac{1}{2}$ times 9 bd. ft., or $13\frac{1}{2}$ bd. ft.

NOTE. 5. It is sometimes more convenient to multiply the length in feet by the width in *inches* (or 12 times the multiplier) to find the number of board feet in a 1-in. plank; and in correction to divide the resulting product by 12.

EXAMPLES FOR PRACTISE

Find the number of board feet in a plank which is

	Long	Wide	Thick		Long	Wide	Thick
1.	16 ft.	9 in.	1 in.	**5.**	18 ft. 6 in.	6 in.	2 in.
2.	18 ft.	7 in.	1 in.	**6.**	15 ft. 9 in.	7 in.	$2\frac{1}{4}$ in.
3.	14 ft.	6 in.	$1\frac{1}{2}$ in.	**7.**	22 ft.	5 in.	$1\frac{1}{4}$ in.
4.	20 ft.	8 in.	$1\frac{3}{4}$ in.	**8.**	24 ft.	1 ft. 3 in.	2 in.

9. How many board feet in 5 planks, each 16 ft. long and $1\frac{1}{2}$ in. thick, if their respective widths are 6 in., 7 in., 9 in., 10 in., and 11 in.? [Note 4]

10. How many board feet in a plank 18 ft. long, $1\frac{3}{4}$ in. thick, and 8 in. wide at one end and 6 in. wide at the other? [Note 3]

11. What is the cost of an invoice of 2-in. planks 18 ft. long, at \$24.75 per thousand feet, if their united width is 45 ft. 7 in.?

12. What is the cost of 24 beams each 26 ft. long, 10 in. wide, and 3 in. thick, at \$21.50 per thousand feet?

COMPUTATION OF VOLUMES

190. By **volume** is meant the contents included within the bounding surfaces of a given amount of space.

NOTE 1. A *rectangular volume* is one which is bounded by six rectangular surfaces [182]. In addition to the *length* and *breadth* of rectangular surfaces, rectangular volumes have a third dimension, *thickness*.

NOTE 2. A *cube* is a rectangular volume bounded by six *equal* squares, called *faces*. All the edges of a cube are of equal length. Thus, a cubic inch is a rectangular volume each edge of which is one linear inch, and each of the six faces of which is one square inch; and a cubic foot is a rectangular volume each edge of which is one linear foot, and each face of which is one square foot.

191. The **contents** or **volume** of a rectangular solid is the amount of space included within its six faces, expressed in cubic units of any convenient denomination.

Thus, to find the contents of a rectangular body which is 5 ft. long, 4 ft. wide, and 3 ft. deep, take any convenient measuring unit, say, one cubic foot (Fig. 1) as the standard. The contents can then be found as follows:

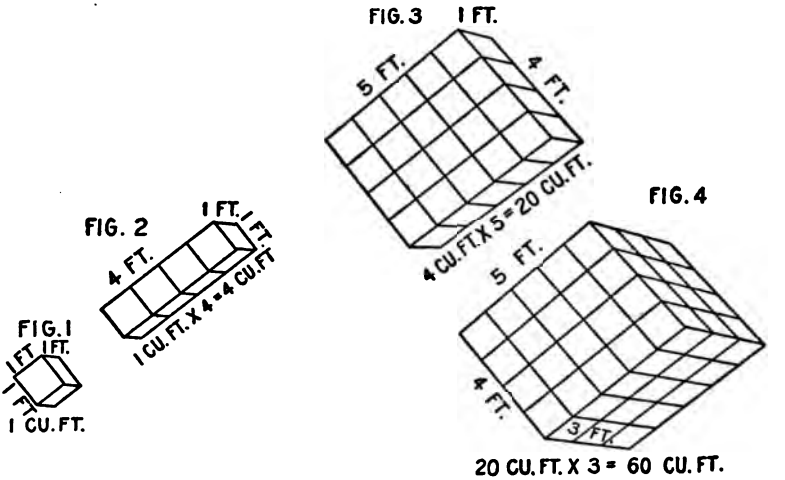


Fig. 1 is the measuring unit, which is 1 ft. long, 1 ft. wide, and 1 ft. deep, or 1 cu. ft.

Fig. 2 shows the number of these cubic measuring units in one row of the upper layer of Fig. 4 (4 cu. ft.).

Fig. 3 shows the number of these cubic measuring units in all the five rows of the upper layer of Fig. 4 (4 cu. ft. $\times$ 5 = 20 cu. ft.).

Fig. 4 shows the number of these cubic measuring units in all three of the layers of the given body (20 cu. ft. $\times$ 3 = 60 cu. ft.).

Hence, a rectangular body 5 ft. long, 4 ft. wide, and 3 ft. deep, contains 5 cu. ft. $\times$ 4 $\times$ 3, or 60 cu. ft.

SUMMARY. (a) *The number of cubic measuring units in one layer* (adjoining *one face* of a rectangular body) *is the multiplicand.* (b) *The number of such layers is the multiplier.* (c) *The number of the cubic measuring units in all the layers* (the *contents* of the rectangular body) *is the product.*

NOTE 1. Before finding the contents of a rectangular body, its length, breadth, and thickness should be reduced to the *same* linear denomination, if not already *so.*

ILLUSTRATIVE EXAMPLE

How many cubic feet of earth are in a rectangular embankment 16 ft. 6 in. long, 10 ft. 9 in. wide, and 6 ft. high?

SOLUTION

$$16\tfrac{1}{2} \text{ cu. ft.} \times 10\tfrac{3}{4} \times 6 = 1064\tfrac{1}{4} \text{ cu. ft.}$$

EXPLANATION. The number of cu. ft. in the row adjoining the *long edge* of the *lower section* of the embankment ($16\frac{1}{2}$ cu. ft.) multiplied by the number of such rows in the lower section ($10\frac{3}{4}$) will produce the number of cu. ft. in all the rows of the lower section ($177\frac{3}{4}$ cu. ft.); and the number of cu. ft. in all the rows of the lower section of 1 ft. in height ($177\frac{3}{4}$) multiplied by the number of such sections in 6 ft. of height (6) will produce the number of cu. ft. in all the sections, or in the entire embankment ($1064\frac{1}{4}$ cu. ft.).

EXAMPLES FOR PRACTISE

1. How many cubic feet of wood are in a pile 28 ft. long, 6 ft. high, and 4 ft. wide?

2. How many cubic yards of air are in a room 16 ft. long, 12 ft. wide, and 10 ft. high?

3. How many cubic feet of wood are in a squared log 18 ft. long, 9 in. wide, and 9 in. thick?

4. How many cubic yards of stone are in a rectangular pile which is 20 ft. long, 10 ft. wide, and 5 ft. high?

5. What is the cost of digging a rectangular excavation 24 ft. long, 18 ft. wide, and 12 ft. deep, at 45¢ per cu. yd.?

6. What is the cost of a pile of cord wood which is 35 ft. long, 6 ft. high, and 3 ft. wide at $7.15 per cord?

7. A cellar is 18 ft. long and 16 ft. wide, and contains 96 cu. yd. What is its depth?

SOLUTION

$$18 \text{ cu. ft.} \times 16 = 288 \text{ cu. ft.} \quad [\text{Summary, } a.]$$
$$27 \text{ cu. ft.} \times 96 = 2592 \text{ cu. ft.} \quad [\text{Summary, } c.]$$
$$2592 \text{ cu. ft.} \div 288 \text{ cu. ft.} = 9 \text{ times (1 ft. deep).}$$

EXPLANATION. A cellar which is 18 ft. long, 16 ft. wide, and 1 ft. deep, contains 18 cu. ft. × 16 × 1, or 288 cu. ft. Therefore a cellar 18 ft. long and 16 ft. wide to contain 96 cu. yd. or 2592 cu. ft. must be as many times 1 ft. deep *as 288 cu. ft. are contained* times in 2592 cu. ft., that is, 9 times 1 ft. deep.

NOTE. 2. If necessary, reduce the given dimensions to linear units, and the given volume to cubic units, of the *same* denomination, before finding the required dimension [Prin. 1, **34**].

8. A room 48 ft. long and 32 ft. wide contains 23040 cu. ft. What is its height?

9. A contractor dug a cellar 36 ft. long and 12 ft. deep, and removed 336 cu. yd. of earth. What was its width?

10. A purchaser paid $59.50 for a pile of cord wood 28 ft. long and 4 ft. wide, bought at $8.50 per cord. How high was the pile?

11. A wagon-body 12 ft. long and 4 ft. 4 in. wide contains 78 cu. ft. What is its height?

12. How long must a rectangular embankment be to contain 35 cu. yd. 5 cu. ft., if it is 6 ft. 3 in. wide, and 9 ft. 6 in. high?

COMPUTATION OF CAPACITIES

192. By **capacity** is meant the amount of space within a containing vessel which enables it to hold a given quantity of water, grain, fruit, vegetables, coal, etc.

NOTE 1. In finding the amount of any given quantity of grains, seeds, or berries, *stricken measure* is used. By stricken measure is meant the measuring vessel *even* full and stricken off with a rule or striker. A stricken bushel contains 2150.42 cu. in.

NOTE 2. In finding the amount of any given quantity of coal, lime, unshelled corn, vegetables, or the larger fruits, *heaped measure* is used. Heaped measure means the measuring vessel heaped in the form of a cone. A heaped bushel contains 2747.71 cu. in. 4 heaped bushels are usually considered equivalent to 5 stricken bushels.

NOTE 3. A gallon, liquid measure, contains 231 cu. in.; and a gallon, dry measure (¼pk.), 268.8 cu. in.

EXAMPLES FOR PRACTISE

1. A rectangular bin is 6 ft. long, 5 ft. wide, and 4 ft. deep. How many bushels of grain will it hold?

SOLUTION

6 cu. ft. × 5 × 4 = 120 cu. ft.

1728 cu. in. × 120 = 207360 cu. in.

207360 cu. in. ÷ 2150.42 cu. in. = 96.42 + times 1 bushel.

EXPLANATION. As grain is measured by stricken bushels [Note 1] divide the capacity of the bin (207360 cu. in.) by the contents of 1 stricken bushel (2150.42 cu. in.) to find how many times 1 bu. of grain the bin will hold.

2. A rectangular bin is 8 ft. long, 6 ft. wide, and 3 ft. 9 in. deep. How many bushels of potatoes can it hold? [Note 2].

3. How many bushels of oats can be stored in a rectangular bin 10 ft. 3 in. long, 4 ft. wide, and 3 ft. 6 in. deep?

4. A cart-body is 7 ft. long, 4 ft. 3 in. wide, and 2 ft. 9 in. deep. How many bushels of shelled corn can it hold?

5. A wagon-body is 10 ft. long, 4 ft. 2 in. wide, and 1 ft. 8 in. deep. How many bushels of apples can it hold?

6. How many gallons of water can a rectangular cistern hold if it is 6 ft. 9 in. long, 4 ft. 6 in. wide, and 10 ft. deep.

SUGGESTION. Capacity of cistern in cu. in. ÷ capacity of 1 liquid gallon in cu. in. [Note 3] = capacity of cistern in liquid gallons.

7. How many gallons of water will fill a rectangular cistern 12 ft. long, 12 ft. wide, and 4 ft. 3 in. deep?

8. How many gallons of water can be placed in a tank which is capable of holding 35 bushels of rye?

9. How many bushels of potatoes can be stored in a vessel capable of holding 265 gallons of water?

10. A cellar 30 ft. long, 18 ft. wide, and measuring 8 ft. from the street level to the bottom of the cellar, was filled by an overflow from a neighboring stream. How many gallons of water did it contain?

11. What must be the depth of a bin 8 ft. long and 6 ft. wide, to hold 96 bushels of oats?

SUGGESTION. Capacity of 1 bu. (2150.42 cu. in.) × number of bu. (96) = capacity of the bin. Then find required dimension as in solution of Ex. 7, **191.**

12. What must be the width of a wagon-body 10 ft. long and 2 ft. deep, to hold 53 bu. of turnips?

13. What must be the depth of a rectangular cistern 6 ft. 4 in. long and 5 ft. 2 in. wide, to hold 2500 gallons of water?

NOTE 4. A sufficiently close approximation to meet ordinary requirements may be obtained by multiplying the capacity of the containing vessel in cu. ft. by .8 to find its capacity in stricken bushels, or by .63 to find its capacity in heaped bushels.

14. *A farmer* has a rectangular bin 10 ft. long, 4 ft. wide, and

4 ft. deep. What is the approximate number of bushels of wheat which it can hold?

15. A man had a rectangular bin built in his cellar to hold coal. If the bin was 10 ft. long, 6 ft. wide, and 4 ft. deep, what approximate weight of coal will it hold, allowing 80 pounds to a bushel and 2240 pounds to a ton?

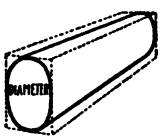
16. Allowing 28 bu. to a long ton of 2240 lb., how long must be a rectangular coal bin 16 ft. wide and 4 ft. deep to hold, approximately, 6 tons of coal?

17. A rectangular bin is 12 ft. long, 8 ft. wide, and 4 ft. deep. What approximate number of bushels of potatoes can it hold?

NOTE 5. If a crib or bin has uniformly sloping sides, its average width is one-half the sum of its top and bottom widths [Note 3, **189**]. Thus, the contents of a granary 18 ft. long, 12 ft. high, 10 ft. wide at the bottom and 14 ft. wide at the top, are 18 cu. ft. $\times$ 12 $\times$ $\frac{1}{2}$ of (10 + 14), or 2592 cu. ft.

18. How many bushels of shelled corn can be stored in a crib which is 12 ft. long, 8 ft. deep, 6 ft. wide at the bottom and 10 ft. wide at the top, using the standard of Note 1?

19. How many bushels of turnips can be stored in a bin 8 ft. long, 6 ft. deep, 4 ft. wide at the bottom and 6 ft. wide at the top, approximating by Note 4?



A CYLINDER

NOTE 6. A containing vessel is said to be *cylindrical* if it is circular, is of uniform diameter throughout its depth, and its ends are parallel. The capacity of a cylindrical receptacle is .7854 of that of a rectangular vessel of the same depth, and having each of its two other dimensions equal to the diameter of the cylindrical receptacle.

20. A cylindrical cistern is 12 ft. deep and 4 ft. in diameter. How many gallons of water can it hold?

SOLUTION

12 cu. ft. $\times$ 4 $\times$ 4 = 192 cu. ft., capacity of a rectangular cistern of equal depth (12 ft.) and a length (4 ft.) and a width (4 ft.) equal to the diameter of the cylindrical cistern. Hence, 192 cu. ft. $\times$ .7854 = 150.7968 cu. ft., capacity of the given cylindrical cistern.

(1728 cu. in. $\times$ 150.7968) $\div$ 231 cu. in. = 1128.0384 times 1 gal. [Note 3].

21. A cylindrical cistern is 6 ft. in diameter and 14 ft. in depth. How many *gallons* of water is it capable of holding?

22. A well is 4 ft. in diameter and the water in it is 6 ft. deep. How many gallons of water are in the well?

23. A cylindrical bin which is 7 ft. high and 3 ft. 6 in. in diameter is filled with wheat. How many bushels does it contain?

24. A circular railroad tank is 8 ft. in diameter and contains 3500 gallons of water. How deep is the water?

NOTE 7. A receptacle whose sides are uniformly round but *tapering*, and whose ends are parallel but *unequal* circles, is equivalent to a cylindrical receptacle of equal depth and of a uniform diameter of one-half the sum of its two extreme diameters. Thus, such a receptacle if 8 ft. in depth, 6 ft. in diameter at one extreme and 4 ft. in diameter at the other extreme, is equal to a cylindrical vessel which is 8 ft. in depth and has a uniform diameter of $(6 + 4) \div 2$, or 5 ft.

25. How many cubic feet in a log 20 ft. long, 18 in. in diameter at one end, and 12 in. in diameter at the other end?

26. How many cu. in. in a bucket which is 2 ft. in depth, 12 in. in diameter at the bottom, and 16 in. in diameter at the top?

27. How many gallons of water will a cistern hold which is 18 ft. in depth, 10 ft. in diameter at the bottom, and 6 ft. in diameter at the top?

NOTE 8. Owing to the lack of uniformity in the bulge of a barrel or cask, its average diameter is approximately found by increasing its head diameter by $\frac{4}{5}$ of the difference between its bung diameter and its head diameter if its staves are much curved, or by $\frac{2}{3}$ of such difference if its staves are slightly curved. Thus, the capacity of a barrel whose length is 36 in., bung diameter 30 in., and head diameter 24 in. will approximately equal that of a cylinder 36 in. long and a uniform diameter of 24 in. $+ \frac{4}{5}$ of (30 in. $-$ 24 in.), or 28 in.

28. How many gallons of cider will a barrel hold which is 40 in. long, bung diameter 32 in., and head diameter 27 in?

29. What is the capacity in gallons of a cask 3 ft. 4 in. long, head diameter 24 in., bung diameter 30 in.?

30. What is the capacity in gallons of a barrel 36 in. long, bung diameter 28 in., head diameter 21 in.?

PERCENTAGE

193. INTRODUCTION. All possible arithmetical terms [Note, 5] in all their various relations to each other, and the means for completing those relations, have now been considered [**48-53** for integers, and **107-109** for fractions]. The successive subjects from **109** to the end of this volume are simply applications of these comprehensive relationships to *new topics* which are of sufficient importance to justify special treatment.

In **109**, the comparison of quantities was made without reference to any particular numerical form in which the result of the comparison was to be expressed. In percentage, however, the result of the comparison is limited in expression to a particular form, *that of hundredths.* The title "percentage" suggests this limitation, being derived from *per*, by; *centum*, hundred; *age*, that which. Hence, *percentage* is the name of that specific class of calculations, whether in addition, subtraction, multiplication, or division, *which is by hundredths.* Thus, to add .03 to .06, to subtract .03 from .06, to multiply 6 by .03, or to divide 6 by .03, may properly be called operations in percentage. The name percentage, however, is ordinarily restricted to problems which involve *comparisons* of quantities the results of which are expressed *in hundredths*, as in the following: \$5 are how many hundredths of \$25? 20 hundredths of how many dollars equal \$5? 20 hundredths of \$25 equal how many dollars?

SUGGESTION. The learner is referred to *a*, *b* and *c* of INTRODUCTION, **109**, for a full exposition of the general treatment of *all* numerical comparisons, which will serve as a suitable preparation for considering the special application of the same general principles to particular comparisons *by hundredths.*

194. Operations in percentage embrace all calculations which involve a specific comparison by hundredths.

195. The **base** is the quantity taken as the standard with which some other quantity is to be compared.

NOTE. Any required comparison of a quantity must include a definite basis or standard, expressed or understood, with which the comparison is to be made. Thus, a given quantity, say \$5, may be *equal to*, or *more than*, or

less than, according to the standard with which it may be compared. That is, \$5 will be *equal to*, if compared with \$5; it will be *more than*, if compared with \$4; or it will be *less than*, if compared with \$6.

196. The **rate per cent.** denotes what comparative part of the base, expressed in hundredths, is the quantity which is contrasted with the base.

NOTE 1. The term *rate* when not followed by the phrase *per cent.* or by the symbol (%) which means "per cent." may signify *any part* of the standard or base. Thus, in the expressions 4 times \$600, $\frac{2}{3}$ of \$600, .005 of \$600, and .08 of \$600, 4, $\frac{2}{3}$, .005, and .08 are rates; the first (4) being an integral rate per unit, the second ($\frac{2}{3}$) a common fractional rate *per third*, the third (.005) a decimal rate *per thousandth*, and only the fourth (.08) a rate *per hundredth* or *per cent.*

NOTE 2. To make a comparison of one quantity with another taken as the standard, some graduated measuring unit is necessary. In percentage, the accepted measuring unit is 1, which represents the entire standard of comparison and which for convenience is conceived to be graduated or scaled in hundredths. Thus, in comparing \$2 with \$5, the measuring standard or base is \$5, represented by the measuring unit 1 (meaning 1 time \$5, or the entire \$5); and this measuring unit 1 is conceived to be divided or scaled into 100 equal parts. If \$2 are compared to the standard (\$5) by means of this graduated scale, it will be found to measure up to the *fortieth* of its one hundred graduations and therefore \$2 are said to be 40 hundredths or 40 % of the standard (\$5).

197. The **percentage** is the quantity which is compared with the base.

NOTE. A percentage expresses the value of a given or required number of hundredths of a certain quantity taken as the base. Thus, in the consummated comparison "\$200 are 25 % of \$800," \$800 is the indicated base or standard of comparison; \$200 is the percentage or quantity compared with the standard; and 25 % is the rate % or result of the comparison, denoting what part \$200 are of \$800, when expressed in hundredths.

198. Identification of terms. In certain classes of problems, particularly in "applied percentage," there is a uniformly accepted base, and it can therefore be "understood." In an important class of percentage problems, however, there is no uniformly recognized standard, and any base may be arbitrarily assumed, as the author of the problem may direct. All bases which are not understood must necessarily be indicated. (*a*) The conventional form for indicating arbitrary bases is the expression "of" (mean-

ing *times*) placed immediately *after* the rate % and immediately *before* the base. (*b*) It is sometimes necessary to indicate the base with the purpose of having it increased [**109**, *b*,] or decreased [**109**, *c*] by a given or required percentage, in which case the phrases "more than" (implying increase) or "less than" (implying decrease), are substituted immediately after the rate % and before the indicated base. (*c*) After the base has been identified, any quantity which is placed in comparison with the indicated base with the view of ascertaining how many hundredths of the given base it is, or of what required base it is a given number of hundredths, is called a percentage. Thus, in the consummated comparisons, "\$20 are 25% of \$80," "\$20 are 75% less than \$80," the indicated base is \$80, and the percentage \$20; and in the consummated comparisons, "\$30 are 120% of \$25," "\$30 are 20% more than \$25," the indicated base is \$25, and the percentage \$30.

199. Relation of terms. (*a*) The *base* is a *multiplicand* because it expresses the value of *one* quantity taken as the standard measuring unit; (*b*) the *rate* % is a *multiplier* because it expresses the *decimal number* (hundredths) of standard measuring units; (*c*) the *percentage* is a *product*, because it expresses the value of *all* the considered decimal number (hundredths) of standard measuring units [Prin. 1, 2 and 3, **107**].

Hence, (*d*), if the *base* (multiplicand) and the *rate* % (multiplier) are given, multiply these factors to find the *percentage* (product); (*e*) if the *base* (multiplicand) and the *percentage* (product) are given, divide the given product by its given multiplicand factor to find its required multiplier factor or *rate;* (*f*) if the *rate* (multiplier) and the *percentage* (product) are given, divide the given product by its given multiplier factor to find its required multiplicand factor or *base*.

NOTE. Though all rates are multipliers and all percentages are products, learners are cautioned against dividing *any* product by *any* multiplier to find a *particular* multiplicand. A product must be divided by *its* multiplier factor, that is, by the particular multiplier which was used to produce it, in order to obtain *its* multiplicand factor, that is, the particular multiplicand which was used to produce it.

200. Convenient common fractional rates should be substituted for inconvenient rates %, particularly for complex rates %

[117]. The rates % in common use which can be advantageously changed to equivalent common fractional rates are shown in the following

TABLE OF EQUIVALENTS:

$1\frac{1}{4}\% = \frac{1}{80}$	$10\% = \frac{1}{10}$	$25\% = \frac{1}{4}$	$62\frac{1}{2}\% = \frac{5}{8}$
$2\frac{1}{2}\% = \frac{1}{40}$	$12\frac{1}{2}\% = \frac{1}{8}$	$33\frac{1}{3}\% = \frac{1}{3}$	$66\frac{2}{3}\% = \frac{2}{3}$
$5\% = \frac{1}{20}$	$16\frac{2}{3}\% = \frac{1}{6}$	$37\frac{1}{2}\% = \frac{3}{8}$	$75\% = \frac{3}{4}$
$6\frac{1}{4}\% = \frac{1}{16}$	$20\% = \frac{1}{5}$	$50\% = \frac{1}{2}$	$87\frac{1}{2}\% = \frac{7}{8}$

NOTE. The tendency to change indiscriminately all rates %, whether convenient or inconvenient, to the form of common fractions is unbusinesslike.

BASE AND RATE GIVEN, TO FIND PERCENTAGE

ILLUSTRATIVE EXAMPLES

201. 1. Find 9 % of $385.42.

SOLUTION

$385.42
.09
34.6878

EXPLANATION. $385.42 is the indicated *base* or multiplicand, being thus arbitrarily shown by the conventional indicator "of" [198, *a*]. 9 % is the rate or multiplier, because it denotes the decimal number of times the base which is to be expressed by the required percentage, or product. Hence, multiply these two given factors of the required percentage, obtaining $34.69−.

2. A man's salary is $875 per annum. If his total expenses are 65 % of his salary, how much does he save?

SOLUTION

100 %, rate % of salary
−65 %, rate % of expenses,
35 %, rate % saved.

$875, indicated base.
× .35, rate % saved.
4375
2625
$306.25, percentage saved.

EXPLANATION. "His salary" is arbitrarily indicated as the *verbal* base by being placed after " % of," and $875 is the numerical expression of this verbal base and therefore the numerical base or multiplicand.

65 % is the rate or multiplier of the base to produce the percentage or product of *total expenses*, but not the rate or multiplier of the base to produce *the required percentage* or product *saved*. If 100 % is the rate of his salary

[Note 2, **196**] and 65 % is the rate of his total expenses, then 100 % − 65 % or 35 %, must be the specific rate or multiplier of the base ($875) to produce the percentage or product saved ($306.25).

3. On a certain farm were harvested 425 bushels of wheat, and 48% more oats than wheat. How many bushels of oats were harvested?

SOLUTION

100 %, rate % of wheat.
48 %, rate % of excess of oats.
148 %, rate % of oats harvested.

425 bu., indicated base.
1.48 rate % of required oats.
3400
1700
425
629.00 bu., percentage of oats.

EXPLANATION. The bushels of wheat harvested is the arbitrarily indicated *verbal* base, because it follows " % more than" [**198**, *b*.] 425 bu. is the numerical expression of the bushels of wheat harvested and therefore the numerical base or multiplicand. 48 % is the expressed rate or multiplier for the excess of the yield of oats over that of the wheat, but not the specific rate or multiplier for the required number of bushels of oats harvested. Expressing the indicated base (425 bu.) upon a scale of hundredths as shown in Note 2, **196** (100 %), the bu. of oats harvested expressed upon the same scale will be 48 % more than 100 %, or 148 %. Hence, multiply the indicated base or multiplicand (425 bu.) by the specific rate % or multiplier of the oats harvested (1.48), obtaining 629 bu. as the specific percentage or product of the oats harvested.

RULE. *To find any desired percentage, multiply the base by the specific rate % of the desired percentage.*

NOTE 1. Any given rate should be identified not only generally as *a* rate or multiplier, but also specifically as *the* rate or multiplier of the indicated base to produce a particular percentage or product. Thus, in Ill. Ex. 2, 100 % is the specific rate of *his salary*, 65 % is the specific rate of *his expenses*, and 35 % is the specific rate of *what is saved*. In Ill. Ex. 3, 100 % is the specific rate of the *bu. of wheat harvested*, 48 % is the rate of *so much of the bu. of oats harvested as is more than the bu. of wheat harvested*, and 148 % is the rate of *the oats harvested*.

NOTE 2. Two or more rates must be expressed in terms of (upon the same graduated scale as) the indicated base before they can be added [Prin. 1, **18**] or subtracted [Prin. 1, **22**].

EXAMPLES FOR PRACTISE

1. .07 times \$800 = how much? **2.** 8 % of \$300 (that is, .08 times \$300) = what percentage (that is, what product by hundredths)? **3.** 28 % of 650 lb. = how many pounds? **4.** 84 % of 850 men = how many men? **5.** $66\frac{2}{3}$ % of 471 sheep = how many sheep? **6.** 125 % of 584 gal. = how many gallons? **7.** $\frac{1}{8}$ % of \$6400 = how much? **8.** $\frac{3}{4}$ % of 800 tons = how many tons? **9.** 28 % of £618 2s. 6d. = what percentage? **10.** Find 38 % more than 650 gallons. **11.** Find 25 % more than 716 sheep. **12.** Find 50 % more than 128 horses. **13.** Find 20 % less than 315 days. **14.** Find $33\frac{1}{3}$ % less than 714 miles. **15.** Find $16\frac{2}{3}$ % less than 468 yards.

16. A grocer purchased 625 pounds of coffee and sold 64 % of his purchase. How many pounds of coffee did he sell?

17. A ranchman had 475 cattle, but lost 28 % of them in a blizzard. How many cattle did he save?

18. By installing new machinery, a factory was enabled to produce 38 % more than at first. If its former capacity was 6500 yards of cloth per week, what is its present capacity?

19. A man's taxable property is assessed at \$28500, 43 % of his assessment being upon his real estate, and the remainder upon his personal property. What is the assessed value of each?

20. In a certain graded school 375 pupils are enrolled, 28 % of the pupils being in the primary department, and the remainder in the intermediate. How many pupils were enrolled in the intermediate department?

21. By the census of 1910, it was found that the population of a certain city was 43 % greater than by the census of 1900. If its population in 1900 was 46800, what was it in 1910?

22. A farm contains 700 acres, 25 % of it being sown in wheat, 32 % being planted in corn, and the remainder set apart for pasturage. How many acres of pasture land does the farm contain?

23. A man died, leaving an estate worth \$128500, 54 % of *his estate having* been bequeathed to his wife, 32 % to his

children, and the remainder to an asylum. If the cost of administering upon the estate was 5% of its value, how much did the asylum receive?

BASE AND PERCENTAGE GIVEN, TO FIND RATE

ILLUSTRATIVE EXAMPLES

202. 1. What % of \$428 are \$184.04?

SOLUTION

\$184.04 ÷ \$428 = .43 = 43%

EXPLANATION. \$428 is the arbitrarily indicated base or multiplicand, being placed immediately after " % of" (meaning hundredths times). \$184.04 is the percentage or product, it being the quantity which is placed in comparison with the indicated base to find what part of the base it is when expressed in hundredths [197]. If stated in more familiar language, the problem would read, "How many hundredths times \$428 are \$184.04." Hence, divide the given percentage or product (\$184.04) by its given base or multiplicand factor (\$428) to find its required rate or multiplier factor, carrying the division to hundredths (by so arranging the percentage that it will have *two more decimal places* than those of the base), obtaining .43 or 43 %.

2. A has \$350 and B. \$448. What % more than A has B?

SOLUTION

\$448 − \$350 = \$98, percentage more.
\$98.00 ÷ \$350 = .28, or 28% (more).

EXPLANATION. A's money is the indicated verbal base or multiplicand, as it is placed immediately after " % more than" [198, *b*], and \$350 is the numerical expression of A's money or the numerical base. \$448 is a percentage or product, it being the quantity which is placed in comparison with the indicated base to find what % more than the base it is. Though B's money is *a* percentage, it is not *the* percentage or product of which it is required to find the rate or multiplier. Therefore, subtract A's money (\$350) from B's money (\$448) to find the *percentage more* (\$98), or the specific percentage or product of which it is required to find the specific rate % or multiplier. Then divide the percentage or product more (\$98) by its indicated base, or multiplicand factor (\$350), to find its required rate or multiplier factor, carrying the division to hundredths as in Ill. Ex. 1, to obtain the rate in hundredths (28 %).

RULE. *Divide the specific percentage or product of which it is required to find the rate, by its* base or multiplicand factor,

and the quotient carried to hundredths will be its required rate % or multiplier factor.

NOTE 1. Percentages should be identified so specifically as to distinguish each percentage from all other percentages by naming its individual characteristic. Thus, in Ill. Ex. 2, $448 and $98 are not only to be regarded as percentages generally, but $448 as *the* specific percentage of B's money, and $98 as the specific percentage *more* that B's money is than A's.

NOTE 2. To obtain a rate in the particular form of a %, the percentage should contain *exactly* two decimal places more than the base. If the percentage contains fewer than the required number of decimal places, supply the deficiency by annexing decimal 0's; and if the percentage contains more than the required number of decimal places, offset the excess by annexing decimal 0's to the base.

NOTE 3. To find the rate of a given percentage means simply to find *what multiplier of the indicated base will produce the given percentage.*

EXAMPLES FOR PRACTISE

1. $30 are how many times $6? **2.** $20 are how many hundredths times $80? **3.** $40 are what % of (that is, how many hundredths times) $120? **4.** $23.29 are what % of $68.50? **5.** $88.69 are what % of $253.40? **6.** What % of $872.60 are $741.71? **7.** What % of $5249.50 are $3254.69? **8.** What % of $528 are $1.32? **9.** What % more than $68.40 are $85.50? **10.** $322.02 are what % more than $268.35? **11.** $275.17 are what % less than $786.20? **12.** What % less than $687.50 are $522.50?

13. A grazier owned 350 sheep, and wolves destroyed 14 of them. What % of his flock did he lose?

14. A clerk's salary is $1200 per annum. If he pays $20 per month for board and room rent, $156 per annum for clothing, and $300 per annum for other expenses, what % of his salary does he save?

15. A grocer sold 1653 lb. sugar and had 522 lb. remaining. What % of his original stock of sugar did he sell?

16. A son inherited 920 acres from his father, and afterwards sold 138 acres. What % of his inheritance did he have remaining?

17. A man owns real estate valued at $23223.75 and personal property valued at $17650.05. What % less than the value of *his real estate is the* value of his personal property?

18. Three men formed a copartnership, A investing \$11900, B \$14875, and C \$15725. What % of the capital of the firm did each invest?

19. The sailing distance from one seaport to another is 8750 miles. What % of the voyage remained after a vessel had sailed 21 days at an average speed of 150 miles per day?

20. A merchant engaged in business with a capital of \$42500 and at the end of the first year found that he had gained \$7650. What % of his original capital did he gain?

21. A man borrowed \$4500 and made three partial payments thereon of \$1820, \$1230, and \$415. What % of his original debt remained unpaid?

PERCENTAGE AND RATE GIVEN, TO FIND BASE

ILLUSTRATIVE EXAMPLES

203. 1. A man bought a house and paid 45 % of the purchase money in cash. At what price did he buy the house, if his cash payment was \$3825?

SOLUTION

\$3825.00 ÷ .45 = \$8500

EXPLANATION. The purchase money is indicated as the verbal base or multiplicand by being placed immediately after " % of." \$3825 is a percentage or product because it is placed in comparison with the indicated verbal base (the purchase money). 45 % is the specific rate or multiplier for \$3825 because it denotes the result of the comparison of \$3825 with the indicated base (\$3825 being .45 of the purchase money). Hence, divide the given percentage or product (\$3825) by its specific rate or multiplier factor (.45) to find its specific base or multiplicand factor (\$8500). Expressed in more familiar terms, the question is "45 hundredths times what purchase money equal \$3825?"

2. A has \$296.80, which is 36 % less money than B has. How much money has B?

SOLUTION

100 % = rate of B's money, the indicated verbal base.

36 % = rate which A's money is less than B's.

64 % = specific rate of A's money, expressed in terms of B's.

\$296.80 (A's money) ÷ .64 (specific rate of A's money) = \$463.75 (B's money, or the base).

EXPLANATION. B's money is indicated as the required verbal base by being placed immediately after " % less than" [198, *b*]. $296.80 is *a* percentage or product because it is placed in comparison with the indicated base (B's money); and specifically it is *the* percentage of A's money. 36 % is the specific rate or multiplier of *what A's money is less than B's*, but not the specific rate or multiplier of the base (B's money) to produce the specific percentage or product of A's money ($296.80). To obtain the specific rate of A's money in terms of B's, let 100 % represent the indicated base, B's money [Note 2, 196]; and as A's money is 36 % less than B's, so 36 % less than 100 %, or 64 %, is the specific rate of A's money. Hence divide the specific percentage or product of A's money ($296.80) by its specific rate (the specific multiplier of B's money to produce A's, 64 %) to find the required base or multiplicand (B's money, $463.75).

RULE. *Divide the given percentage or product by its rate factor* (the multiplier of the base which was used to produce the given percentage), *and the quotient will be its base factor* (the multiplicand which was used to produce the given percentage).

EXAMPLES FOR PRACTISE

1. $63 are 7 times how many dollars? **2.** 72 gallons are .09 times how many gallons? **3.** 42 yd. are 6 % of (.06 times) how many yards? **4.** 252 tons are 45 % of what weight? **5.** 28 % of how much vinegar = 150.64 gal.? **6.** 24 % of what sum of money = $2039.10? **7.** $\frac{3}{8}$ % of what base = $23.82? **8.** $\frac{1}{4}$ % of how much money = $116.91?

9. $633.45 are 64 % more than how much money? **10.** 24 % more than what sum of money = $928.45? **11.** 52 % less than what sum of money = $616.20? **12.** £187 10s. are 76 % less than how much?

13. After selling 414 tons of coal, a dealer found that he had 52 % of his original stock remaining. How many tons did he have at first?

14. A merchant paid $247.25, which was 43 % of his creditor's claim against him. How much of the claim remained unpaid?

15. A's farm contains 783 acres of land, which is 8 % more than the number of acres in B's farm. How many acres does *B's farm contain?*

16. A owns 12 % of the stock of a corporation, B 15 %, C 20 %, D 25 %, and E the remainder. What is the value of B's stock, if E's is worth $7084?

17. A's house is worth 46 % more than B's farm. What is the value of B's farm, if A's house is worth $9855?

18. A man purchased a building lot for $4800, erected a house thereon which cost $5600, expended $2050 in painting and decorating the house, $510 in furnishing it, and had 28 % of his money remaining. How much did he have at first?

19. By an unfortunate investment a merchant lost $16872, which was 37 % of his original capital. What was his subsequent capital?

20. A dealer sold an invoice of goods for 26 % more than he paid for them, and received $818.37. How much did the dealer pay for the goods sold?

COMMON APPROXIMATIONS IN PERCENTAGE

204. INTRODUCTION. (*a*) After obtaining a denominate percentage by **201** or a denominate base by **203**, it is customary among business men to reduce the decimal part of the result to the form of a common fraction as in **125**. If no special reason exists for reducing the decimal to a common fraction of a particular denominator, it is usually reduced to 8ths. After thus reducing the decimal to 8ths, the obtained fraction should be expressed in lowest terms, if not already so.

(*b*) In some lines of business a special denominator is adopted which expresses the number of units of the next lower denomination used in that kind of business which equal 1 integral unit of the obtained result. Thus, coal dealers reduce decimal parts of a result in tons to the denominator which expresses the number of pounds (the next lower denomination to tons used in the coal business) which equal 1 ton, that is, to 2000ths, if short tons, or to 2240ths, if long tons; grain dealers reduce decimal parts of an obtained result in bushels to the denominator which expresses the number of pounds (the next lower denomination to bushels used in the grain business) which equal 1 bushel of that particular kind of grain, that is, to 60ths if wheat, to 56ths if corn, etc. In the same manner fractional parts of other units of merchandise are expressed with still other denominators.

ILLUSTRATIVE EXAMPLES

1. Find 8 % of 419 bu. wheat.

SOLUTION

419
.08
bu. 33.52
60
lb. 31.20

Ans. 33 bu. 31 lb.

EXPLANATION. First find 8 % of 419 bu. by **201**, obtaining 33.52 bu. Retain the integral part of the result (33 bu.) as a part of the final answer, and reduce the decimal part (.52 bu.) to pounds (60ths) by **125**, obtaining 31 + lb., thus making the complete answer 33 bu. 31 lb., usually written 33^{31} bu.

2. A dealer sold 127 gal. molasses which was 43 % of all the molasses he had in stock. How many gallons did he have remaining, reducing any decimal of a gallon to 8ths (pints)?

SOLUTION

.43)127.00(295⅜ gallons, (base)
86
410
387 — 295⅜ gal. at first
230 — 127 gal. sold
215 — 168⅜ gal. remaining
15
8
43)120(3 approximately = ⅜
129

EXPLANATION. Divide the specific percentage sold (127 gal.) by the specific rate % sold (.43), obtaining 295 gal. as the indicated base, or all the molasses he had in stock, and a remainder of 15 (⅛ gal.). Multiply this remainder (15) by the required denominator (8ths) and divide the result (120 eighths) by the original divisor (43) obtaining 2 eighths and more than half of another eighth, or practically ⅜ [**86**], making the complete base 295⅜ gal.

EXAMPLES FOR PRACTISE

1. A man bought 765 bu. potatoes and sold 52 % of his purchase. How many bushels and pounds (60ths) did he have remaining?

2. A coal dealer bought 163 tons of coal and sold 37 % of his purchase at one time, and 45 % of the remainder at another time. How many tons and pounds (2240ths) did he have finally remaining?

3. A grain dealer sold 32 % of his stock of corn and had 416 bushels remaining. How many bushels and pounds (56ths) of *corn did he sell?*

4. A merchant sold 15% of an invoice of muslin and had 473 yards remaining. How many yards of muslin were in the invoice, expressing any resulting fraction of a yard in 4ths?

5. A man sold 785 bu. of oats, which was 62% of his original purchase. How many bushels and pounds (32nds) did he have remaining?

6. The distance between two cities is 268 miles. If a man has traveled 53% of the distance, how many miles and yards (1760ths) has he yet to travel?

7. A woodman delivered 173 cords of wood, which was 72% of the amount specified in his contract. How many cords and cord feet (8ths) remained to be delivered?

APPROXIMATIONS OF RATES %

205. In the practical application of **202** to business calculations, many difficulties will present themselves, methods for surmounting some of which will now be considered.

(*a*) When results are obtained in United States money, they are expressed approximately to the nearest cent. This will frequently cause embarrassment to the learner. Thus, in a business calculation, 5% of \$28.37, or \$1.4185, would be entered in the books of account as \$1.42. If occasion should afterward arise to learn what % of \$28.37 are \$1.42, the exact result (5%) cannot be obtained by the usual process. The *insignificant* remainder (.0015) when compared with the divisor (28.37) would at once remind the experienced calculator that it was caused by counting a fraction of a cent more than a half (.85 of a cent) as a *whole* cent, thus making the dividend (\$1.42) .15 of a cent greater than the *exact* expression, and causing the remainder of .15 of a cent when the exact quotient (5%) is found.

SOLUTION I

\$28.37)\$1.4200(.05
1 4185
15

(*b*) On the other hand, if in a practical calculation, it is required to take 36% of \$78.59, the exact result will be \$28.2924, and its practical expression \$28.29. If for any reason, it is subsequently necessary to learn what rate % had been employed to obtain this percentage, the *exact* result will not be secured, but only 35% with a *very significant* remainder. This significant remainder (7835) when contrasted with the divisor (7859) should remind the calculator

SOLUTION II

6
\$78.59)\$28.2900(35
23 577
4 7130
3 9295
7835

that the remainder was caused by discarding from the percentage ($28.29) a fraction of a cent less than a half (.24 of a cent), thus making the percentage lack .24 of a cent of being exactly 36% of the base, and causing the remainder (.7835) to lack .24 of a cent of exactly containing the divisor 1 more time than 35%, or 36%.

(*c*) Again, it may be necessary to take $6\frac{1}{4}$% of $49.32, of which the exact percentage is $3.0825, or, practically expressed, $3.08. If, afterward, it should become necessary to find what % had been used to obtain $3.08, the result would be 6% and an *intermediate* remainder (that is, neither an insignificant remainder as in *a*, nor a significant remainder as in *b*, but intermediate between the two) of .1208. The calculator should then at once conclude that there was a fraction of a per cent used in the rate. The fractional rates in common use are *halves*, *fourths*, *eighths*, and *thirds*. Of these, the last (*thirds*) will immediately suggest itself by being associated with 16, 33 and 66 in forming the frequently used commercial rates $16\frac{2}{3}$%, $33\frac{1}{3}$%, and $66\frac{2}{3}$%. If thirds are not thus suggested as in the present instance, the remainder (1208) should be reduced to *eighths*, the highest multiple of the remaining commercial denominators, by 86, obtaining 1 eighth and the significant remainder 4732 which lacks 200 (8 times the discarded $.0025) of being another eighth, making $\frac{2}{8}$, or in lowest terms $\frac{1}{4}$; thus showing that the rate used was exactly $6\frac{1}{4}$%.

SOLUTION III

```
$49.32)$3.0800(6¼%
       2 9592
       ------
        1208
          8   2
        ----
        9664(1 = 2/8 = ¼
        4932
        ----
        4732
```

NOTE 1. If in *c*, thirds are suggested by association with 16, 33 or 66, pupils are cautioned against accepting the probability for a *certainty*. The probability should be tested, and only an *insignificant* remainder as in Solution 1, or a *significant* remainder as in Solution 2, can be accepted as conclusive. *The correct fractional rate will never leave an intermediate remainder.*

SUMMARY. *In finding rates %, insignificant remainders should be disregarded; significant remainders which approximate closely to the base require the last quotient figure to be increased by 1: and intermediate remainders, when thirds are not suggested, should be reduced to eighths and the result expressed in lowest terms.*

NOTE 2. If a base expresses many thousands of dollars as in great wholesale transactions, or millions of dollars as in many calculations of railroad companies and other great corporations, rates are usually carried to five decimal places, the first two places after the point expressing %, and the following *three places denoting* thousandths of a per cent.

EXAMPLES FOR PRACTISE

What % of	What % of
1. \$374.86 are \$157.44?	5. \$9248 are \$3479.56?
2. \$968.43 are \$377.69?	6. \$9.65 are \$4.04?
3. \$758.65 are \$216.22?	7. \$67841.35 are \$42740.05?
4. \$4832.72 are \$4687.74?	8. \$6873.24 are \$2019.01?

9. The gross earnings of a railroad for the year 1910 were \$1375487.60, and its operating expenses \$296785.37. What % of the gross earnings were the operating expenses (to 5 decimal places)?

10. A wholesale firm extended credits to the amount of \$3468756.75 during the year 1910, of which it failed to collect \$12378.40. What % of its credits did it collect (to 5 decimal places)?

BASES UNDERSTOOD

206. INTRODUCTION. The idea expressed by the term "base" as employed in percentage differs little from that of the same word in ordinary usage. Thus, the base of any object, say a monument, is the foundation or pedestal, upon which it rests, and without which it cannot stand. So in a percentage problem, if the base is not directly indicated, it is "understood" to be the entire quantity of which the rate % *represents* a part, and of which the percentage *is* a part, and therefore that upon which both these terms depend for their value as a monument depends upon its pedestal for its stability. A base which is thus comprehensible is sometimes called a "normal base" to distinguish it from an "arbitrary base" which has no special characteristic by which it can be identified, but which depends simply upon the arbitrary choice of the author of the problem in selecting it as such. A *normal base* is therefore the quantity from which the percentage naturally derives its origin, that of which one would readily think as the *whole* while considering any *part* of it, that which would naturally suggest itself to the mind as the answer if the interrogative "of what" be placed immediately after the rate %.

NOTE. If there is a possibility of the slightest doubt relative to the identity of the base, it should be indicated by placing *of*, *more than*, or *less than*, or words of equivalent import, immediately after the rate % and immediately before the otherwise doubtful base [198, *a*, *b*, *c*].

EXAMPLES FOR PRACTISE

1. A man's income is \$1800 per annum. If he pays 20 % for board, 15 % for clothing, and 25 % for incidentals, how much does he save?

2. A town has a population of 10500, 55 % being native born, 20 % Germans, 15 % Irish, and the remainder of other nationalities. How many of each does the town contain?

3. A man bought a house for \$6500, paying \$1800 in cash, and agreeing to settle the remainder in annual payments of 25 %. What was the amount of each annual payment?

4. A man died leaving property of the value of \$13500. He bequeathed 45 % to his wife, 35 % to his only child, and the remainder to a hospital. How much did the hospital receive?

5. A farm of 105 acres is only 28 % as large as the farm adjoining it. How many acres in the adjoining farm?

6. If coffee loses 15 % in roasting, how many pounds of green coffee must be bought to produce 1020 pounds when roasted?

7. A is worth \$38500 and B only 47 % as much. What is B worth?

8. A man weighed 190 pounds before an attack of illness, and 133 pounds directly after the attack. What % did he lose in weight?

PERCENTAGES OF INCREASE OR DECREASE

207. INTRODUCTION. No *increase* or *decrease* is possible, except there be first, as a *basis* for such a modification, a quantity to be augmented or diminished. Hence, in all problems of this character, the normal base always understood is the quantity to be increased or diminished, that is, the quantity immediately *before* such a change has been effected; and the normal percentages always understood are the increase or decrease itself, and the quantity *after* such a change has been made. Most of the applications of percentage from **209** to **291**, in which bases are understood, depend upon this important principle in the proper selection of the base; profit being but another name for increase, loss for decrease, the various discounts (bank, true, or trade) for de*creases, etc., etc.*

NOTE. Increase means *more than*, and decrease *less than* [198, *b*]. Hence, when in doubt, the rational answer to the interrogative "more than what" placed after a rate of increase, or "less than what" placed after a rate of decrease, will suggest the base understood.

EXAMPLES FOR PRACTISE

1. At the census of 1900, a city had a population of 37240, and at the census of 1910, its population was 50274. What was the % of increase?

2. During a financial depression, a clerk had his salary diminished 15 %, or \$300. What was his salary after the reduction?

3. By a rise in real estate, a house has increased in value 28 %, and is now worth \$10880. What was its value before the rise?

4. A firm's gross sales last year were \$287300, and this year \$333268. What was the % of increase?

5. By introducing new machinery a factory has increased its output $16\frac{2}{3}$ %, and now produces 182753 yards of sheeting per week more than before. What was the capacity of the factory before installing the new machinery?

6. The capital of a firm was increased 20 % in 1910, and 15 % in 1911, the total increase during the two years being \$17406. What was its capital on Jan. 1, 1910?

7. The net earnings of a corporation were diminished 25 % during the financial depression of 1907, and still further diminished 10 % during the year 1908, but during the year 1909, under improved conditions, they were increased 20 %. If the net earnings for 1909 were \$268764.50, what were the net earnings during 1907?

8. If the net capital of a firm was \$45000 on Jan. 1, 1908, and there was a net increase of 10 % per annum for four successive years, what was its capital on Jan. 1, 1912?

REVIEW OF PERCENTAGE

208. 1. A merchant sold 43 % of his stock of millinery for \$6875.35. What was the value of the remainder of his stock of millinery?

2. A and B are partners, A having invested \$19161 and B \$23419. What % of the capital of the firm did each partner invest?

3. A cistern now contains 741 gallons of water, 24 % having leaked out. How many gallons did it originally contain?

4. A clerk spent \$676.50 of his annual salary, and saved \$523.50. What % of his salary did he spend?

5. If A's age is 25 % greater than B's, what % of A's age is B's?

6. A is worth \$45000 and B \$30000. What % more than B is A worth? What % less than A is B worth?

7. A man inherited \$74500, invested 45 % of it in real estate, 25 % of it in United States bonds, and loaned the remainder to a friend. What was the sum loaned?

8. A bakery used 425 pounds of flour in making 748 loaves of bread, each loaf weighing 12 ounces. What % more than the flour did the bread weigh?

9. A owns 12 % of the stock of a corporation, B 15 %, C 20 %, D 30 %, and E the remainder. If B's investment is \$3600, what is E's?

10. A man sold 40 % of a tract of land at one time, 20 % of the remainder at a second sale, 25 % of what still remained at a third sale, and then had 234 acres finally remaining. How many acres did the original tract contain?

11. In 1910, a merchant's sales amounted to \$185920, and in 1911, his sales were \$158032. What was the per cent. of decrease in 1911?

12. A certain property has depreciated \$3375 in value, and is now worth \$9125. What was the % of depreciation?

13. A man bought one house for \$4500 and another house for 42 % more. What was the cost of both?

14. On Sept. 23, 1911, merchandise was bought for \$1340 on 3 months' credit, and subject to a discount of 3 % if paid within 10 days. What is the proper payment on Oct. 1, 1911?

15. A firm's business was placed in the hands of a receiver who discovered that its total resources after deducting the costs of receivership were \$28500, and that its total liabilities were \$37500. What % of the firm's indebtedness can the receiver pay to the creditors?

16. Two railroads carried 8260 pounds freight at the through

rate of 28 cents per cwt. If one company was entitled to 37 % of the through charge, how much should the other company receive?

17. Two railroads, one 200 miles long and the other 300 miles long, carried 750 barrels flour at the through rate of 18 cents per barrel, which was to be divided *pro rata* between them (that is, in proportion to each road's per cent. of the total mileage). What % of the total freight charge should each road have received? What percentage?

18. Three railroads respectively 200 miles, 250 miles, and 350 miles in length, transported 28375 pounds freight at the through rate of 28 cents per cwt., which was pro-rated among them in accordance with a general traffic agreement. What % of the total distance did each road carry the through freight? What percentage of the freight collection should each road receive?

19. The total cost of an arithmetic and a geography is \$2.10, and the arithmetic cost 32 % less than the geography. What is the cost of each?

EXPLANATION. The required cost of the geography is indicated as the verbal base or multiplicand by being placed immediately after " % less than." \$2.10 is a percentage because it is compared with (by being expressed in terms of) the indicated base. As \$2.10 is the joint cost of the geography (the indicated base) *plus* the cost of the arithmetic (a percentage), its specific joint rate must be the specific rate of the geography or base (100 % of the cost of the geography) *plus* the specific rate of the cost of the arithmetic or percentage (100 % − 32 %, or 68 % of the cost of the geography), that is, 168 % of the cost of the geography. Hence, divide the joint percentage or product (\$2.10) by its joint rate % or multiplier (168 %), obtaining \$1.25 as the common base or multiplicand (the cost of the geography).

Then, the cost of both (\$2.10) minus the cost of the geography (\$1.25) should equal the cost of the arithmetic (\$.85). Or, the obtained common base or multiplicand (\$1.25) multiplied by the specific rate or multiplier for the arithmetic (68 %) will produce the specific percentage or product of the arithmetic (\$.85).

20. A man bought a corner-house and the house adjoining it for \$7752, paying 28 % more for the corner-house than for the one adjoining it. How much did he pay for each?

21. A, B, C, and D are partners. A's investment is 20 % greater than B's, C's investment is 10 % less than A's, and D's

investment is 75% of C's. If the capital of the firm is \$53579, what is each partner's investment?

NOTE. If two or more bases are indicated in the same problem, accept the initial or *root* base as *the* base of the solution, and regard the remaining derivative bases as so many percentages of the root base. There can be no consolidation of two or more specific rates or multipliers except when they are expressed upon *one common base* or multiplicand [Prin. 1, 18].

22. A has 25% more money than B, B 28% more than C, C 40% less than D, and D 25% as much as E. If all five together have \$641.20, how much has each?

23. The sales of a firm were increased 20% per annum for three successive years. If the total increase during the three years was \$36400, what were the sales during the second year?

24. The gross earnings of a railroad company during the year 1910 were \$6781345.30, of which \$2346785.90 was expended for wages, \$385465.20 for repairs, \$275864.20 for fuel, and \$1753285.90 for fixed charges. What % of the gross earnings was each of the above charges (to five decimal places)?

25. A commission merchant sold 10% of a consignment of wheat at one sale, 30% of the remainder at a second sale, 40% of what still remained at a third sale, and then had 1701 bushels remaining. How many bushels of wheat were in the consignment?

26. A owned 20% of the capital stock of a steamboat company, and sold 40% of what he owned for \$4800. At the same appraisement, what was the value of the entire capital stock?

PROFIT AND LOSS

209. INTRODUCTION. Profit and Loss problems are such percentage problems as have special reference to gains or losses in selling. While differing in no respect from other percentage problems, they are grouped together for special treatment because of their relative commercial importance, and the consequent necessity for acquiring thorough familiarity with the general principles of percentage as applied to them, and for cultivating special skill in computing them.

210. The cost is the sum *paid* for the thing purchased.

NOTE 1. The *prime cost* of merchandise is the *first* cost, that is, the cost *before any expense* connected with the purchase, such as freight from place

of purchase, drayage, custom-house charges, commission on purchase, insurance, storage after purchase, etc., has been added. The *gross* cost is the *last* cost, that is, the first cost increased by *all* expenses incidental to the purchase from the moment of purchase to the moment of sale.

NOTE 2. When the term cost is used without a qualifying word, *the actual cost*, or gross cost, is always understood.

211. The **selling price** is the sum *received* for the thing sold.

NOTE 1. The *gross selling price* of merchandise is the total sum received from the sale, before any expense connected with the sale has been deducted. The *net selling price* is the gross selling price diminished by *all* incidental expenses incurred *after the moment of sale*, such as commission on sales, discounts allowed, free delivery charges, etc.

NOTE 2. When the term selling price is used without a modifying adjective, the *actual* or net selling price is understood.

212. The **profit** or **gain** is the excess of the net selling price *above* the gross cost.

213. The **loss** is the deficiency of the net selling price *below* the gross cost.

214. Relation of terms. 1. *A gross cost is a base or multiplicand.* 2. *A % of profit, of loss, or of selling price, is a rate or multiplier.* 3. *A profit, a loss, or a selling price, is a percentage or product.*

NOTE 1. Profit means increase (more than cost) and loss means decrease (less than cost). Calculations in profit or loss are therefore applications of **207**. As the gross cost is uniformly increased by the profit, or diminished by the loss to find the net selling price, the gross cost is a *normal base* or standard of comparison, and does not require to be indicated as is necessary for arbitrary bases. "Of gross cost" is therefore understood after every rate of profit or of loss, when no other base is arbitrarily indicated.

NOTE 2. Since the selling price equals the cost plus the profit, or the cost minus the loss, the specific rate of selling price must equal the specific rate of cost (100 %) plus the specific rate of profit, or minus the specific rate of loss. [Note 1, **201**.]

EXAMPLES FOR PRACTISE

RULE. If the cost (base) and a rate are given, multiply the cost by the specific rate of the required profit, or loss, or selling price.

Goods cost

1. \$57.80 and are sold at 25 % profit. Find profit.
2. \$183.75 and are sold at 18 % loss. Find loss.
3. \$275.80 and are sold at 28 % profit. Find profit.

Goods cost

4. \$628.50 and are sold at 20% loss. Find loss.
5. \$342.16 and are sold at $16\frac{2}{3}$% profit. Find profit.
6. \$486.25 and are sold at $33\frac{1}{3}$% loss. Find loss.
7. \$254.85 and are sold at 35% profit. Find selling price.
8. \$685.90 and are sold at 25% profit. Find selling price.
9. \$816.35 and are sold at 10% profit. Find selling price.
10. \$723.40 and are sold at 5% loss. Find selling price.
11. \$374.85 and are sold at 20% loss. Find selling price.
12. \$436.95 and are sold at $12\frac{1}{2}$% loss. Find selling price.
13. \$783.24 and are sold at 45% gain. Find gain.
14. \$294.35 and are sold at 18% loss. Find loss.
15. \$385.16 and are sold at 24% gain. Find gain.
16. \$416.28 and are sold at 16% loss. Find loss.

RULE. If the cost (base) and a profit or loss or selling price (percentages) are given, divide the specific percentage or product of which it is required to find the rate, by its cost or multiplicand factor, and the quotient carried to hundredths, will be its specific rate % or multiplier factor, applying *a*, *b*, or *c*, **205**, when the division is inexact.

Goods cost

17. \$38.75 and are sold at \$9.69 gain. Find % of gain.
18. \$68.50 and are sold at \$10.96 gain. Find % of gain.
19. \$872.50 and are sold at \$401.35 gain. Find % of gain.
20. \$497.40 and are sold at \$89.53 loss. Find % of loss.
21. \$26.25 and are sold at \$2.10 loss. Find % of loss.
22. \$63.85 and are sold at \$22.99 loss. Find % of loss.
23. \$342.97 and are sold for \$425.28. Find % of gain.
24. \$682.25 and are sold for \$777.77. Find % of gain.
25. \$359.34 and are sold for \$294.66. Find % of loss.
26. \$458.37 and are sold for \$401.07. Find % of loss.
27. \$178.15 and are sold for \$213.78. Find % of gain.
28. \$728.35 and are sold for \$591.78. Find % of loss.
29. \$563.27 and are sold at \$93.88 gain. Find % of gain.
30. \$94.65 and are sold at \$17.98 loss. Find % of loss.

RULE. To find the cost, divide any given percentage or product of profit, of loss, or of selling price, by its specific rate or multiplier factor.

31. The gain is \$48.65 and rate of gain 25%. Find cost.
32. *The gain* is \$150.00 and rate of gain 32%. Find cost.

Find the cost, if

33. The loss is \$346.05 and the rate of loss 45 %.

34. The loss is \$168.65 and the rate of loss 20 %.

35. The selling price is \$487.97 and the rate of gain $16\frac{2}{3}$ %.

36. The selling price is \$607.88 and the rate of loss $12\frac{1}{2}$ %.

37. The selling price is \$423.72 and the rate of gain 15 %.

38. The selling price is \$52.88 and the rate of loss $33\frac{1}{3}$ %.

39. The selling price is \$348.70 and the rate of gain $37\frac{1}{2}$ %.

40. The selling price is \$58.65 and the rate of loss $6\frac{1}{4}$ %.

41. The selling price is \$105.19 and the rate of gain 34 %.

42. The selling price is \$406.50 and the rate of loss 35 %.

43. The selling price is \$519.86 and the rate of gain 10 %. Find gain.

44. The selling price is \$130.58 and the rate of loss 15 %. Find loss.

45. Merchandise was sold for \$48.40. If the gain was \$9.68, what was the rate % of gain?

46. Groceries were sold for \$62.68. If the loss was \$15.67, what was the rate % of loss?

47. A lot of hardware was sold at 25 % gain. If the seller received \$22.80, how much of it was profit?

48. A house was sold for \$2125 by which a loss of 15 % was sustained. How much less than cost was received by the seller?

49. A man bought two horses at \$125 each. He afterwards sold one of them at 25 % gain and the other at 15 % gain. How much did he receive for the two horses?

SUGGESTION. A *common* cost or multiplicand (\$125) multiplied by the specific *joint* rate of selling price, or multiplier for the joint product (125 % + 115 %) = joint selling price or product.

50. A speculator in real estate bought three houses at \$2400 each, and afterwards sold one of them at a profit of 35 %, the second at a profit of 30 %, and the third at a profit of 15 %. How much did he receive for the three houses?

51. Two houses were sold for \$2400 each, by which the seller gained 20 % on one house and lost 20 % on the other. Did the seller receive what he paid for the two houses? If not, did he gain or lose, and how much?

52. Three farms were bought for $3500 each, and they were afterwards sold respectively at 25% gain, 30% gain, and 18% loss. How much was received for the three farms?

53. A man bought a tract of land for $4500, paid $625 additional for improvements, and then sold it for $6150. What was his % of gain?

54. A farmer bought 48 acres of land for $3600, and afterwards sold 32 acres from his purchase at a profit of 25%. For what sum per acre must he sell the remainder of the purchase to net an average profit of 20% on the entire purchase?

55. Allowing for a loss of 12% in roasting, what should be the selling price of roasted coffee which was bought unroasted at 20 cents per pound, to net a gain of 10% [**207**]?

56. An instalment dealer bought furniture at $18 per set. What must he ask per set to net a profit of 25%, after allowing 20% for bad collections?

57. A wholesale fruit dealer bought a vessel load of watermelons numbering 12000 at $16 per hundred, and classifying them according to size and quality, sold 5000 of them at $25 per hundred, 3000 at $20 per hundred, 2500 at $18 per hundred, 1000 at $15 per hundred, and the remainder for $46. What was his % of profit on the entire purchase?

58. A merchant sold 30% of a purchase at 25% profit, 60% of the remainder at 20% profit, and what still remained at 10% profit. What was his % of gain on the entire purchase?

59. To increase their capital, a firm agreed to permit one-half of the profits to remain in the business for three successive years. The net gains during the first year were 20%, during the second year 25%, and during the third year 10%. If the capital of the firm at the close of the third year was $31185, what was its capital at the beginning of the first year?

60. A firm sells goods in its wholesale department at 12% profit, and in its retail department at 25% advance on its wholesale selling price. What % profit upon original cost is made in the retail department?

61. If 20% of an invoice of fruit is spoiled in shipment, at what % gain must the remainder be sold to net 10% profit on the *original cost*?

62. A dealer sold merchandise for \$48.99 and gained 15%. What should have been the selling price to gain 25%?

63. A retail grocer bought 5 barrels of flour at \$6.20 per barrel, paid 25 cents drayage to have the purchase brought to his store, and afterwards sold one barrel to a customer for \$7.75 and paid 25 cents drayage to have it delivered at his customer's residence. What was the grocer's rate % of profit?

64. If the sales during the year were \$60191, the inventory of stock on hand at end of year \$9625, and the gain \$7851, what were the total purchases, and the % of gain?

65. If the purchases during the year are \$35798.56, the inventory of stock on hand at end of year \$6500, and the rate of gain $12\frac{1}{2}$%, what are the total sales?

66. The books of a manufacturer for the year 1911 show the following costs: productive material \$200000, productive labor \$300000, overhead expenses \$100000. What % of the manufacturing cost is each element of expense? [See *Suggestions* following.]

SUGGESTIONS. (*a*) Manufacturing or factory cost consists of the following elements: productive labor, productive material, and "overhead" factory expense.

(*b*) Productive or direct labor is labor which is employed directly upon the product manufactured *to sell*. The labor of the factory superintendent foremen, bookkeepers, clerks, porters, etc., is called non-productive or indirect labor.

(*c*) Productive material is material which is used directly in the articles manufactured *to sell*. Material used for general purposes around the factory, as repairs, etc., is called non-productive material.

(*d*) Overhead factory expense includes non-productive material; non-productive labor; and any general factory expense as heat, light, power, etc., except such as is incident *to selling* the manufactured product.

67. In the manufacture of a certain make of bicycles, the productive labor was \$82500, productive material \$71000, and overhead expense \$23000. What % of the manufacturing cost is the productive labor? Is the productive material? Is the overhead expense?

68. Factory order No. 16713 for 100 sewing machines shows the following costs: direct labor \$500, productive material \$600. Estimating the overhead expense at 40% of the direct labor, what was the manufacturing cost of each sewing machine?

NOTE 3. In manufacturing cost calculations, though not the uniform practise, it is customary to distribute the indirect, unproductive or overhead expense among the units of product at an estimated % upon the direct labor employed in the production of such units.

69. In a certain factory, shop order No. 2519 shows consumption of productive materials \$83.67, direct labor, \$41.83. Estimating the overhead expense at 40% of the productive labor, what is the factory cost?

70. Shop order No. 1627 of Monumental Machine Works includes the following costs: direct labor \$10248, productive material \$3722, estimated overhead factory expense 25% of direct labor, estimated distributing expense, shipping, etc., 3% on total sales. What is the selling price to net 25% profit on factory cost?

71. Estimating 50% of factory cost for direct labor, what should be the selling price of 75 units of product at $16\frac{2}{3}$% profit, if the direct labor per unit is \$48?

72. The annex for storing the raw materials of a factory with its contents and records pertaining to the same, are destroyed by fire. The general office records show stores on hand at last inventory \$20000, purchases since last inventory \$60000, manufactured and sold since last inventory \$96000, labor and material on unfinished product in the factory at the time of the fire, \$14400. The manufacturing costs are estimated at 50% for direct labor, 30% for productive material, and the remainder for overhead expense. If the manufacturer's selling price is 50% advance on factory cost, what was the value of the raw material in the annex at the time of the fire?

TRADE DISCOUNT

215. INTRODUCTION. Problems in Trade Discount are applications of the general principles of percentage to calculations which have special reference to deductions made by manufacturers and jobbers and merchants upon sales of a certain magnitude to stimulate large purchases; or as a compensation to middlemen for handling merchandise to sell at a stipulated list price; or as an absolute allowance to purchasers on "time prices" if paid for *at any time* within a given period.

216. The **gross price** is the price *before* any deduction has *been made.*

217. The **trade discount** is the deduction which has been made from the gross price.

218. The **net price** is the price *after* all deductions have been made.

219. Identification of terms. 1. *A gross price is a base or multiplicand.* 2. *A % of trade discount or of net price is a rate or multiplier.* 3. *A trade discount or a net price is a percentage or product.*

NOTE 1. Discount (from *dis*, away; *count*, compute) means counting away. In taking *away* from a quantity, it is decreased. Hence, problems in trade discount are applications of **207**, relating to decreases; and the quantity diminished (the gross price) is the normal base *always understood;* and the trade discount (the *decrease*), and the net price (the quantity *after* the decrease) are percentages.

NOTE 2. If two or more rates of discount are allowed, the gross price is the *root base* upon which the first rate of discount is computed; and the proceeds of the first discount should be taken as a *derivative* base upon which the second rate of discount is computed; and the proceeds of the second discount should be regarded as a second derivative base upon which the third rate of discount is computed; etc., etc.

GROSS PRICE GIVEN, TO FIND THE NET PRICE

ILLUSTRATIVE EXAMPLE

220. The gross price is \$278.56, upon which a discount of 28 %, 20 %, and 3 % is allowed. Find the net price.

SOLUTION

\$278.56, gross price or root base
.72, (100 % − 28 %)

55712
194992

\$200.5632, proceeds of 1st discount
40.1126, 20 % or $\frac{1}{5}$ of \$200.563

\$160.4506, proceeds of 2d discount
4.8135, 3 % of \$160.4506

\$155.6371, proceeds of 3d discount

EXPLANATION. Multiply the gross cost or root base (\$278.56) by the specific rate of proceeds of the first discount (100 % − 28 % = 72 %) to find the proceeds of the first discount (\$200.5632).

Then multiply the obtained derivative base (\$200.5632) by the second rate of discount (20 % = $\frac{1}{5}$) obtaining \$40.1126, and subtract, leaving \$160.4506 as the proceeds of the second discount.

Then multiply the last obtained derivative base (\$160.4506) by the third rate of discount (3 %), obtaining \$4.8135, and subtract, leaving \$155.6371 as the proceeds of the third discount, or required net price.

NOTE 1. An *aliquot rate* is one which is an exact divisor of 100 %, as 20 % in the Ill. Ex.; and a non-aliquot rate is one which is *not* an exact divisor of 100 %, as 28 % in the Ill. Ex. Aliquot rates of discount are more conveniently handled by first finding the discount and then subtracting; but non-aliquot rates of discount, by at once multiplying the base by the specific rate of net price, as by 100 % − 28 %, or 72 %, in the Ill. Ex.

NOTE 2. A rate of discount consisting of a single figure, as 3 % in the Ill. Ex., is best computed as follows: Commencing with the cents' order of the base, multiply it and each of its successive higher orders by the rate of discount, and write the successive orders of the product *two* places to the right of the orders multiplied, first multiplying the rejected orders at the right of the cents' order of the base to obtain the correct carrying figure to the retained product.

NOTE 3. All results should be carried to *four decimal places*, to insure accuracy in the cents' order of the final result.

NOTE 4. A fraction of a half or more of the lowest retained order *is not* usually counted in computing a discount (which is against the seller), and *is* counted as another unit of the lowest retained order in computing a net price (which is in favor of the seller). Not counting in the former instance is equivalent to counting in the latter.

EXAMPLES FOR PRACTISE

Find the net price		Find the net price	
Gross Price	*Rates of Disc't*	*Gross Price*	*Rates of Disc't*
1. $375.85	25 %, 10 %, 5 %	**6.** $285.94	32 %, 24 %, 5 %
2. $582.90	25 %, 20 %, 10 %	**7.** $723.56	25 %, 16 %, 9 %
3. $873.62	10 %, 10 %, 10 %	**8.** $938.25	$16\frac{2}{3}$ %, $12\frac{1}{2}$ %, 2 %
4. $462.75	20 %, 10 %, 7 %	**9.** $427.36	10 %, 7 %, 1 %
5. $629.38	10 %, 5 %, 3 %	**10.** $246.78	5 %, 3 %, 2 %

11. Goods are invoiced at $658.90 and 25 %, 5 %, and 3 % off. What is the net cost to the purchaser?

12. Bought 2 doz. pocket knives, No. 278, at $4.50 per dozen; ½ doz. butcher knives, No. 715, at $7.50 per dozen; and 3 doz. locks, No. 645, at $3.75 per dozen. What was the net cost, after allowing a discount of 20 %, 5 %, and 2 %?

13. Bought May 16, 1912, a bill of merchandise amounting to $738.65 gross and payable in 90 days, upon which a merchandise discount of 10 %, 10 %, and 5 % was allowed, with a time discount of 3 % if paid within 10 days. What should be the *payment if made* in full on May 25, 1912?

REDUCTION OF RATES OF DISCOUNT

ILLUSTRATIVE EXAMPLES

221. 1. Reduce the discount series of 20 %, 10 %, and 5 % to an equivalent single discount.

SOLUTION

100 % = gross price
20 % = 1st discount
80 % = proceeds of 1st discount
($\frac{1}{10}$ of 80) 8 % = 2d discount
72 % = pro. of 2d discount
($\frac{1}{20}$ of 72) 3.6 % = 3d discount
68.4 % = { pro. of 3d discount / or rate of net price }
100 % − 68.4 % = 31.6 %, single discount

EXPLANATION. Subtract the rate of the first discount (20% of the gross price) from the rate of gross price (100 %) to find the rate of 1st proceeds (80 % of the gross price).

Next subtract the rate of the 2d disc't ($\frac{1}{10}$ of 80 % of the gross price or 8 % of the gross price) from the proceeds of the 1st disc't (80 % of the gross price) to find the proceeds of the 2d disc't (72 % of the gross price).

Next subtract the rate of the 3d disc't ($\frac{1}{20}$ of 72 % of the gross price or 3.6 % of the gross price) from the proceeds of the 2d disc't (72 % of the gross price) to obtain the rate of proceeds of the last discount (68.4 % of the gross price).

The difference between 100 % of the gross price and the rate of final proceeds (68.4 % of the gross price), or 31.6 % of the gross price must therefore be the total of the three discounts expressed in a required single discount upon the gross price.

2. Reduce the discount series of 32 %, 24 %, and 16 % to an equivalent single discount.

SOLUTION

100 % − 32 % = 68 %
100 % − 24 % = 76 %
100 % − 16 % = 84 %
.68 × .76 × .84 = .434112
100 % − 43.4112 % = 56.5888 %

EXPLANATION. If 32 % of the gross price (100 %) is taken off, then 68 % of the gross price will remain.

If 24 % of what remains of the gross price is then taken off, 76 % of (times) 68 % of the gross price will yet remain.

If 16 % of what yet remains of the gross price is finally taken off, then 84 % of 76 % of 68 % of the gross price, or 43.4112 % of the gross price, will finally remain as the rate of net price.

Then subtract the rate of final net price (43.4112 %) from the rate of the original *gross price* (100 %) to find the rate of total discount (56.5888 %).

NOTE 1. The solution of Ill. Ex. 1 is preferable, if the rates of discount are aliquots [Note 1, **220**]; but when the rates of discount are non-aliquots, the solution of Ex. 2 will be the more convenient. If the same problem contains both classes of rates, employ the method of Ex. 1 for aliquot rates, and of Ex. 2 for the non-aliquot rates.

Note 2. The value of a series of discounts will not be changed by any variation in the order of their arrangement. Thus, a discount series of 10 %, 5 % and 3 % upon any gross price will produce the same net price as 5 %, 3 % and 10 %, or as 3 %, 10 % and 5 %; for the product of $(100 - 10) \times (100 - 5) \times (100 - 3)$ will be the same, irrespective of the order in which they may be multiplied.

EXAMPLES FOR PRACTISE

Find the rate of net price, if the rates of discount are

1. 20 %, 10 %
2. 25 %, 10 %, 5 %
3. 10 %, 5 %, 3 %
4. 20 %, 20 %, 10 %
5. 25 %, 20 %, 10 %
6. 10 %, 10 %, 10 %
7. 35 %, 24 %, 8 %
8. 20 %, 15 %, 10 %, 2 %
9. 10 %, 5 %, 3 %, 1 %

Reduce the following to equivalent single discounts:

10. 25 %, 10 %
11. 20 %, 3 %
12. 20 %, 5 %, 3 %
13. 10 %, 10 %, 5 %
14. 20 %, 10 %, 10 %
15. 5 %, 3 %, 2 %

TO FIND WHAT PER CENT. OF DISCOUNT WILL NET A GIVEN PRICE

ILLUSTRATIVE EXAMPLE

222. What % of discount must be allowed to net \$14.60, if the merchandise is marked \$18.25?

SOLUTION

\$18.25 = gross price
14.60 = net price
3.65 = discount
\$3.6500 ÷ \$18.25 = 20 %

EXPLANATION. The solution of this example is an application of **202**. Hence,

Divide the specific percentage of which it is required to find the %, that is, the percentage or product of discount (\$3.65) by its base or multiplicand factor (the gross price, \$18.25) to find the specific rate of discount or multiplier factor (20 %).

NOTE. If the division is not exact, as is highly probable, apply *a*, *b*, *c*, **205**, to insignificant, significant and intermediate remainders; and apply Note 1, **205**, to *suggestive* rates.

RULE. *Divide the specific percentage of discount* (product) *by the gross price* (its base or multiplicand factor) *and the quotient carried to hundredths will be the rate % of discount* (its multiplier *factor).*

EXAMPLES FOR PRACTISE

What % of discount must be allowed to net

1. $34.01, if goods are marked $35.80?
2. $60.32, if goods are marked $75.40?
3. Cost, if goods cost $43.83 and are marked $48.70?
4. Cost, if goods cost $58.80 and are marked $67.20?
5. 25 % profit, if goods cost $75 and are marked $133.93?
6. 20 % profit, if goods cost $85.35, and are marked $122.90?
7. 5 % loss, if goods cost $15.80, and are marked $17.66?
8. Goods are marked $38.25. What second rate of discount must be used with 20 %, to net $29.07?

THE NET PRICE BEING GIVEN, TO FIND THE GROSS PRICE

ILLUSTRATIVE EXAMPLE

223. At what price must goods be marked to net $5.25 after allowing a discount of 25 %, 5 %, and 3 %?

SOLUTION

	1.00	= rate of gross price
	.25	= rate of 1st discount
	.75	= rate of 1st proceeds
($\frac{1}{20}$ of .75)	.0375	= rate of 2d discount
	.7125	= rate of 2d proceeds
(.7125 × .03)	.021375	= rate of 3d discount
	.691125	= rate of net price

```
 321                 60
.691125)$5.2500($7.59
         48379
          4121
          3456
           665
           622
            43 = more than half of .69
```

EXPLANATION. Divide the specific percentage or continued product of net price ($5.25) by its specific rate of net price or continued multiplier factor (.691125), found as shown in **221**, obtaining $7.60 as the required gross or marked price, its multiplicand factor.

The short method for dividing by a decimal divisor of many places is fully explained in **130**.

The small index figures are placed over the lowest decimal order of the successive divisors; that is, the first divisor is $.691\overset{1}{1}$, the second $.69\overset{2}{1}$, the third and last $.6\overset{3}{9}$.

RULE. *Divide the specific percentage of net price* (product) *by the specific rate of net price* (its multiplier factor), *and the quotient will be the gross or marked price* (its multiplicand factor).

EXAMPLES FOR PRACTISE

At what price must goods be marked to net

1. \$36.50 after allowing a discount of 25 %?
2. \$18.75 after allowing a discount of 15 % and 10 %?
3. \$62.50 after allowing a discount of 25 %, 10 %, and 7 %?
4. \$213.63 after allowing a discount of 25 %, 20 %, 5 %, 3 %?
5. \$85.42 after allowing a discount of 30 %, 20 %, 10 %, 4 %?
6. 25 % profit? Cost, \$24.36. Discounts 10 %, 10 %, 5 %.
7. 16⅔ % profit? Cost, \$45.60. Discts., 20 %, 10 %, 3 %, 1 %.
8. 10 % loss? Cost, \$16.85. Discounts, 10 %, 10 %, 4 %, 2 %.
9. 12½ % loss? Cost, \$9.60. Discounts, 25 %, 5 %, 5 %, 3 %.
10. What payment on May 28, 1912, will be in full of the following

INVOICE

BALTIMORE, May 20, 1912.

Mr. R. M. BROWNING,

Bought of F. A. SADLER & Co.

Terms: 60 days, net; 10 days, 2 %.

3 doz. Butcher Knives \$4.50	**	**			
25 %, 20 %, 10 %	*	**	*	**	
\$4.25 \$5.75 \$7.50					
2 " Pocket Knives ea. 345, 460, 624	**	**			
10 %, 10 %, 2 %	*	**	**	**	
\$6.50 \$7.25					
½ " Hatchets ea. 783, 825			*	**	
\$5.00 \$6.25 \$7.15					
5 " Spades ea. No. 1 No. 2 No. 3	**	**			
\$4.50 \$6.00					
3 " Shovels ea. No. 1 No. 2	**	**			
	***	**			
50 %, 10 %	**	**	**	**	
			**	**	
Packing and Drayage				50	

INTEREST

224. INTRODUCTION. (*a*) In the "Introduction" of **193**, it was shown that Percentage is an application of the general principles which govern the relation of *all* quantities. These same general principles will now be shown to apply to quantities which include a hitherto unconsidered element, that of time.

(*b*) Interest is a compensation for the use of money in the same way as rent is a compensation for the use of property, or wages for the use of personal labor. In all of these it is evident that the amount of the compensation will vary with the length of time they are used. Hence, the necessity of establishing a fixed unit of time by which to graduate the amount of compensation, as wages at so many dollars *per day*, rent at so many dollars *per month*, and interest at so much % of the principal *per year*.

(*c*) It is thus seen that the element of time is not a new term of percentage, but a necessary modifier of one of the three already existent terms (the rate) which is no longer absolute as before, but conditioned upon *one year's use*, as its specific name "rate *per annum*" implies. Hence, at 5% per annum, the corresponding rate for two years will be two times 5%, or 10%; for three years will be three times 5%, or 15%; for 73 days will be $\frac{73}{365}$, or $\frac{1}{5}$ of 5%, or 1%; for 146 days will be $\frac{146}{365}$, or $\frac{2}{5}$ of 5%, or 2%, etc.

225. **Interest** is a compensation for the use of money.

226. The **principal** is the sum of money loaned, or the sum of money due but unpaid, for which a compensation is charged.

NOTE. The sum of the principal and its accrued interest is called the *amount*.

227. The **rate of interest** is the % of the principal which is charged for using it one year.

NOTE. The *specific rate* of interest for less than one year is such a part of the rate per annum, as the given time in days is part of the number of days in a year. Thus, counting 12 months of 30 days each, or 360 days to a year, as is ordinarily done in interest calculations, at 6 % per annum, it will be $\frac{1}{6}$ of 6 %, or 1 % for $\frac{1}{6}$ of 360 days, or 60 days; and if 1 % of the principal is the rate of interest for 60 days, the rate of interest for twice 60 days, or 120 days must be twice 1 %, or 2 %; for $\frac{1}{4}$ of 60 days, or 15 days, must be $\frac{1}{4}$ of 1 %, or $\frac{1}{4}$ %; and, generally, the specific rate of interest for any number of days at 6 % per annum is such a part of 1 % (the rate of interest for 60 days), as the given time in days is part of 60 days; or, stated in a form for convenient use, is equal to the quotient of the given days, regarded as hundredths, divided by 60. The

specific rate for less than 1 year at other rates per annum than 6 % is similarly obtained; that is, *the specific rate of interest for any given time and at any given rate % per annum must equal* 1 % *of the given time expressed in days divided by such a part of* 360 *days as* 1 % *is part of the given rate per annum.* Hence, if the specific rate of interest is sought for 216 days at 5 % per annum, divide 1 % of 216 da. (2.16 da.) by $\frac{1}{5}$ of 360 da. (72 da.) obtaining 2.16 ÷ 72, or .03; to find the specific rate of interest for 225 da. at 8 % per annum, divide 1 % of 225 da. (2.25 da.) by $\frac{1}{8}$ of 360 da. (45 da.), obtaining 2.25 ÷ 45, or .05; etc.

228. Simple interest is a compensation for the use of the original principal, or for any unpaid part of the original principal.

229. Ordinary interest is interest computed in the *ordinary* manner for less than one year upon the assumption that the entire year consists of 12 months of 30 days each, or of 360 days.

230. Identification of terms. 1. *The principal is the base or multiplicand.* 2. *The specific % for the given time* [Note, **227**], *is the rate or multiplier.* 3. *The interest or the amount for the given time is the percentage or product.*

PERCENTAGE METHOD FOR COMPUTING INTEREST AT 6 %

ILLUSTRATIVE EXAMPLES

231. 1. Find the interest of \$685.74 at 6 % for 193 days.

SOLUTION

\$685.74
.193
205722
617166
68574
6)132.34782
\$22.057

EXPLANATION. \$685.74 is a *normal* base understood [**206**] or the multiplicand [**230**, 1]; 6 % is the rate or multiplier, for one year's interest, but not the specific rate for 193 da. At 6 % per annum of 360 da., the corresponding rate will be $\frac{1}{6}$ of 6 %, or 1 %, for $\frac{1}{6}$ of 360 da., or 60 da. The specific rate of interest for 193 da. must therefore be as many times 1 % of the principal as 60 da. are contained times in 193 da., or $\frac{193}{60}$ of 1 % of the principal. As $\frac{193}{60}$ of $\frac{1}{100}$ of the principal equal $\frac{193}{6000}$ of the principal, or $\frac{1}{6}$ as many thousandths of the principal as there are days, that is, $\frac{1}{6}$ of 193 thousandths of the principal, therefore,

Multiply the principal or multiplicand (\$685.74) by 6 times the specific rate or multiplier (.193), obtaining 6 times the specific percentage of interest or product (\$132.34782); and divide this interest by 6 to find the correct interest (\$22.06−).

2. Find the interest of \$417.28 for 2 yr. 228 da. at 6 %

SOLUTION

2 yr. of 360 da. = 720 da.
Fraction of a yr. = 228 da.
6).948
Specific rate.... .158

Principal = $417.28
Specific rate = .158
333824
208640
41728
$65.93024

EXPLANATION. There are twice 360 da. or 720 da. in the 2 complete years, and 228 da. in the fraction of a year, or 720 da. + 228 da. (948 da.) in the given time. Hence, $\frac{1}{6}$ of .948 of the principal, or .158 of the principal must be the specific rate of interest for the given time. Therefore,

Multiply the principal or multiplicand ($417.28) by the specific rate of interest or multiplier for 2 yr. 228 da. (.158) to find the specific percentage of interest or product ($65.93 +).

RULE. *Multiply the principal by one-thousandth of the time expressed in days, and divide the product by 6.*

NOTE 1. If the days are exactly divisible by 6, as in Ill. Ex. 2, it will be found more convenient to multiply the principal by one-sixth of the days expressed in thousandths, thus minifying the multiplication and altogether avoiding the division by 6.

NOTE 2. The interest at any higher or lower rate than 6 % may be found by proportionally increasing or diminishing the interest obtained by the preceding process. Thus, as 7 % is $\frac{7}{6}$ of 6 %, so the interest at 7 % must be $\frac{7}{6}$ of the interest at 6 %; as 5 % is $\frac{5}{6}$ of 6 %, so the interest at 5 % must be $\frac{5}{6}$ of the interest at 6 %; etc.

NOTE 3. If the time is expressed in months and days, or years, months and days, reduce it to days by allowing 360 days to each year and 30 days to each month. Though the ordinary interest for the *exact time* between two dates is required, if that interval contains one or more years, each year should *uniformly* be regarded as containing 360 days.

EXAMPLES FOR PRACTISE

At 6 % per annum, find the interest of

1. $748.35 for 48 da.
2. $813.72 for 174 da.
3. $258.37 for 253 da.
4. $521.63 for 217 da.
5. $1253.18 for 119 da.
6. $613.28 for 324 da.
7. $924.13 for 7 mo. 23 da.
8. $416.25 for 5 mo. 16 da.
9. $2358.16 for 1 yr. 8 mo. 13 da.
10. $758.30 for 3 yr. 10 mo. 24 da.

PERCENTAGE METHOD FOR COMPUTING INTEREST AT ANY PER CENT

ILLUSTRATIVE EXAMPLE

232. Find the interest of \$675.83 at 5 % for 223 days.

SOLUTION

\$675.83
2.23
202749
135166
135166
8)1507.1009
9)188.3876
\$20.9319

EXPLANATION. \$675.83 is the principal or base (multiplicand); and 5 % is the rate or multiplier for one year, but not the specific rate of interest or multiplier for 223 da.

At 5 % per annum of 360 da., the corresponding rate must be $\frac{1}{5}$ of 5 %, or 1 %, for $\frac{1}{5}$ of 360 da., or 72 da. Therefore the specific rate of interest or multiplier for 223 da. must be as many times 1 % of the principal as 72 da. are contained times in 223 da., or $\frac{223}{72}$ of 1 % of the principal. As $\frac{223}{72}$ of $\frac{1}{100}$ of the principal equal $\frac{223}{7200}$ of the principal, or $\frac{1}{72}$ as many hundredths of the principal as there are days, that is, $\frac{1}{72}$ of 223 hundredths of the principal, therefore,

Multiply the principal or multiplicand (\$675.83) by 72 times the specific rate or multiplier (2.23), obtaining 72 times the specific percentage of interest or product (\$1507.1009); and divide by 72 (= 8 × 9) to find the correct interest (\$20.93 +).

RULE. *Multiply the principal by one-hundredth of the time expressed in days, and divide the product by such a part of 360 as 1 % is of the given rate per annum.*

NOTE 1. In finding the interest by the above method, it will materially lessen the mechanical labor to cancel any factor which is common to both multiplier and divisor. Thus, to find the interest at 8 % for 63 days, instead of multiplying the principal by .63 and dividing by 45 (360 ÷ 8), it will be easier to multiply by $\frac{1}{9}$ of .63 or .07, and to divide by $\frac{1}{9}$ of 45, or 5.

NOTE 2. If the given rate per annum is not an exact divisor of 360, first find the interest at 6 %, and apply Note 2, **231**.

EXAMPLES FOR PRACTISE

Find the interest of

1. \$568.73 at 8 % for 97 da.
2. \$842.95 at 9 % for 265 da.
3. \$673.48 at 5 % for 173 da.
4. \$498.25 at 4 % for 82 da.
5. \$725.38 at 3 % for 141 da.
6. \$284.83 at 10 % for 327 da.
7. \$916.40 at 12 % for 285 da.

Find the interest of

8. \$372.50 at $4\frac{1}{2}$ % for 280 da.
9. \$563.75 at $3\frac{1}{2}$ % for 297 da.
10. \$728.34 at 7 % for 264 da.
11. \$815.93 at 8 % for 7 mo. 24 da.
12. \$425.30 at 5 % for 5 mo. 12 da.
13. \$785.90 at 4 % for 1 yr. 279 da.
14. \$893.25 at 3 % for 3 yr. 175 da.

TO COMPUTE INTEREST AT 6 % BY "THE ALIQUOT METHOD"

233. INTRODUCTION. The following method is a modification of the aliquot method in common use which will enable the calculator to verify the mechanical accuracy of the final result without reviewing all the figures used in obtaining that result. By this method, the aliquots are so arranged that each successive aliquot is an aliquot part of the aliquot *immediately preceding it*, and that the last aliquot is also an aliquot part of the initial statement. This arrangement will cause the verification of the few figures in the last aliquot to include the verification of *all* the preceding aliquots; for if the last aliquot, first obtained by an unbroken succession of derivations from the initial statement, is found to be correct by being obtainable by direct derivation from the same initial statement, then the succession of aliquots from which it was originally derived must also be correct.

ILLUSTRATIVE EXAMPLES

1. Find the interest of \$283.75 at 6 % for 210 days.

SOLUTION

\$2	83.75	= int. for		60	days
8	51 25	= "	"	180	"
1	41 87	= "	"	30	"
9	93 12	= "	"	210	"

EXPLANATION. At the rate of 6 % per annum of 360 da., the corresponding rate must be $\frac{1}{6}$ of 6 %, or 1 % of the principal for $\frac{1}{6}$ of 360 da., or 60 da.

Therefore, by means of an elongated decimal point, cut off two figures from the right of the dollars, which is equivalent to taking 1 % of the principal [Prin. 5, **120**], thus obtaining the initial interest for 60 da. (Note, **227**.)

Next find the interest for the *greatest* exact dividend of 60 da. (180 da.) which is found in the given number of days (210); and as 180 da. are 3 times 60 da., the interest for 180 da. must be 3 times the interest for 60 da. (\$2.8375 × 3), or \$8.5125.

Next find the interest for the *greatest* aliquot part of 180 da. (the *last* obtained result) which can be found in the remaining 30 da. of the given time (210 da.). As 30 is itself an aliquot part of 180 (30 = $\frac{1}{6}$ of 180), therefore $\frac{1}{6}$ of the interest for 180 da. ($\frac{1}{6}$ of \$8.5125), or \$1.4187, must be the interest for 30 da.

The interest for 30 da. added to the interest for 180 da. must equal the interest for 210 da. (\$9.93 +).

VERIFICATION. Observe if the last aliquot (\$1.4187), *supposed* to be the interest for 30 da., is one-half of the initial statement (\$2.8375) which is known

to be the interest for 60 da. If so, it should be accepted as correct, as the last aliquot ($1.4187) has now been obtained from two independent sources. If the interest for 30 da. is correct, then the interest for 180 da. must also be correct, as the former was originally derived from the latter. As a further precaution, add the two parts of the time (180 + 30) to see if they equal the entire time (210 da.). Lastly prove the addition by Note 1, **19**.

2. Find the interest of $875.93 at 6 % for 4 yr. 285 da.

SOLUTION

$8	75.93	=	int.	for	60 da.
35	03 72	=	"	"	240 "
5	83 95	=	"	"	40 "
	72 99	=	"	"	5 "
35	03 72	=	"	"	4 yr.
175	18 6				
$251	82 98	=	"		4 yr. 285 da.

EXPLANATION. If, as shown in Ex. 1, $8.7593 is the interest of $875.93 for 60 da., then the interest of $875.93 for 4 times 60 da., or 240 da. (the greatest dividend of 60 da. found in 285 da.) must be 4 times $8.7593, or $35.0372.

If the interest for 240 da. is $35.0372, the interest for $\frac{1}{6}$ of 240 da., or 40 da. (the greatest aliquot part of the immediately preceding 240 da. which is found in the remaining 45 da. of the given 285 da.) must be $\frac{1}{6}$ of $35.0372, or $5.8395.

If the interest for 40 da. is $5.8395, the interest for $\frac{1}{8}$ of 40 da., or 5 da. (the greatest aliquot part of the immediately preceding 40 da., which is found in the remaining 5 da. of the originally given 285 da.) must be $\frac{1}{8}$ of $5.8395, or $.7299.

If 6 % is the rate of interest for 1 yr., 4 times 6 %, or 24 % will be the corresponding rate for 4 yr. As the initial statement ($8.7593) is 1 % of the principal, 24 times $8.7593 (= 4 times the initial statement, or $35.0372; and 20 times the initial statement, or $175.186) must be the interest for 4 years.

The sum of the interest for these several parts of the full time is therefore the full required interest.

VERIFICATION. As a precaution against a possible mechanical error observe if the interest for 5 da. ($.7299) is $\frac{1}{12}$ of the initial interest for 60 da. ($8.7593). If it is, not only is it correct, but also all the preceding aliquots; for the interest for 5 days was originally obtained by successive derivations through each of the preceding aliquots, and a correct result cannot be derived from an incorrect.

RULE. 1. *Cut off two figures from the right of the dollars in the principal by an elongated decimal point, and regard the result as the interest for 60 days.*

2. *Take as many times the interest for 60 days as the greatest dividend of 60 days in the given days, is times 60 days.*

3. *Take such an aliquot part of the last obtained interest as will equal the greatest aliquot part of the last obtained days which is found in the remaining days of the given time.*

4. *Thus continue until the required interest for all the days of the given time has been found.*

NOTE 1. If care is taken to have the terminating aliquot days also an aliquot of 60 da., it will only be necessary to test the last aliquot interest by comparison with the initial interest for 60 days, to verify not only the last result, but also all the intermediate aliquots between it and the initial statement.

NOTE 2. If the given time contains years and days, first find the interest for the days, and verify. Then take as many times 6 times the initial statement (1 % of the principal) as the given years are times 1 year, and verify. At 6 % the interest for years is most conclusively verified by comparing one partial product with the other. Thus, the multiplier for 4 yr., is 4 times 6 %, or 24 %, hence, if the partial product by 4, when obtained, should be twice the partial product by 2, it will prove both partial products to be correct.

NOTE 3. Should any given interval in days lack an aliquot part of 60 da. of being an exact dividend of 60 da., it will usually be more convenient to increase the multiplier by 1, and deduct the aliquot excess of interest from the resulting product. Thus, to find the interest for 117 da., first find the interest for 120 da. and diminish the result by the interest for 3 da. ($\frac{1}{40}$ of 120, that is, $\frac{1}{4}$ of the interest for 120 da. written one place to the right of the orders divided).

EXAMPLES FOR PRACTISE

Verifying results, find the interest at 6 %.

	Principal	Time		Principal	Time
1.	$648.15	240 da.	**10.**	$425.93	204 da.
2.	$375.34	186 "	**11.**	$537.52	15
3.	$827.43	222 "	**12.**	$875.40	262 "
4.	$516.20	345 "	**13.**	$647.52	25 "
5.	$285.95	168 "	**14.**	$9246.30	146 "
6.	$416.62	85 "	**15.**	$17893	78 "
7.	$742.85	160 "	**16.**	$9.50	50 "
8.	$924.50	275 "	**17.**	$68.75	2 yr. 153 da.
9.	$618.80	42 "	**18.**	$87.25	7 yr. 311 da.

NOTE 4. By *compound subtraction*, the interval between two dates is most conveniently found as follows: From any date in the fifth month (say, May 19, 1912) to the same date in the eleventh month (Nov. 19, 1912) are 6 mo. of

30 da. each or 180 da., therefore to Nov. 27, or 8 da. later, the time will be 8 da. more than 180, or 188 da., and to Nov. 13, or 6 da. earlier the time will be 6 da. less than 180, or 174 da. If the interval is greater than one year, first find the number of *complete* years in the given interval, and then as above described, find the time in the remaining fractional part of a year. Thus, from Sept. 25, 1908, to July 18, 1912, are 3 yr. 293 da.; that is, from Sept. 25, 1908, to the latest corresponding date in the interval (Sept. 25, 1911) are 3 years, and from Sept. 25, 1911 (9th mo. of 1911) to July 25, 1912 (7th mo. of 1912, or the equivalent of 7 + 12, or the 19th mo. of 1911) are 10 mo. of 30 da. each, or 300 da.; and to July 18, 1912, or 7 da. earlier than July 25, 1912, are 7 da. less than 300 da., or 293 da.

NOTE 5. To find the exact number of days in any fractional part of a year, first find the time by the 30-day method [Note 4,] and add one day for each 31-da. mo. in the given interval and subtract two days if the interval includes the last day of February of a common year, or subtract one day if it includes the last day of February of a leap year.

Finding the time by Note 4, what is the interest at 6 %

19. Of \$129.60 from Sept. 18, 1910, to June 16, 1911?

20. Of \$673.45 from Mch. 23, 1908, to Nov. 13, 1912?

21. Of \$827.64 from Dec. 12, 1905, to May 6, 1911?

22. Of \$285.96 from Oct. 31, 1903, to June 17, 1911? [Note 2, 163.]

23. Of \$875.49 from May 13, 1905, to Oct. 31, 1911? [Note 2, 163.]

24. Of \$5168.35 from Nov. 27, 1902, to July 12, 1912?

25. Of \$87.90 from July 31, 1907, to Oct. 19, 1911? [Note 2, 163.]

26. Of \$8.53 from Jan. 16, 1906, to Nov. 25, 1911?

27. Of \$62.70 from June 30, 1908, to Feb. 29, 1912? [Note 2, 163.]

28. Of \$85378.15 from Feb. 21, 1908, to May 17, 1912?

29. Of \$73.92 from May 18, 1909, to Sept. 3, 1911?

30. Of \$2354.70 from June 28, 1910, to Jan. 31, 1912?

31. Of \$825.37 from Dec. 28, 1902, to Nov. 18, 1911?

32. Of \$497.85 from Apr. 24, 1907, to July 18, 1911?

33. Of \$745.93 from Aug. 18, 1905, to Sept. 24, 1912?

Finding the exact time by Note 5, what is the interest at 6 %

34. Of \$978.42 from May 23, 1911, to Dec. 16, 1911?

35. *Of* \$427.65 from Oct. 15, 1911, to Mch. 31, 1912?

36. Of $695.80 from Feb. 3, 1911, to Nov. 6, 1911?
37. Of $584.75 from June 30, 1911, to Aug. 5, 1911?
38. Of $752.35 from Dec. 15, 1911, to Dec. 22, 1911?
39. Of $859.40 from Jan. 20, 1908, to Nov. 26, 1908?
40. Of $348.59 from May 1, 1910, to Sept. 20, 1910?
41. Of $2853.60 from July 3, 1911, to Feb. 6, 1912?
42. Of $695.73 from Mch. 28, 1910, to Oct. 12, 1910?
43. Of $196.82 from Nov. 17, 1910, to Apr. 5, 1911?
44. Of $378.45 from Apr. 12, 1911, to Aug. 24, 1911?
45. Of $518.62 from Dec. 19, 1910, to Feb. 23, 1911?
46. Of $35.76 from Oct. 8, 1911, to May 23, 1912?
47. Of $9.20 from Sept. 29, 1907, to Aug. 6, 1908?
48. Of $18275.65 from Aug. 13, 1909, to Jan. 25, 1910?
49. Of $598.25 from May 16, 1904, to Nov. 12, 1909?
50. Of $643.50 from July 28, 1908, to Mch. 3, 1912?
51. Of $2468.75 from June 24, 1905, to Apr. 6, 1911?
52. Of $525.12 from Feb. 25, 1908, to Aug. 31, 1911?
53. Of $716.53 from Dec. 19, 1903, to May 25, 1908?

TO COMPUTE INTEREST AT ANY PER CENT. BY THE 6% METHOD

ILLUSTRATIVE EXAMPLE

234. Find the interest of $758.35 at 9% for 215 days?

SOLUTION

$7	58.35	=	int. at	6%	for	60	da.
22	75 05	=	"	"	"	180	"
3	79 17	=	"	"	"	30	"
	63 19	=	"	"	"	5	"
27	17 41	=	"	"	6%	"	215	"
13	58 70	=	"	"	3%	"	215	"
$40	76 11	=	"	"	9%	"	215	"

EXPLANATION. First find the interest of the given principal for the given time at 6% by **233**, obtaining $27.1741.

As 9% is one-half of 6% (3%) added to 6%, so the interest at 9% must be one-half of the interest at 6% ($13.5870) added to the interest at 6% ($27.1741), or $40.76 +.

RULE. *First find the interest at 6%, then regard whatever is over or under 6% in the given rate, as an aliquot of 6%, and increase or diminish the interest at 6% by the corresponding aliquot part of the interest at 6%.*

NOTE. If the given rate is more than one aliquot of 6 % over or under 6 %, employ Note 2, **231**.

EXAMPLES FOR PRACTISE

Verifying each result, find the interest of

1. \$578.25 at 7 % for 165 da.
2. \$297.62 at 9 % for 273 da.
3. \$815.40 at 5 % for 137 da.
4. \$765.53 at 10 % for 315 da.
5. \$654.25 at 3 % for 198 da.
6. \$485.60 at 4 % for 143 da.
7. \$816.12 at 8 % for 85 da.
8. \$3487.25 at 4½ % for 118 da.
9. \$1852.70 at 3½ % for 256 da.
10. \$96.85 at 5½ % for 174 da.
11. \$5.90 at 7½ % for 1 yr. 216 da.
12. \$816.35 at 12 % for 4 yr. 297 da.

DIRECT ALIQUOT METHOD OF FINDING THE INTEREST AT ANY PER CENT.

ILLUSTRATIVE EXAMPLE

235. Find the interest of \$647.28 at 5 % for 205 days.

SOLUTION

\$6	47.28	=	int. at 5 % for				72	da.
12	94 56	=	"	"	"	"	144	"
4	31 52	=	"	"	"	"	48	"
1	07 88	=	"	"	"	"	12	"
	8 99	=	"	"	"	"	1	"
\$18	42 95	=	int. at 5 % for				205	da.

EXPLANATION. At the rate of 5 % of the principal per annum of 360 da., the corresponding rate must be ⅕ of 5 % of the principal, or 1 % of the principal for ⅕ of 360 da., or 72 days. Therefore, by means of an elongated decimal point, cut off two orders from the right of the dollars, which is equivalent to taking 1 % of the principal [Prin. 5, **120**] and thus to obtaining the interest for 72 da. at 5 % per annum [Note, **227**].

Next find the interest for the *greatest* dividend of 72 da. (144 da.) which is found in the given number of days (205 da.); and as 144 da. are 2 times 72 da., the interest for 144 da. must be 2 times the interest for 72 da. (\$6.4728), or \$12.9456.

Next find the interest for the *greatest* aliquot part of 144 da. (the last obtained result) which can be found in the remaining 61 da. of the given time, or for 48 da. (⅓ of 144 da.). If \$12.9456 is the interest for 144 da., ⅓ of \$12.9456 or \$4.3152 must be the interest for ⅓ of 144 da., or 48 da.

Next find the interest for the *greatest* aliquot part of 48 da. (the last *obtained result)* which can be found in the remaining 13 da. of the given time,

or for 12 da. ($\frac{1}{4}$ of 48 da.). If \$4.3152 is the interest for 48 da., $\frac{1}{4}$ of \$4.3152, or \$1.0788, must be the interest for $\frac{1}{4}$ of 48 da., or 12 da.

Next find the interest for the remaining 1 da. of the given time, which is $\frac{1}{12}$ of the last obtained interest for 12 da.; or \$0.899.

Lastly add these several partial interest results, obtaining \$18.43– as the complete interest.

VERIFICATION. Observe if the last aliquot (\$.0899) is $\frac{1}{72}$ ($= \frac{1}{8}$ of $\frac{1}{9}$) of the initial statement (\$6.4728). If so, the last aliquot must be correct, and consequently all the preceding aliquots through which the last aliquot was successively derived, must also be correct. As a further precaution, add the aliquot days to see if they equal 205; and prove the final addition by Note 1, **19**.

RULE. 1. *If 360 be divided by as many times 1 as the given rate is times 1 %, the quotient will be the initial number of days for which 1 % of the principal is the initial interest.*

2. *Take as many times the initial interest as the greatest exact dividend of the initial days in the given days, is times the initial days.*

3. *Take such a part of the last obtained interest as will equal the greatest aliquot part of the last obtained days which is found in the remaining days of the given time.*

4. *Continue the last preceding process upon each following result until all the given time has been considered.*

VERIFICATION. If the successive aliquot days have been so arranged that the terminating aliquot days are also an aliquot of the initial days, by observing if the last obtained aliquot interest is also derivable from the initial interest, it will not only be a verification of itself, but also of all the aliquots which precede it.

NOTE 1. If the given time is expressed in *years* and *days*, first find the interest for the days, and verify. Then take as many times the initial interest (1 % of the principal) as will equal the rate per annum multiplied by the number of years.

NOTE 2. If the given interval in days lacks an aliquot of the initial days of being 1 more time the initial days, it will usually be more convenient to increase the multiplier of the initial interest by 1, and deduct the excess of interest from the resulting product. Thus, to find the interest for 177 da. at 8 %, first find the interest for 180 da. (4 times 45 da.) and deduct therefrom the interest for 3 da. ($\frac{1}{60}$ of 180 da.).

NOTE 3. If the given rate is not an exact divisor of 360 apply **234**. Or, find the interest at some convenient rate which is a divisor of 360; thus, at 3½ %, if more convenient, first find the interest at 3 % and add $\frac{1}{6}$ of itself; or find the interest at 4 % and deduct $\frac{1}{8}$ of itself.

EXAMPLES FOR PRACTISE

Verifying each result, find the interest of

1. \$875.90 at 9 % for 135 da.
2. \$462.75 at 4 % for 210 da.
3. \$695.82 at 5 % for 261 da.
4. \$715.60 at 8 % for 318 da.
5. \$297.50 at 3 % for 165 da.
6. \$935.75 at 10 % for 204 da.
7. \$392.18 at 12 % for 316 da.
8. \$1586.35 at 4½ % for 205 da.

Obtaining the time by the 30-day method, find the interest of

9. \$3272.24 from July 20, 1902, to Feb. 17, 1908, at 4½ %.
10. \$1873.16 from Jan. 26, 1908, to Sept. 20, 1911, at 10 %.
11. \$1384.20 from Apr. 9, 1906, to Nov. 13, 1911, at 3 %.
12. \$278.15 from Nov. 30, 1905, to Aug. 10, 1908, at 3½ %.
13. \$48.73 from Apr. 28, 1906, to Nov. 26, 1910, at 9 %.

Obtaining the exact days, find the interest of

14. \$435.76 from Oct. 13, 1905, to July 30, 1911, at 12 %.
15. \$2896.37 from Aug. 19, 1897, to May 7, 1912, at 5 %.
16. \$9183 from June 16, 1909, to Mch. 24, 1912, at 4 %.
17. \$4238.75 from May 31, 1904, to July 16, 1911, at 4½ %.
18. \$37.95 from Sept. 24, 1903, to Dec. 13, 1910, at 8 %.

INTERCHANGEABILITY OF PRINCIPAL AND TIME

236. INTRODUCTION. (*a*) In **232** it was seen that the principal and the time in days are uniform factors in finding the interest at any rate per annum. As the product of any two factors will be the same in whatever order they may be arranged, it must follow that the product of the time in days multiplied by one-hundredth of the principal will be the same as that of the principal multiplied by one-hundredth of the time in days; that is, *the principal and the time in days may be interchanged at the convenience of the calculator without affecting the correctness of the final result.* Thus, the interest of \$180 for 240 da. at any rate per annum will be the same as the interest \$240 for 180 da. at the same rate. Advantage should be taken of this fact whenever it is seen that the principal regarded as days can be separated into fewer aliquot parts of the initial days at the given rate than can the actual time in days.

(*b*) This interchange of principal and time may be made more frequently advantageous by amplifying the principles underlying *the processes* of **233** and **235**. Thus, if cutting off two orders

from the right of the dollars in any principal will produce the interest of that principal for such a part of 360 days as 1 % is of the given rate per annum, then cutting off *three* orders from the right of the dollars in the principal (being equivalent to taking one-tenth of the preceding initial interest) must produce the interest for one-tenth of the preceding initial days; and cutting off *one* order from the right of the dollars in the principal (being equivalent to taking ten times the same initial interest) will produce the interest for ten times the same initial days; and cutting off *no* order from the right of the dollars in the principal (being equivalent to taking one hundred times the same initial interest) will produce the interest for one hundred times the same initial days; etc.; etc.

(*c*) Thus, if cutting off two figures from the right of the dollars in the principal will produce the interest of that principal at 6 % for 60 da., then cutting off *three* figures must produce the interest for one-tenth of 60 da., or 6 da.; cutting off *one* figure, for 10 times 60 da., or 600 da.; cutting off *no* figure, for 100 times 60 da., or 6000 da. Similarly at 9 % per annum, cutting off three orders from the right of the dollars in the principal, will produce the interest for 4 da.; cutting off two orders, for 40 da.; one order for 400 da.; and *no* order, for 4000 da.; etc.; etc.

(*d*) To differentiate these processes, cutting off *three* orders is suggestively called the *units' rule* (3 units of days at 12 %); cutting off *two* orders, the *tens' rule* (3 tens of days); cutting off *one* order, the hundreds' rule (3 hundreds of days); and cutting off no order, the thousands' rule (3 thousands of days).

NOTE 1. As all ordinary interest is computed upon the uniform basis of 360 days to a year, if the given time is expressed in years and days, the years should be reduced to days upon the same basis, even when dates are given and the exact number of days is found in the fractional part of a year in the given interval.

NOTE 2. If the given principal contains cents, no advantage will be gained by interchanging, for the cents will then have to be regarded as such a fraction of a day as they are of one dollar.

ILLUSTRATIVE EXAMPLES

1. Find the interest of $685.72 at 4 % for 27 days. **2.** Of $275.83 at 6 % for 150 days. **3.** Of $5000 at 8 % for 123 days.

SOLUTIONS

(1)

	| $685.72	=	9 days' int.
$2	| 057 16	=	27 " "

(2)

$27	| 5.83	=	600 days' int.
$6	| 8 95	=	150 " "

(3)

$123	=	4500 days' int.
⅛ of $123 = 13	666 =	500 " "
$136	67 – =	5000 " "

EXAMPLES FOR PRACTISE

Employing the fewest figures, find the interest of

1. $496.75 at 6 % for 24 da.
2. $715.25 at 6 % for 54 da.
3. $563.70 at 6 % for 18 da.
4. $758.62 at 6 % for 27 da.
5. $435.80 at 6 % for 150 da.
6. $847.35 at 6 % for 200 da.
7. $6000 at 6 % for 327 da.
8. $800 at 6 % for 171 da.
9. $7500 at 6 % for 192 da.
10. $400 at 6 % for 259 da.
11. $400 at 8 % for 319 da.
12. $800 at 5 % for 153 da.
13. $4500 at 7 % for 137 da.
14. $300 at 9 % for 273 da.
15. $600 at 4½ % for 169 da.
16. $2000 at 12 % for 317 da.
17. $1800 at 10 % for 159 da.
18. $2500 at 6 % for 3 yr. 253 da.
19. $7200 at 5 % for 4 yr. 198 da.
20. $1800 at 3 % for 2 yr. 147 da.

ACCURATE INTEREST

237. Accurate interest is interest computed upon the accurate basis of 365 days to a common year, or of 366 days to a leap year.

NOTE 1. All disbursements of interest by the governments of the United States and of foreign nations, all calculations of interest by foreign commercial houses, and many computations by business houses in the United States, especially by American importers of foreign goods, are made upon the accurate basis.

ILLUSTRATIVE EXAMPLE

Find the accurate interest of $297.83 for 189 da. at 8 %.

SOLUTION

$297.83
.08
23.8264
189
2144376
1906112
238264
365)4503.1896($12.3375

EXPLANATION. First find the interest of the given principal for 1 year at the given rate, obtaining $23.8264. As the accurate interest for 1 day is $\frac{1}{365}$ of the accurate interest for 1 year, the accurate interest for 189 da. must be 189 times $\frac{1}{365}$, or $\frac{189}{365}$, of the accurate interest for one year, obtained by multiplying $23.8264 by the numerator (189) and dividing by the denominator (365).

RULE. *Multiply the interest for one year at the given rate per annum by the exact number of days in the given fraction of a year, and divide the result by 365 if the interval is part of a common interest year, or by 366 if part of a leap interest year.*

NOTE 2. If the given interval is more than one year, first find the accurate interest for the days in the fraction of a year, and then find the interest for the years, and add the two results.

NOTE 3. Always obtain the accurate time [**164**, or Note 5, **233**] between two dates when computing accurate interest.

NOTE 4. The learner is cautioned against the common error of confounding the *calendar year* with the *interest year*. A calendar year commences with Jan. 1 and terminates with Dec. 31, both inclusive, but an interest year commences with the date on which the debt commences to draw interest and terminates at the corresponding date of the next following year. Thus, an interest-bearing debt contracted May 18, 1911, and paid July 3, 1911, has accrued interest for 46 days of the 366-day year commencing May 18, 1911, and terminating May 18, 1912; while an interest-bearing debt contracted May 18, 1912, and paid July 3, 1912, has accrued interest for 46 days of the 365-day year commencing May 18, 1912, and terminating May 18, 1913. The leap year is so called because it includes the *leap day* (Feb. 29); therefore any *interest year* which includes the leap day must be a leap interest year, though the leap day itself may not be included in the given interval of that leap interest year.

EXAMPLES FOR PRACTISE

Find the accurate interest of

1. \$963.75 at 4 % for 82 da.
2. \$725.62 at 3 % for 174 da.
3. \$286.40 at 5 % for 230 da.
4. \$437.52 at 8 % for 161 da.
5. \$582.37 at 6 % for 271 da.
6. \$345.28 at 9 % for 168 da.
7. \$296.75 at 7 % for 93 da.
8. \$583.92 at 4½ % for 335 da.
9. \$857.42 at 3½ % for 252 da.
10. \$728.19 at 5 % for 3 yr. 87 da.
11. \$649.50 at 4½ % for 1 yr. 279 da.
12. \$484.32 at 4 % for 4 yr. 176 da.
13. \$2983.52 from Aug. 18, 1903, to May 18, 1910, at 9 %.
14. \$786.25 from Mch. 14, 1907, to Nov. 28, 1907, at 10 %.
15. \$685.97 from Apr. 23, 1904, to Dec. 23, 1912, at 8 %.
16. \$925.82 from May 14, 1905, to Sept. 20, 1911, at 12 %.
17. \$716.25 from Jan. 3, 1909, to Feb. 18, 1912, at 3½ %.
18. \$1586.73 from June 19, 1895, to Mch. 17, 1900, at 7 %.
19. \$6178.30 from May 23, 1910, to Aug. 16, 1913, at 5%.
20. \$875.45 from Oct. 12, 1909, to July 17, 1912, at 3½ %.
21. \$2495.18 from Feb. 29, 1908, to Oct. 27, 1912, at 4%.

NOTE 5. Reduce the lower denominations of English money to a decimal of a £ as in Ill. Ex. 2, **160**, compute the interest in the usual manner, and reduce the decimal part of a £ in the obtained answer to integers of the lower denominations as in Ill. Ex., **159**.

22. Find the accurate interest of £178 15s. 6d. from May 3, 1908, to Nov. 18, 1913, at 4% per annum.

23. What is the accurate interest of £3487 2s. 6d. from Sept. 25, 1908, to June 10, 1913, at 3% per annum?

24. What is the accurate interest of £625 18s. 3d., at 5%, from Mch. 21, 1910, to Sept. 6, 1913?

25. Find the accurate interest at 3½% per annum of £345 12s. 6d. from Nov. 3, 1905, to May 26, 1913.

RELATION OF THE FOUR ELEMENTS OF INTEREST

238. INTRODUCTION. (*a*) A problem in interest is one in which *three* of the four elements of interest *are given*, and it is required to find the omitted fourth element. The four elements (principal, rate per annum, time, interest or amount) have a uniform relation to each other; so that when any three of them are given, it is possible to complete the relation by finding the necessary omitted element. The most useful methods for finding the interest when it is the omitted element have already been presented [**231–236**]. It now remains to consider problems in which the interest is not required as heretofore, but is given, together with any two of the three remaining elements (principal, rate, time), and it is required to find the omitted remaining element. The following general process is applicable to all such problems:

(*b*) Find the interest or the amount by any one of the preceding methods which is most favored, for all the units of the given elements, and *computed for 1 unit of the required element;* then divide the given interest or amount for all the units of the required element by the obtained interest or amount for *1* unit of the required element, to find the number of units in the required element.

SUMMARY. 1. *The total interest or amount of the total principal for the total time and at the total rate % per annum, is a product.* 2. *The total interest or amount of any two of the three remaining elements for 1 unit of the required element, is the multiplicand.* 3. *The number of units in the required element is the multiplier.*

PRINCIPAL, INTEREST AND TIME GIVEN, TO FIND THE RATE

ILLUSTRATIVE EXAMPLE

239. At what rate per annum will \$758.36 produce \$13.65 interest in 216 days?

SOLUTION

\$7	58.36	= int. for	60	da. at	6	%	
22	75 08	= " "	180	"	"	"	
3	79 18	= " "	30	"	"	"	
	75 83	= " "	6	"	"	"	
6)27	30 09	= " "	216	"	"	"	
4	55 01	= " "	216	"	"	1	%

\$4.5501)\$13.6500 { 2, practically 3 times } { the int. at 1 %, or 3 % }

9 1002

4 5498 = significant remainder

EXPLANATION. First find the interest of the given principal (\$758.36) for the given time (216 da.) at 6 %, obtaining \$27.3009 and divide by 6 to find the interest at 1 % (\$4.5501).

As the given interest (\$13.65) is 2 times the interest at 1 %, and a very significant remainder almost equal to the divisor [**205**, *b*], it should be regarded as practically 3 times the interest at 1 %, or 3 %.

SUMMARY. *The required rate % per annum will be as many times 1 %, as the given interest is times the obtained interest at 1 % of the same principal for the same time.*

NOTE 1. If the remainder from the final division is *insignificant*, follow **205**, *a*; if *significant*, follow **205**, *b*; and if *intermediate*, follow **205**, *c*. In practise, the division is rarely exact.

NOTE 2. If the amount [Note, **226**] is given, diminish it by the principal to find the interest, and then proceed as in the preceding solution. *The amount at* 3 % (103 %) *is not* 3 *times the amount at* 1 % (101 %).

EXAMPLES FOR PRACTISE

At what rate % per annum will

1. \$596.85 produce \$19.70 interest in 132 days?
2. \$1237.29 produce \$43.99 interest in 256 days?
3. \$3278.45 produce \$163.92 interest in 225 days?
4. \$45.97 produce 62 cents interest in 108 days [**205**, *c*]?
5. \$257.30 amount to \$273.31 in 320 days?
6. \$768.25 amount to \$781.79 in 141 days [**205** *c*]?

PRINCIPAL, INTEREST AND RATE GIVEN, TO FIND THE TIME

ILLUSTRATIVE EXAMPLE

240. In what time will \$568.75 produce \$76.78 interest at 5 % per annum?

SOLUTION

\$568.75 × .05 = \$28.4375, 1 yr.'s int.

\$28.4375)\$76.7800(2 years
56 8750
19.9050
360
1194 3000
5971 50
2
28.4375)7165.8000(251 days
5687 50
1478 300
1421 875
56 4250
28 4375
27 9875, significant

2 yr. 252 da., Ans.

EXPLANATION. The interest of the given principal (\$568.75) at the specific rate per annum (5 %) equals \$28.4375 for 1 year. Therefore it will require as many years to produce \$76.78 interest as \$28.4375 are contained times in \$76.78, or 2 yr. and $\frac{19905}{113750}$ of a year.

As ordinary interest is computed upon the basis of 360 da. to a year, there will be as many days in $\frac{19905}{113750}$ of a yr., as it contains 360ths. Hence, reduce the fraction of a year to 360ths by **86**, obtaining 251 da. and a very significant remainder almost equal to the divisor which should be counted as another day [**205**, *b*,] making the required time 2 yr. 252 da.

SUMMARY. *The required time will be as many times one year or one day, as the given interest is times the interest for one year or for one day, of the same principal and at the same rate per annum.*

NOTE 1. Apply **205**, *a*, to all insignificant remainders from the final division; and **205**, *b*, to all significant remainders.

NOTE 2. If the given interest is less than the interest for one year, divide it at once by the interest of the same principal for 1 day.

EXAMPLES FOR PRACTISE

In what time will

1. \$958.63 produce \$32.59 interest at 8 % per annum?
2. \$7285.15 produce \$166.65 interest at 4½ % per annum?
3. \$815.30 produce \$14.90 interest at 7 % per annum?
4. \$3825.75 produce \$130.50 interest at 4 % per annum?
5. \$8270.75 amount to \$10236.43 at 6 % per annum?

INTEREST, TIME, AND RATE GIVEN, TO FIND THE PRINCIPAL

ILLUSTRATIVE EXAMPLE

241. What principal loaned for 279 days at 8 % per annum, will produce $57.50 interest?

SOLUTION

2.79 ÷ 45 = .062, int. of $1 at 8 % for 279 da.
$57.50 ÷ .062 = 927.42 times $1, or $927.42.

EXPLANATION. 8 % per annum equals 8 cents per annum for each dollar of the required principal. If the interest of $1 for one year of 360 da. is 8 cents, the interest of $1 for $\frac{1}{8}$ of 360 da., or for 45 da., must be $\frac{1}{8}$ of 8 cents, or 1 cent. If the interest of $1 is 1 cent for each 45 da. of the given time, it will be as many cents for 279 days as 45 da. are contained times in 279 da., or 6.2 cents. Hence, the required principal must be as many times $1, as the given interest of the required principal ($57.50) is times the interest of $1 of principal ($.062), for the same time and at the same rate per annum, or 927.42 times $1.

SUMMARY. *The required principal will be as many times $1, as the given interest of the required principal is times the interest of $1 for the same time and at the same rate % per annum.*

NOTE. To find the interest of $1 for any given time and at any given rate per annum, *divide the given days regarded as cents by such a part of* 360 *as 1 % is of the given rate per annum.* Thus, the interest of $1 for 152 da. at 9 % per annum equals 1.52 ÷ (360 ÷ 9) or $.038 [Note, **227**]. If the given rate is not an exact divisor of 360, first find the interest of $1, at some other convenient rate which is an exact divisor of 360, and add to, or take from, the obtained result as the given rate is greater or less than the assumed rate.

EXAMPLES FOR PRACTISE

What principal will produce

1. $4.08 interest at 3 % in 84 days?
2. $295.58 interest at 8 % in 324 days?
3. $169.87 interest at 6 % in 247 days?
4. $1524.75 interest at 9 % in 1 yr. 282 da.?
5. $5.58 interest at 5 % in 96 days?
6. $202.71 interest at 7 % in 144 days?
7. $445.73 interest at 4 % in 2 yr. 315 da?

AMOUNT, TIME, AND RATE GIVEN TO FIND THE PRINCIPAL

ILLUSTRATIVE EXAMPLE

242. What principal will amount to \$349.59 in 51 days at 4 % per annum?

FIRST SOLUTION

\$.51 ÷ 90 = \$.005⅔, int. of \$1 at 4 %
\$1 + \$.005⅔ = \$1.005⅔, amt. of \$1 at 4 %
\$1.005⅔)\$349.59

3 3
3.017)1048.770(347.62 times \$1
905 1
143 67
120 68
22990
21119
18710
18102
6080
6034
46

FIRST SOLUTION. The interest of \$1 for 51 da. at 4 % is \$.005⅔ [Note, 241]. This, added to \$1, will be the amount of \$1 for the same time and at the same rate. As the given amount (\$349.59) is 347.62 times the amount of \$1 of principal (\$1.005⅔) for the same time and at the same rate, so the principal which produced the given amount must be 347.62 times \$1, or \$347.62.

SECOND SOLUTION

\$349.59
90
\$90.51)\$31463.10(347.62 times \$1
27153
43101
36204
68970
63357
56130
54306
18240
18102
138

SECOND SOLUTION. If it is seen that the interest of \$1 for the same time and at the same rate [Note, 241] will include an inconvenient fraction, as it usually does, the second solution will be found preferable.

To multiply both divisor and dividend by the same number will not affect the value of the quotient [Prin. 5, 34]. Therefore, instead of dividing the given amount (\$349.59) by the amount of \$1 at the same rate and for the same time (\$1 + $\frac{51}{90}$) as in the first solution, it will usually involve less labor to divide 90 times the given amount (\$349.59 × 90 = \$31463.10)

by 90 times the amount of $1 for the given time and at the given rate ($1 \times 90 + .\overline{51} \times 90 = 90.51$). Hence, multiply the given *amount by such a part of 360 as 1 % is of the given rate per annum* (in the Ill. Ex., by $360 \div 4$, or 90); *and divide the result by the same quotient* (90) *increased by the days regarded as hundredths* (.51), *or by 90.51.*

SUMMARY. *The required principal is as many times $1 of principal, as the given amount of the required principal is times the obtained amount of $1 of principal for the same time and at the same rate % per annum.*

NOTE. If, in using the second solution, the given rate (as 7 %, 3½ %, etc.) is not an exact divisor of 360, the resulting fraction may be avoided *by multiplying the given amount by 360, and dividing the result by 360 increased by as many times the days of prepayment regarded as hundredths as the given rate is times 1 %.*

Thus, in the Ill. Ex., ($349.59 \times 360) \div (360 + .\overline{51} \times 4) = $347.62. That is, instead of dividing 90 times the given amount by 90 times the amount of $1, as in the *second solution*, by this process 4 times 90 times, or 360 times, the given amount is divided by 4 times 90 times the amount of $1 ($\overline{90 \times 4} + .\overline{51} \times 4$) or by 362.04 [Prin. 5, **34**].

EXAMPLES FOR PRACTISE

What principal will amount to

1. $628.73 in 243 days at 8 % per annum?
2. $1965.37 in 162 days at 6 % per annum?
3. $748.26 in 117 days at 3 % per annum?
4. $3562.80 in 327 days at 7 % per annum [See Note]?
5. $523.42 in 1 yr. 257 da. at 4½ % per annum?
6. $2685.72 in 2 yr. 168 da. at 3½ % per annum?
7. What principal will amount to $986.35 on May 16, 1911, if loaned on Aug. 28, 1910, at 5 % (exact days)?
8. What sum of money loaned Apr. 16, 1911 at 4 %, will amount to $1628.15 on Nov. 27, 1911 (time by 30-da. method)?

COMPOUND INTEREST

243. Compound interest is interest computed in the usual manner, but which, at the end of a stipulated period of time, is *compounded* with (that is, added to) the principal for that period, to form a new principal for the next following period.

ILLUSTRATIVE EXAMPLE

Find the interest of \$678.45 for 1 yr. 320 da., if compounded semi-annually at 6% per annum.

SOLUTION

\$678.45	(first principal)
20.3535	(3% of \$678.45)
698.8035	(first amount)
20.9641	(3 % of \$698.8035)
719.7676	(second amount)
21.5930	(3 % of \$719.7676)
741.3606	(third amount)
17.2984	(int. of \$741.36 for 140 da.)
758.6590	(final amount)
678.45	(original principal)
80.21 −	(comp. int. for 1 yr. 320 da.)

EXPLANATION. At 6 % per annum, the corresponding rate is ½ of 6 %, or 3 %, for one-half a year.

1 yr. = 2 half-years; and 320 da. = 1 half year (½ of 360 da. = 180 da.) and 140 da. of a fourth half year. Hence, 1 yr. 320 da. = 3 half-years and 140 days.

To secure accuracy in the final result, carry all intermediate results to *four* decimal places. As hundredths × hundredths = ten-thousandths, so to obtain the necessary four decimal places, multiply the cents' order of the principal (hundredths) by the rate % per period (3 hundredths), and write the first order of the product underneath the principal but two places to the right of the cents' order of the principal (that is, in the ten-thousandths' order). Add the obtained interest for the first period (\$20.3535) to the first principal (\$678.45), to find the first amount (\$698.8035), which will constitute a new principal for the second half-yearly interest period.

Continue this process as many times as there are complete interest periods (3 times). Then find the interest of the amount for the last complete period (\$741.3606) for the fraction of the fourth period (140 da.) at 6 % per annum by **233**, obtaining \$17.2984, which added to the last obtained amount (\$741.3606) will produce the final amount (\$758.66 −).

The difference between the original principal (\$678.45) and the final amount (\$758.66) will be the required compound interest (\$80.21 −).

NOTE 1. If the given time contains a fraction of a year expressed in days, regard 180 days thereof (if it contains so many) as another half-year if the compounding of interest is semi-annual; or regard each 90 days thereof as another quarter, if the compounding of interest is quarterly. If dates are given, and there is a fraction of a year, regard each successive six calendar months thereof, irrespective of the number of days they may contain, as another half-year if the compounding is semi-annual, or each three calendar months thereof as another quarter if the compounding is quarterly; and compute the time for any possible remaining fractional part of an interest *period in the customary* manner.

EXAMPLES FOR PRACTISE

Find the compound interest of

1. $4375.80 for 3 yr. at 6% per annum, comp. annually.
2. $965.25 for 4 yr. at 4% per annum, comp. half-yearly.
3. $1825.30 for 2 yr. at 5% per annum, comp. quarterly.
4. $2469.75 for 4 yr. at 7% per annum, comp. annually.
5. $6138.26 for 2 yr. at 8% per annum, comp. quarterly.
6. $594.65 for 1 yr. 270 da. at 6%, comp. half-yearly.
7. $763.40 for 1 yr. 195 da. at 4% per annum, comp. quarterly.
8. $298.84 for 2 yr. 78 da. at 3%, comp. half-yearly.
9. $3297 for 1 yr. 54 da. at 5% per annum, comp. quarterly.
10. $682 for 2 yr. 173 da. at 4½% per annum, comp. annually.
11. $456.75 for 2 yr. 135 da. at 9%, comp. half-yearly.
12. $791.62 for 1 yr. 198 da. at 8%, comp. quarterly.
13. No payment of interest having been previously made, what is the amount due Oct. 23, 1911, upon a debt of $578.60, contracted May 12, 1908, at 5% per annum, interest to be compounded annually if not paid (time by compound subtraction)?
14. What payment on June 17, 1912, will discharge a debt of $1837.15, incurred Nov. 25, 1910, drawing interest at 4%, payable quarterly, if the debtor agreed to compound all deferred interest payments, and had permitted the second and fourth interest payments to lapse (time in exact days)?
15. All the interest payments having lapsed, what was the balance due Sept. 10, 1910, upon a note of $725.65, dated June 21, 1908, drawing interest at 4½% per annum, payable semi-annually and specifying that the interest is to be compounded if not paid (time by comp. subtraction)?
16. What was the amount due Aug. 17, 1912, upon a note of $648.75, dated Feb. 17, 1911, drawing interest from date at 5% per annum, and to be compounded quarterly if not paid, if no payment had been made previous to final settlement (time by compound subtraction)?

NOTE 2. Much labor in computing compound interest may be saved by the use of a complete compound amount table. The following abridgment will sufficiently illustrate its use:

COMPOUND AMOUNT TABLE

Periods	1%	2%	3%	4%	5%	6%	Periods
1	1.0100 000	1.0200 0000	1.0300 0000	1.0400 0000	1.0500 000	1.0600 000	1
2	1.0201 000	1.0404 0000	1.0609 0000	1.0816 0000	1.1025 000	1.1236 000	2
3	1.0303 010	1.0612 0800	1.0927 2700	1.1248 6400	1.1576 250	1.1910 160	3
4	1.0406 040	1.0824 3216	1.1255 0881	1.1698 5856	1.2155 063	1.2624 770	4
5	1.0510 101	1.1040 8080	1.1592 7407	1.2166 5290	1.2762 816	1.3382 256	5
6	1.0615 202	1.1261 6242	1.1940 5230	1.2653 1902	1.3400 956	1.4185 191	6
7	1.0721 354	1.1486 8567	1.2298 7387	1.3159 3178	1.4071 004	1.5036 303	7
8	1.0828 567	1.1716 5938	1.2667 7008	1.3685 6905	1.4774 554	1.5938 481	8
9	1.0936 853	1.1950 9257	1.3047 7318	1.4233 1181	1.5513 282	1.6894 790	9
10	1.1046 221	1.2189 9442	1.3439 1638	1.4802 4428	1.6288 946	1.7908 477	10
11	1.1156 683	1.2433 7431	1.3842 3387	1.5394 5406	1.7103 394	1.8982 986	11
12	1.1268 250	1.2682 4179	1.4257 6089	1.6010 3222	1.7958 563	2.0121 965	12
13	1.1380 933	1.2936 0663	1.4685 3371	1.6650 7351	1.8856 491	2.1329 283	13
14	1.1494 742	1.3194 7876	1.5125 8972	1.7316 7645	1.9799 316	2.2609 040	14
15	1.1609 690	1.3458 6834	1.5579 6742	1.8009 4351	2.0789 282	2.3965 582	15
16	1.1725 786	1.3727 8570	1.6047 0644	1.8729 8125	2.1828 746	2.5403 517	16
17	1.1843 044	1.4002 4142	1.6528 4763	1.9479 0050	2.2920 183	2.6927 728	17
18	1.1961 475	1.4282 4625	1.7024 3306	2.0258 1652	2.4066 192	2.8543 392	18
19	1.2081 090	1.4568 1117	1.7535 0605	2.1068 4918	2.5269 502	3.0255 995	19
20	1.2201 900	1.4859 4740	1.8061 1123	2.1911 2314	2.6532 977	3.2071 355	20

DIRECTIONS. 1. To find the compound amount, multiply the amount of $1 for the given number of periods and at the rate % per period, as given in the above table, by the given principal.

2. To find the compound interest, deduct $1 from the amount of $1 for the given number of periods and at the rate % per period, as given in the above table, to find the compound interest of $1; and multiply by the given principal.

CAUTION. The learner is warned against the error of supposing that the compound amount or compound interest at one rate per period is proportioned to the compound amount or compound interest at some other rate for the same time; that is, that the compound amount or compound interest at 7 % is one-sixth more than the compound amount or compound interest at 6 %. For, $1 + the interest of that $1, at 7 %, cannot evidently be one-sixth more than another $1 + the interest of that other $1, at 6 %; that is, that ($1 × 1.07 × 1.07 × 1.07) − $1, or the compound interest of $1 at 7 %, for 3 years, can be one-sixth more than ($1 × 1.06 × 1.06 × 1.06) − $1, or the compound interest of $1 at 6 % for the same number of years. If their multiplicands

are the *same*, simple products will be proportioned to their respective multipliers; but *continued products* have constantly varying multiplicands, and cannot therefore be thus proportioned.

17. Find the compound amount of \$785.30 for 8 years at 6 % per annum compounded semi-annually.

SOLUTION

8 years = 16 half-years; 6 % per annum = 3 % per half-year.

\$1.60470644 = amt. of \$1 for 16 periods at 3 % per period.

$\times$ 785.30 = given number of dollars.

\$1260.175967332 = comp. amt. of \$785.30 for 16 periods at 3 %.

Find from the table the compound

18. Amount of \$968.35 for 13 yr. at 6 % per yr., comp. yearly.

19. Amount of \$1873.42 for 9 yr. at 5 % per yr., comp. yearly.

20. Amount of \$628.95 for 6 yr. at 8 %, comp. half-yearly.

21. Amount of \$716.25 for 8 yr. at 4 %, comp. half-yearly.

22. Interest of \$845.73 for 3 yr. at 12 %, comp. quarterly.

23. Interest of \$467.28 for 4 yr. at 4 %, comp. quarterly.

24. Interest of \$2875.50 for 7 yr. at 6 %, comp. yearly.

25. Interest of \$943.25 for 5 yr. at 10 %, comp. half-yearly.

TRUE DISCOUNT

244. INTRODUCTION. The sum to be paid in discharge of a debt will vary with the time of payment. If paid on the due date, the sum to be paid will be the debt itself, without any increase for interest or decrease for discount; if paid after it is due, the sum to be paid will be the original debt increased by the accrued interest thereon from its due date to the date of postpayment; and if paid before it is due, the sum to be paid will be the original debt diminished by the interest on the prepayment from the date of prepayment to the due date of the debt. That is, interest is a compensation for the use of money [**224**, *b*]. Therefore, if money is used *after* the stipulated time of payment, the debtor should pay compensatory interest thereon for every day during which the creditor has been deprived of the use of what was lawfully his; and if money is paid *before* the stipulated time, the debtor should receive compensatory interest thereon for every day during which he has deprived himself of the use of that which was

legally his. In either case, the question is, what sum paid at a preceding or subsequent date, is equivalent to a given sum which is legally payable at a certain fixed date; or, with particular reference to a prepayment as in true discount, *what sum of money placed at interest on the date of prepayment at the current rate per annum will amount to the same sum as the given debt on the date upon which that debt is due.*

245. The **worth** of a debt is its value as expressed in the legal currency of the country in which it is estimated.

246. The **present worth** of a debt is its value on the day it is paid.

NOTE. The true or correct value of a debt due at a certain future time and paid before its maturity, that is, the *present worth* of that debt by *true discount*, must be such a principal [**226**] as loaned on the day of prepayment at the current rate per annum, will amount to that debt on the day of maturity [**242**]; for the creditor will then have received the principal and the use (interest) of that principal for the days of prepayment, and if the two combined (amount) are equal to the debt at maturity, it must be an equitable discharge of that debt.

247. The **future worth** of a debt is its value on the day it is due.

NOTE. Future refers to *any* time yet to come; but as here used, it is restricted to that *specific future* time at which a debt falls due. Hence, the future worth of a non-interest bearing debt must be the debt itself; but the future worth of an interest bearing debt must be the debt plus the accrued interest thereon from the date upon which it commenced to draw interest to its due date.

248. **True discount** is the deduction from the future worth of a debt which is equal to the interest upon its present worth for the days of prepayment.

NOTE. True discount is so-called because it is the *true* or correct deduction; that is, it is the compensatory interest upon the *exact* sum of the use of which the debtor has deprived himself by paying the debt before it was legally due.

ILLUSTRATIVE EXAMPLE

249. A debt of $764.50, legally due Oct. 19, 1911, was paid June 25, 1911. What was the sum paid, allowing true discount at 6%? What was the true discount?

SOLUTION

June 25 to Oct. 19 = 116 da.

60. $764.50

1.16 60

61.16)45870.00($750 Present Worth.

42812

30580

30580

0

$764.50 − $750. = $14.50, true disct.

EXPLANATION. The present solution is similar to the second solution of Ill. Ex., **242**, to which the student is referred for a full explanation.

Divide 60 times the percentage of future worth ($764.50 × 60 = $45870) by 60 times its specific rate ($1 + \frac{1}{6}.1^{6}$) × 60, or 61.16, obtaining $750 as the base, or required present worth.

If preferable, the first solution of Ill. Ex., **242**, may be used.

VERIFICATION. Find the interest of the obtained present worth ($750) for the days of prepayment (116), and add this interest ($14.50) to the present worth ($750). If the result ($764.50) equals the future worth, the solution may be regarded as mechanically correct.

250. SUMMARY. *True discount* is an application of **242**, the present worth of a debt corresponding to the *principal;* the *true discount*, to the *interest* of the principal for the days of prepayment; the *future worth*, to the *amount;* the *days of prepayment* to the *time;* and the *rate % of discount*, to the *rate*. Hence, to find the present worth:

RULE. *Multiply the future worth by such a part of 360 as 1 % is part of the given rate per annum, and divide the product by the same part of 360 increased by the days of prepayment regarded as hundredths.*

NOTE 1. To avoid fractions, if the rate per annum of discount is not an aliquot part of 360, multiply the future worth by 360, and divide the resulting product by 360 increased by as many times the days of prepayment regarded as hundredths, as the given rate is times 1 %. Thus, in the Ill. Ex., ($764.50 × 360) ÷ (360 + 1.16 × 6) = $750 [Note, **242**].

NOTE 2. To find the present worth of an interest-bearing debt, first find its future worth (amount due at maturity, Note, **247**); and then proceed as with other future worths.

NOTE 3. If the time of prepayment is expressed in years or months, reduce to days upon the basis of 360 days to each year, or of 30 days to each month. If dates are given, find the days of prepayment in accordance with local usage.

EXAMPLES FOR PRACTISE

Find the present worth and the true discount of

1. \$968.75 at 6%, if paid 49 days before maturity.
2. \$723.46 at 5%, if paid 127 days before maturity.
3. \$2965.80 at 4½%, if paid 213 days before maturity.
4. \$7816.35 at 8%, if paid 53 days before maturity.
5. \$527.90 at 7%, if paid 165 da. before maturity [Note 1].
6. \$845.12 at 3½%, if paid 87 days before maturity.
7. \$693.50 at 3%, paid 2 yr. 139 da. before maturity [Note 3].
8. \$378.65 at 4%, paid 1 yr. 173 da. before maturity.
9. Find the true discount on \$1945.80 at 5%, if paid 98 days before it is due.
10. What is the true discount on a debt of \$635.70, at 6%, if paid 253 days before maturity?
11. A house was bought for \$3600, payable one-half cash, and the remainder in 6 months without interest. What equivalent single cash payment can be made on date of purchase, allowing true discount on the deferred payment at 5% per annum?
12. The same kind of broadcloth can be bought from A at \$3.50 per yard and 2 months' credit, and from B at \$3.58 per yd. and 6 months' credit. Of whom will it be more advantageous to buy; and how much will be gained by purchasing 95 yards at the more favorable of these terms, allowing true discount at 6% per annum?
13. Merchandise amounting to \$726.85 was bought on May 23, 1911, at 90 days, upon which a trade discount of 25%, 20%, and 10% was allowed. What cash payment on July 7, 1911, will be in full payment of the purchase, allowing true discount at 4½% per annum, and computing the exact time in days?
14. The same brand of flour can be bought from one dealer at \$6.25, and 4 months' credit, and from another dealer at \$6.20 cash. How much per barrel will be saved by purchasing at the more advantageous of these terms, allowing true discount at 5% per annum?
15. A debt of \$1785.90, bearing interest at 5% from Apr. 17, 1911, and legally due Nov. 13, 1911, was paid Aug. 6, 1911.

What was the sum paid, allowing true discount at 6 %, and computing the exact time in days?

16. Merchandise amounting to $948.60 was bought on 3 months' credit. What payment made 1 mo. 12 da. after the date of purchase will discharge the debt, allowing true discount at 4 %?

BANK DISCOUNT

251. Bank discount is a deduction from the future worth of a debt which is equal to the interest upon that future worth for the number of days it is paid before maturity.

NOTE. As true discount is the difference between the present worth of a debt (the principal in true discount, **248**) and its future worth (the principal in bank discount), so the difference between the two discounts must be the interest upon the difference between the present worth and the future worth, that is, the interest upon the true discount.

252. The **maturity** of a debt is the date upon which it is legally due.

NOTE 1. By the law of a few States, if a debt is expressed in the form of a note or draft, 3 days, called *days of grace*, are added to the nominal time (time named upon the face of a note or draft) in determining its maturity. The *nominal maturity* of a note or draft is therefore the expiration of the time named upon its face; and the *legal maturity*, if grace is allowed, will be 3 days thereafter. If no grace is allowed, as in most States, the two maturities fall upon the same date.

NOTE 2. In finding the maturity of commercial paper which is drawn payable a given number of *months* after date, if it should fall nominally due on the 29th, 30th, or 31st of a month which contains fewer than that number of days, it will be nominally due on the *last day* of such a month; and if grace is allowed, it will be legally due on the third day of the next following month [Note 2, **163**].

253. The **term of discount** is the number of days between the date of discount and the date of legal maturity.

NOTE. In computing the term of discount, the custom of many cities is to include within the interval both the day of discount and the day of maturity, which is equivalent to calculating discount for 1 day more than the time as ordinarily computed.

254. The **proceeds** of a note or draft is its *cash value* on the day upon which it is discounted.

NOTE. The *proceeds* is what remains of the future worth after deducting the interest thereon for the days of prepayment.

255. Identification of terms. 1. *The sum due at maturity is the principal* (multiplicand). 2. *The specific rate of discount or of proceeds is the rate* (multiplier). 3. *The discount or the proceeds is the percentage* (product).

NOTE. As the proceeds of a debt equals the sum due at maturity *minus* the interest thereon for the days of prepayment [Note, **254**], so the specific rate of proceeds must equal the rate of the sum due at maturity or base (100 %) *minus* the specific rate of interest for the days of prepayment at the given rate of interest per annum [Note, **227**].

TO FIND THE BANK DISCOUNT OR THE PROCEEDS

ILLUSTRATIVE EXAMPLE

256. A note of \$875.90, dated May 24, 1912, legally payable 90 days after date, was discounted on July 5, 1912, at 6 %. Find the date of maturity, the term of discount, the discount, and the proceeds.

SOLUTION

May 24 + 90 days = Aug. 22, maturity.
July 5 to Aug. 22 = 48 da., term of discount.

$$\begin{array}{r|l} & \$875.90 \\ \hline 7 & 007\ 2 \end{array} \begin{array}{l} = \text{discount for 6 da. at 6 \%.} \\ = \text{discount for 48 da. at 6 \%.} \end{array}$$

\$875.90 − \$7.01 = \$868.89, proceeds

EXPLANATION. 90 days after the date of the note (May 24) will require the 7 remaining days of May + the 30 da. of June + the 31 da. of July (= 68 da.), and 22 da. of Aug., thus making Aug. 22, the expiration of the 90th day, or the date of maturity.

From July 5 (the date of discount) to Aug. 22 inclusive (the date of maturity) = 48 da. (the days of prepayment, or term of discount).

Interest of the sum due at maturity (face of note, \$875.90) for the time of prepayment (48 da.) equals the compensatory discount for paying the note before it is legally due (\$7.01 −).

The worth at maturity (\$875.90) minus the discount for prepayment (\$7.01) equals the proceeds or present worth of the note on the date of discount (*\$868.89*).

SUMMARY. 1. *Bank Discount is the interest upon the face* of a non-interest-bearing note, or upon the amount due at maturity of an interest-bearing note, or upon the future worth of any form of debt, for the number of days it is paid before it is legally due.* 2. *The proceeds equal the sum due at maturity minus the bank discount.*

* NOTE 1. The *face value*, or as it is commonly abbreviated, the *face* of a note, is the value expressed in the body of the note, *exclusive of interest.*

NOTE 2. An interest-bearing note has *three* values: its face value, its value at maturity (future worth), and its cash value on the day it is discounted or paid (proceeds). In non-interest-bearing notes, the face value also expresses the value at maturity or future worth.

NOTE 3. In allowing interest upon interest-bearing notes, when the time of payment is expressed in months, most banks compute only 30 days to each of the expressed months; but, in computing the term of discount upon such notes, the *exact* number of days of prepayment is found.

EXAMPLES FOR PRACTISE

[TO THE STUDENT. In the following examples, allow no grace and find the exact number of days in the term of discount, unless otherwise directed by the Teacher to conform with a differing local usage such as Note 1, **252** or Note, **253**.]

Find date of maturity, term of disc., disc., and proceeds.

	Face of note	Date of note	Term of note	Date of disc.	% of disc.
1.	$596.75	May 17, 1911	90 days	June 12, 1911	5 %
2.	$842.90	Nov. 25, 1911	4 months	Jan. 7, 1912	6 %
3.	$285.32	Feb. 15, 1912	60 days	Mch. 9, 1912	4½ %
4.	$1763.25	Dec. 3, 1911	3 months	Feb. 18, 1912	7 %
5.	$934.65	Oct. 19, 1910	30 days	Oct. 23, 1910	4 %
6.	$2481.12	Mch. 31, 1912	3 months	Apr. 20, 1912	3 %
7.	$672.85	Sept. 24, 1911	90 days	Oct. 2, 1911	3½ %
8.	$486.50	Jan. 8, 1911	4 months	Feb. 16, 1911	5 %
9.	$3674.15	Apr. 20, 1912	60 days	May 5, 1912	8 %
10.	$398.48	June 12, 1911	6 months	Aug. 8, 1911	5½ %
11.	$725.30	Aug. 18, 1911	90 days	Sept. 3, 1911	6 %
12.	$486.28	July 31, 1911	4 months	Aug. 24, 1911	9 %

13. Find the proceeds of the following note, if discounted July 6, 1912, at 8%:

$2576.^{00}$ BOSTON, MASS., May 26, 1912.

Three months after date, value received, I promise to pay to Fairman A. Sadler, or order, Two Thousand Five Hundred Seventy-six Dollars.

R. MORTIMER BROWNING.

14. Find the proceeds of the following note, if discounted Nov. 17, 1911, at 5%:

$845.75 CHICAGO, ILL., Oct. 18, 1911.

Ninety days after date, value received, I promise to pay Philip A. Smith, or bearer, Eight Hundred Forty-five $\frac{75}{100}$ Dollars, with interest at 6%.

JAMES J. DAVIES.

15. Find the proceeds of the following note, if discounted May 23, 1912, at 4½%, applying Note 3:

$713\frac{55}{100}$ BALTIMORE, MD., March 3, 1912.

Four months after date, value received, I promise to pay to Allen S. Will, or order, Seven Hundred Thirteen $\frac{55}{100}$ Dollars, with interest at 5%.

GEORGE C. ROUND.

16. Find the proceeds of the following draft, if accepted July 17, 1912, and paid Aug. 20, 1912, allowing discount at 4%:

$491\frac{25}{100}$ NEW ORLEANS, LA., July 13, 1912.

At sixty days sight, pay to the order of Henry A. Griesemer & Co., Four Hundred Ninety-one $\frac{25}{100}$ Dollars, value received, and charge to the account of

GEORGE W. WHITESIDE.

To BROWN BROS. & CO.,
BALTIMORE, MD.

17. Find the proceeds of the following draft, if discounted Sept. 18, 1911, at 4½%:

$874\frac{55}{100}$ NEW YORK, Aug. 12, 1911.

Ninety days after date, pay to the order of Joseph S. Sinclair & Co., Eight Hundred Seventy-four $\frac{55}{100}$ Dollars, value received, and charge to the account of

EDMUND BERKELEY.

To W. BANKHEAD THORNTON,
ALEXANDRIA, VA.

18. On Oct. 15, 1911, merchandise to the amount of $917.65 was bought at 3 months. What payment on Nov. 8, 1911, will discharge the indebtedness, allowing bank discount at 5 %?

19. What payment on Feb. 13, 1912, will discharge a debt of $2173.85, due May 24, 1912, allowing bank discount at 6 %?

20. At 5 %, what are the proceeds of a 60-day note of $468.75, dated June 17, 1911, and discounted July 10, 1911?

TO FIND THE FACE OF A NOTE TO PRODUCE A GIVEN PROCEEDS

ILLUSTRATIVE EXAMPLE

257. What must be the face of a note payable 90 days after date, which, if discounted on day of issue at 5 %, will produce $647.52 as proceeds?

SOLUTION

$$
\begin{array}{rrr}
 & \$647.52 & \\
 & 72 & \\
\hline
72. & 129504 & \\
.90 & 453264 & 2 \\
\hline
71.10) & 46621.44 & (\$655.72 \\
 & 4266 & \\
\hline
 & 3961 & \\
 & 3555 & \\
\hline
 & 4064 & \\
 & 3555 & \\
\hline
 & 5094 & \\
 & 4977 & \\
\hline
 & 1170 & \\
 & 711 & \\
\hline
 & 459 &
\end{array}
$$

EXPLANATION. $647.52 is a percentage of proceeds, or a product [**255**, 3]. 5 % is the rate of discount per annum, or *a* multiplier, but not *the* specific rate or multiplier to produce $647.52 proceeds [Note, **255**].

As the given percentage of proceeds ($647.52) equals the required face of the note or future worth *minus* the interest thereon for 90 days at 5 % per annum, so its specific rate or multiplier must be the specific rate of the required face or future worth (100 %) *minus* the specific rate of discount for 90 da. at 5 % per 360 da. [Note, **227**], or $1 - \frac{90}{720}$.

Hence, divide the given percentage of proceeds, or product ($647.52) by its specific rate or multiplier factor ($1 - \frac{90}{720}$), or, clearing the divisor of its fraction, divide 72 times the given percentage ($647.52 × 72 = $46621.44) by 72 times its specific rate (72 times 1, or 72) − (72 times $\frac{90}{720}$, or .90), that is, by 72 − .90, or 71.10, obtaining $655.72 as its multiplicand factor, or the required face or future worth.

VERIFICATION. Find the interest on the obtained future worth ($655.72) for the given term of discount (90 da.), obtaining $8.20 — as the discount, and subtract the discount ($8.20) from the obtained future worth

($655.72). If the remainder equals the given proceeds ($647.52), the solution may be accepted as mechanically correct.

RULE 1. *To find the future worth or base in bank discount, divide the given percentage of proceeds by 100% minus the specific rate of discount for the days of prepayment* [Note, 227]. Or, to avoid fractions,

2. *Multiply the proceeds by such a part of 360 as 1% is part of the given rate per annum, and divide the resulting product by the same part of 360 after diminishing it by 1% of the days of prepayment.*

NOTE. If the given rate per annum of discount is not an aliquot of 360, multiply the given proceeds by 360, and divide the resulting product by 360 diminished by as many times 1 % of the days of prepayment as the given rate per annum is times 1 %. Thus, in the Ill. Ex., ($647.52 × 360) divided by 360 − (.90 × 5), or by 355.50 = $655.72. That is, instead of dividing the specific percentage of proceeds ($647.52) by its specific rate ($1 - \cdot\frac{9}{72}$), preferably divide 360 times the specific proceeds by 360 times its specific rate, or by (360 times 1) − (360 times $\cdot\frac{9}{72}$).

EXAMPLES FOR PRACTISE

Find the face of a note which will produce

1.	$648.73 proceeds,	if discounted 85 days	before maturity at 5%.
2.	$945.85	125	6 %.
3.	$1374.50	62	8 %.
4.	$500.	145	$4\frac{1}{2}$ %.
5.	$6258.25	97	3 %.
6.	$4000.	132	7 %.
7.	$693.40	76	$3\frac{1}{2}$ %.
8.	$2500.	231	4 %.
9.	$1257.85	5 mo. 16 da	6 %.

10. I owe $695.80 payable to-day; and in settlement I give my creditor a 60-day note, the proceeds of which if discounted at bank to-day at 6%, will exactly discharge my indebtedness. What was the face of the note?

11. What must be the face of a 90-day note, which, if discounted on day of issue at 5%, will produce $486.25 as proceeds?

12. What should be the face of a note payable 60 days after date, which given to a creditor is equivalent to a cash payment of $500, the rate of discount at the creditor's bank then being $4\frac{1}{2}$%?

PARTIAL PAYMENTS

258. INTRODUCTION. **Partial payments** is the process for finding the balance due at a given date upon an interest-bearing debt when one or more partial payments thereon have been previously made. There are several methods in use, only two of which have more than a local importance. The principal of the two is the method of finding the balance due at the time of each successive payment, adopted by the Supreme Court of the United States, and hence called

THE UNITED STATES RULE FOR PARTIAL PAYMENTS

ILLUSTRATIVE EXAMPLE

A note of $2346.75, dated May 16, 1909, drawing interest from its date at 6 %, has the following indorsements: paid Oct. 23, 1909, $486.30; paid July 18, 1910, $76.50; paid Feb. 25, 1911, $500; paid June 17, 1911, $865. What is the balance due Nov. 2, 1911?

SOLUTION

Face of note or original principal		$2346.75
Interest from date of note, May 16, 1909, to date of first payment, Oct. 23, 1909 (time by 30-day method = 157 da.), is $61.41; which is deducted from the first payment ($486.30) leaving a balance of $424.89 to apply to the reduction of the principal	424.89	
Balance due after making the first payment		$1921.86
Interest of unpaid principal ($1921.86) from its date, Oct. 23, 1909, to the date of the next following payment, July 18, 1910 (265 da.), is $84.88, or $8.38 more than the second payment $76.50). This unpaid interest ($8.38) is carried forward to the accrued interest at the date of the third payment, to find the total interest then due.		
Interest of same principal ($1921.86) from the date of the last considered payment, July 18, 1910, to the date of the next following payment, Feb. 25, 1911 (217 da.), is $69.51. To this, add the unpaid interest from the last operation ($8.38), making the total unpaid interest on Feb. 25, 1911, equal $77.89, which is deducted from the third payment ($500), leaving a credit balance of $422.11 to apply to the reduction of the unpaid principal	422.11	
Balance due after making the third payment		$1499.75

Balance due after making the third payment	$1499.75
Interest of new principal ($1499.75) from its date, Feb. 25, 1911, to the date of the next following payment, June 17, 1911 (112 da.), is $28, which is deducted from the fourth payment ($865), leaving a credit balance of $837 to apply to the reduction of the unpaid principal	837.
Balance due after making the fourth payment	$ 662.75
Interest of new principal ($662.75) from its date, June 17, 1911, to the date of final settlement, Nov. 2, 1911 (135 da.), is	14.91
Balance due at date of final settlement, Nov. 2, 1911	$ 677.66

RULE. 1. *From the first payment, deduct the accrued interest upon the principal to the date of said payment, and diminish the principal by the remainder, if any, to find the unpaid principal.*

2. *Deduct from the second payment the accrued interest upon the unpaid principal to the date of the second payment, and diminish the unpaid principal by the remainder, if any, to find how much of the interest-bearing principal is still unpaid.*

3. *If the interest due at the date of any payment should prove to be greater than said payment, deduct the payment from the interest and add the unpaid interest to the accrued interest for the next following interval to find the total unpaid interest at the end of the next following interval; then proceed in the usual manner.*

4. *Continue this process consecutively until every payment has been considered.*

5. *The final balance will be the balance due at the last payment, increased by the interest thereon from the date of the last payment to the date of the final settlement.*

NOTE 1. By the U. S. rule, any payment is first applied to the discharge of the accrued interest to the date of such payment, and the remainder of the payment, if any, is then applied to the reduction of the interest-bearing principal; and any deficiency of a payment to discharge the accrued interest is carried forward to the interest for the next following interval to find the total unpaid interest at the end of the next following interval.

NOTE 2. The unpaid balance of the principal at any intermediate settlement should never exceed the immediately preceding unpaid balance. Only unpaid interest could have caused it to be greater; and to compute interest upon such a balance would be virtually to compute interest upon the unpaid interest it contains (compound interest) to which the U. S. Rule is opposed.

NOTE 3. The Supreme Court uniformly employs the 30-day method [**163**; *Note* 4, **233**] in finding the time between two dates.

EXAMPLES FOR PRACTISE

1. A note of \$3750, dated Sept. 26, 1908, drawing interest from date at 6 %, has the following payments indorsed thereon: May 9, 1910, \$575; Jan. 17, 1911, \$750. Find the balance due July 21, 1911.

2. A note of \$2975, dated Aug. 20, 1909, drawing interest at 5 %, was credited with the following payments: Nov. 12, 1910, \$473.85; Mch. 21, 1911, \$678.35; Sept. 9, 1911, \$296.80. What was the balance due Dec. 23, 1911?

3. What was the balance due Apr. 12, 1912, on a note of \$816.25, dated Oct. 18, 1908, and drawing interest at 4½ %? Payments: June 24, 1909, \$256.25; Mch. 8, 1910, \$176.80; Nov. 20, 1910, \$156.25; May 25, 1911, \$150.

4. A note of \$1285.60, dated Oct. 25, 1907, drawing interest at 6 %, has the following indorsements: Mch. 13, 1909, \$200; Feb. 12, 1910, \$352.75; Dec. 29, 1910, \$148; Apr. 20, 1911, \$275. What was the balance due July 8, 1911?

5. A note of \$6500, dated May 23, 1910, drawing interest at 5 %, was indorsed as follows: Nov. 17, 1910, \$125; June 20, 1911, \$400; Feb. 16, 1912, \$100. What was the balance due Sept. 12, 1912?

6. A note of \$5000, dated Feb. 17, 1910, drawing 6 % interest, was paid in full on Oct. 10, 1912. What was the final payment, if the following partial payments had been previously made: Dec. 19, 1910, \$200; Sept. 16, 1911, \$200; May 25, 1912, \$200?

7. I have a note of \$658.25, drawing interest at 6 %, and dated July 24, 1910, upon which I have received the following payments: Oct. 12, 1910, \$45; Apr. 28, 1911, \$10. What was the balance due Sept. 16, 1911?

8. A note of \$956.25, dated Jan. 16, 1912, drawing 6 % interest, had a payment of \$375.80, dated May 14, 1912, indorsed thereon. On Aug. 12, 1912, the maker of the note made a second payment of \$250, and gave a new interest-bearing note for the remainder. What was the face of the new note?

MERCHANTS' RULE FOR PARTIAL PAYMENTS

259. The **merchants' rule** is so called because it conforms to the usage of merchants in obtaining the balances of all accounts which contain past-due (that is, interest-bearing) items, as in

finding the "cash balance" of any account current [299], or as in finding the balance due at a given date "per average" [296].

ILLUSTRATIVE EXAMPLE

A debt of $478.30, drawing interest at 6% from May 14, 1911, was credited with the following partial payments: July 7, 1911, $178.35; Sept. 20, 1911, $200; Oct. 25, 1911, $75.50. Find the balance due Dec. 3, 1911.

SOLUTION

The principal, or debt		$478.30
Interest of $478.30 from May 14 to Dec. 3 (exact time, 203 da.)		16.18
Indebtedness at settlement if no payments had been made		$494.48
First payment on July 7, 1911	$178.35	
Interest of $178.35 from July 7, to Dec. 3 (149 da.)	4.43	
Second payment on Sept. 20, 1911	200.00	
Interest of $200 from Sept. 20 to Dec. 3 (74 da.)	2.47	
Third payment on Oct. 25, 1911	75.50	
Interest of $75.50 from Oct. 25 to Dec. 3 (39 da.)	.49	461.24
Debit balance on Dec. 3		$ 33.24

RULE. 1. *Increase the debt by its accrued interest from the date it commenced to draw interest to the date of final settlement.*

2. *Find the interest of each payment from the date of its payment to the date of final settlement.*

3. *Subtract the sum of the payments and their accrued interest from the sum of the debt and its accrued interest.*

NOTE 1. The above is simply the common practise among merchants of subtracting the total credits at any given date from the total debits at that same date, to find the debit balance at that date.

EXAMPLES FOR PRACTISE

Direction: In the following problems, find the exact time in days [Note 5, 233], though usage in this respect is not uniform in all cities of the United States.

1. What was the balance due Sept. 28, 1912, on a note of $792.50, dated May 20, 1912, drawing interest from date at 6%, and containing the following indorsements: July 12, 1912, $150; Aug. 8, 1912, $200.

2. A note of $1873.25, dated Nov. 17, 1910, drawing interest *from date at* 5%, was credited with the following payments:

Jan. 28, 1911, $375.80; May 16, 1911, $468.25; June 25, 1911, $268.10. What was the balance due Aug. 12, 1911?

3. Find balance due Dec. 28, 1912, on a bill of merchandise amounting to $398.75, purchased Feb. 16, 1912, on two months' credit, computing interest at 6% from the due date of the purchase. Payments: May 13, 1912, $85.30; July 27, 1912, $49.75; Oct. 3, 1912, $158.60.

4. At 6%, find balance due Jan. 6, 1912, on a bill of groceries amounting to $678.35, purchased Mch. 3, 1911, on three months' credit without interest. Payments: Apr. 20, 1911, $275; July 26, 1911, $150; Nov. 16, 1911, $145.80.

SUGGESTION. If payments are made *before* a non-interest bearing purchase falls due (as in the payment of $275 on April 20, 1911), first find the balance due at maturity by subtracting the prepayments plus their accrued interest to the due date (that is, plus their true discount), from the non-interest bearing purchase, to find the debit balance at maturity, which draws interest from the due date of the purchase; after which proceed as usual with the postpayments, if any.

5. At 6%, find balance due Nov. 25, 1910, upon a purchase of $846.25 on May 18, 1910, at 90 days' credit without interest. Payments: June 10, 1910, $175.30; July 14, 1910, $200; Sept. 28, 1910, $250.

6. At 8%, find balance due Jan. 29, 1912, upon a purchase of $328.65 on Apr. 7, 1911, at 4 months without interest. Payments: May 18, 1911, $75.40; June 23, 1911, $183.25; July 10, 1911, $45.80.

7. At 9%, find balance due Feb. 20, 1912, upon a purchase of $419.25 on July 16, 1911, at 30 days without interest. Payments: Aug. 15, 1911, $175.90; Oct. 10, 1911, $85.30; Jan. 6, 1912, $65.70.

8. A purchase of $956.30 on April 13, 1910, and payable 3 months after date, had the following payments credited thereon: May 20, 1910, $200; Sept. 13, 1910, $300; Oct. 25, 1910, $150. Find balance due Dec. 13, 1910, at 6%.

9. At 6%, find balance due June 12, 1913, upon a purchase of $715.25 on Nov. 20, 1912, at 60 days without interest. Payments; Dec. 10, 1912, $158.60; Jan. 19, 1913, $175; Mch. 3, 1913, $148.65; Apr. 25, 1913, $125.90; May 1, 1913, $100.

COMMISSION AND BROKERAGE

260. INTRODUCTION. *Commission* or *brokerage* is the application of the principles of percentage, to any business transaction performed by one person, called an *agent*, for the benefit of another person, called a *principal*. Thus, it may be inconvenient as well as unnecessarily expensive to travel a long distance to the best market for buying a particular kind of merchandise, or one may not possess the requisite skill or knowledge of local conditions to buy or sell under the most favorable circumstances; or one may not have access to the floor of the "Exchange" or "Chamber of Commerce," or "Board of Trade" at which the fluctuations in prices are posted; and it may therefore be so highly desirable to secure the services of one who possesses these advantages, as to justify the payment of a commission, or remuneration, to him for the service rendered.

NOTE. What has been said applies not only to merchandise, but to real estate, or stocks, or bonds, which are bought or sold by an agent; or to collections of money; or to any other authorized special mercantile service rendered by an agent to his principal.

261. **Commission** or **brokerage** is a charge made by an agent for buying or selling merchandise, or real estate, or other property, or for collecting or disbursing moneys.

NOTE 1. The *amount* of commission will vary with the *extent* of the service rendered, and in buying or selling is usually estimated at a certain % of the sum paid by the agent for the merchandise bought (its prime cost), or received by the agent for the merchandise sold (its gross proceeds). In many instances, however, it is computed at a certain price per unit, irrespective of the sum paid or received for it, as grain per bushel, flour per barrel, cotton per bale, leaf tobacco per hogshead, stocks per share, small fruits per crate, etc.

NOTE 2. If a consignment is sold on credit, in addition to the commission for selling, a certain % of the credit sale, called *guaranty*, is charged by the agent for assuming responsibility for its final payment.

NOTE 3. Merchandise forwarded to an agent to be sold is called a *consignment*, the person who sends the merchandise being called the *consignor* or *shipper*, and the agent to whom it is sent, the *consignee*.

262. The **gross proceeds** of a sale is the total sum received by the agent from the buyer of the consignment before deducting the expenses incidental to the sale.

NOTE. After all the expenses of the sale have been deducted from the gross proceeds, the remainder is called the *net proceeds* of the sale.

263. The **prime cost** of a purchase is the sum paid by the agent to the seller thereof, and does not include any expense incidental to the purchase.

NOTE. After the incidental expenses have been added to the original purchase money, the result is called the *gross cost.*

264. Identification of terms. 1. *The prime cost of a purchase, or the gross proceeds of a sale, is the base or multiplicand.*

2. *The specific rate of any given or required percentage of commission, of brokerage, of guaranty, of net proceeds of a sale,* or of gross cost of a purchase,† is the rate or multiplier.*

3. *The commission, the brokerage, the guaranty, the net proceeds of a sale, or the gross cost of a purchase, is a percentage or product.*

* NOTE 1. As the net proceeds of a sale equal the gross proceeds or base *minus* all the expenses, so the specific rate of net proceeds must equal the rate of gross proceeds or base (100 %) minus the rate of all the expenses.

† NOTE 2. As the gross cost of a purchase equals the prime cost or base *plus* all the expenses, so the specific rate of gross cost must equal the rate of prime cost or base (100 %) plus the rate of all the expenses.

NOTE 3. If an expense incidental to a purchase or to a sale is a non-percentage, that is, if it has not been computed at a certain % of the base, as drayage, freight, storage, cooperage, advertising, etc., any net proceeds of a sale or gross cost of a purchase which includes one or more such non-percentage or non-product charges, must itself be a non-percentage or non-product, and can therefore have no specific rate or multiplier. Hence, add all the non-percentage charges to a given non-percentage net proceeds, or subtract them from a given non-percentage gross cost, to reduce it to a percentage or product, after which apply the principles of percentage in the usual manner.

EXAMPLES FOR PRACTISE

1. If the prime cost of a purchase is \$625.80 and the rate of commission for buying is 3 %, what is the commission?

Prime cost or base × specific rate of commission = commission.

Find the commission, if the

2. Prime cost is \$378.25, and the rate of commission 3 %.

3. Gross proceeds are \$234.70, and the rate of commission 2 %.

4. Prime cost is \$1348.62, and the rate of commission $2\frac{1}{2}$ %.

5. Gross proceeds are \$6358.20, and the rate of brokerage $\frac{1}{8}$ %.

6. Prime cost is \$5786.90, and the rate of brokerage $\frac{5}{8}$ %.

7. If the gross proceeds of a sale are \$875.30, the rate of commission 2 %, and the drayage \$3.75, what are the net proceeds?

Base − (base × rate of com.) − other charges = net proceeds.

Find the net proceeds of a sale, if the gross proceeds are

8. \$586.75, the commission 3 %, and the other expenses \$6.35.

9. \$2783.60, the com. $1\frac{1}{2}$ %, and the other expenses \$37.20.

10. \$8625, the brokerage $\frac{3}{16}$ %, and the other expenses \$136.50.

11. \$7594.16, the brok. $\frac{7}{8}$ %, and the other expenses \$213.75.

12. If the prime cost of a purchase is \$763.42, the rate of commission $1\frac{1}{4}$ %, and the other charges \$35.90, what is the gross cost?

Base + (base × rate of com.) + other charges = gross cost.

Find the gross cost of a purchase, if the prime cost is

13. \$853.75, the com. $1\frac{5}{8}$ %, and the other charges \$25.60.

14. \$2346.50, the com. $1\frac{1}{4}$ %, and the other charges \$17.25.

15. \$18250.75, the brok. $\frac{1}{8}$ %, and the other charges \$362.75.

16. \$9580, the brokerage $\frac{3}{4}$ %, and the other charges \$237.15.

17. If the prime cost of a purchase is \$687.50, and the gross cost \$701.25, what is the rate % of commission?

EXPLANATION. Divide the specific percentage or product of commission (\$701.25 − \$687.50 = \$13.75, commission) by its base or multiplicand factor (\$687.50), obtaining 2 % as the specific rate of com., or multiplier factor.

Find the rate % of commission, if the

18. Gross proceeds are \$685.75, and the commission \$27.43.

19. Prime cost is \$468.50 and the commission \$9.37.

20. Gross pro. are \$768.35, and the commission \$23.05 [**205**, *a*].

21. Prime cost is \$18750, and the commission \$70.31 [**205**, *c*].

22. Gross proceeds are \$672.85, and the net proceeds \$645.94.

23. Prime cost is \$296.45, gross cost \$312.52, and drayage \$1.25.

24. What is the prime cost of a purchase, if the commission is \$20.57, and the rate of commission 3 %?

Given percentage ÷ its specific rate = its specific base.

Find the gross proceeds or the prime cost, if the

25. Commission for buying is \$10.69, and the rate of com. $2\frac{1}{2}$ %.

26. Commission for selling is \$27.48, and the rate of com. 3 %.

27. Brokerage for buying is \$42.50, and the rate of brok. $\frac{1}{4}$ %.

28. Brokerage for selling is \$37.60, and the rate of brok. $\frac{5}{8}$ %.

29. Net proceeds are \$947.36, and the rate of commission 3 %.

30. Net proceeds are \$658.56, and the rate of commission 2 %.

31. Gross cost is \$5018.75, and the rate of brokerage $\frac{3}{8}$ %.

32. Gross cost is \$972.36, and the rate of commission $1\frac{1}{2}$ %.

33. Net proceeds are \$624.25, the com. 3 %, the drayage \$6.25.

(Net proceeds + non-percentage charges) ÷ specific rate = gross proceeds.

34. Net pro. are \$487.25, com. $2\frac{1}{2}$ %, other expenses \$1.50.

35. Net pro. are \$855.24, com. 2 %, other charges \$2.75.

36. Net pro. are \$446.85, com. 4 %, other charges \$3.15.

37. Gross cost is \$751.09, com. 3 %, other charges \$1.25.

(Gross cost − non-percentage charges) ÷ specific rate = prime cost.

38. Gross cost is \$269.52, commission 2 %, other expenses 75 ¢.

39. Gross cost is \$585.45, commission $1\frac{1}{4}$ %, freight \$2.25.

40. Gross cost is \$874.41, commission $1\frac{3}{4}$ %, drayage \$7.50.

41. If the net proceeds are \$376.35, and the rate of commission $2\frac{1}{2}$ %, what is the commission?

42. If the gross cost is \$425.39 and the rate of commission $3\frac{1}{4}$ %, what is the commission?

43. An agent received \$375.18 to invest in wheat. If his rate of commission was 3 %, what was his commission?

44. An agent remitted \$462.95 to his principal as the net proceeds of a sale. If the agent's rate of commission was $1\frac{1}{2}$ %, what were the gross proceeds of the sale?

45. An agent sold 9873 pounds of sugar at $4\frac{3}{8}$ cents per pound, charged $1\frac{5}{8}$ % commission for selling, and \$2.90 for other expenses. What were the net proceeds of the sale?

46. An agent bought 5278 pounds bacon at $13\frac{5}{8}$ cents per pound, charged $2\frac{3}{4}$ % commission for buying, and 75 cents for drayage. What sum should his principal remit in settlement?

47. An agent received \$348.25 to invest in potatoes. If his commission was 3 %, how many barrels could he buy at \$2.15 per barrel, and what was the unexpended balance, if any?

SUGGESTION. In business practise, no fraction of a barrel is bought; nor is the agent entitled to a commission on any possible unexpended balance

of a remittance. Hence, divide the remittance ($348.25) by the gross cost of 1 barrel, to find the number of barrels which can be bought with the remittance, and regard any remainder from the division as an unexpended balance.

48. An agent sold 495 bushels wheat at $1.03 per bushel, and with the proceeds bought flour at $5.95 per barrel, the commission for buying and selling being 2 %. How many barrels of flour did the agent buy, and what was the unexpended balance?

49. An agent received $298.40 to invest in wheat. If his commission was $2\frac{1}{2}$ %, how many bushels could he buy at 98 cents per bushel?

SUGGESTION. Unlike merchandise bought by the barrel, in buying grain there is no unexpended balance; but the remainder from the division should be expressed as a fraction, the denominator of which equals the number of pounds in one bushel of the particular kind of grain under consideration [Note 1, **142**]. In this problem, to obtain pounds or 60ths of a bushel, multiply the remainder from the division by 60, and divide the resulting product by the original divisor [**86**].

50. An agent sold 75 barrels of flour at $6.25 per barrel net cash, and 50 barrels at $6.40 per barrel, payable in 3 months. His charges were 2 % for selling, $1\frac{1}{2}$ % for guaranty, and $2.50 for other expenses. What sum should he remit to his principal in settlement?

51. At what price shall an agent be ordered to buy flour at 2 % commission that, after allowing 5 cents per barrel for freight and 2 cents per barrel for drayage, the flour can be sold at $5.94 per barrel and net 20 % profit?

52. A broker sold 5600 bushels of wheat at $1.09 per bushel, and charged $\frac{1}{8}$¢ per bushel for selling. What were the net proceeds of the sale?

53. An agent sold 28356 pounds leaf tobacco at $15.60 per hundredweight, charged $1\frac{1}{4}$ % commission for selling, and $7.50 for other expenses. What were the net proceeds of the sale?

54. An agent bought 18725 pounds wheat at $1.15 per bushel, charged 2 % for commission, 25 cents per load of 36 bushels for drayage, and 6 cents per cwt. for freight prepaid. What sum should his principal remit to him in settlement?

55. An agent received $348.72 to invest in onions. His charges were 3 % for commission, 2 cents per barrel for drayage,

5 cents per barrel for freight, and $7.60 for other expenses. How many barrels of onions could he buy at $1.95 per barrel, and what was the unexpended balance?

SUGGESTION. First diminish the remittance by the sum of the charges which were not estimated per barrel, and divide the remainder by the prime cost per barrel increased by the charges which were directly or indirectly estimated per barrel. The quotient will denote the number of barrels. Regard any remainder from the division as an unexpended balance.

56. An agent received $427.95 to invest in apples. He bought the apples at $2.75 per barrel and charged 3% commission for buying, 25 cents per load of 10 barrels for drayage, 6 cents per barrel for freight, and $2.95 for advertising. How many barrels of apples could the agent buy with the remittance, and what was the unexpended balance?

SUGGESTION. Divide the drayage per load by the number of barrels per load to find the drayage per barrel; and proceed as in Ex. 55 to find the number of barrels. Then multiply the drayage per barrel for *full loads* by the number of barrels which the last load lacks of being a full load, to find the extra drayage (in excess of the former estimated drayage per barrel for *full loads*) incident to the last incomplete load; and deduct it from what would otherwise have been the unexpended balance, to find the correct unexpended balance.

57. An agent received $485.30 to invest in corn. His charges were 2% for commission, 40 cents per load of 50 bushels for drayage, and 5 cents per cwt. for freight prepaid. How many bushels and pounds (56ths of a bushel) did he buy at 65 cents per bushel?

58. An agent received $472.10 to invest in potatoes. He charged 3% commission for selling, 35 cents per load of 12 barrels for drayage, 4 cents per barrel for freight, and $2.40 for other expenses. How many barrels of potatoes could the agent buy at $1.95 per barrel, and what was the unexpended balance?

STOCKS AND BONDS

265. INTRODUCTION. (*a*) A *corporation* is an association of persons which is authorized by a special legal instrument called a *charter* to transact business under certain limitations and obligations as if it were a person.

(*b*) The *stock* of a corporation is so much of its capital as is represented by its shares. A *share* of a corporation is one of the

equal parts into which its *capital stock* is divided. As a convenience in expressing its market value, in declaring dividends or assessments, in computing the premium or discount at which it is bought or sold, the capital stock is usually divided into as many shares of $100 each, or of some multiple or aliquot thereof, as will equal the required capital. The owner of one or more of these shares is called a *stockholder*, and the instrument issued by a corporation to each stockholder to certify how many shares he holds is called a *stock certificate*.

(*c*) The *bonds* of a corporation are so much of its capital as is represented by its bonded indebtedness. For convenience in making interest calculations, etc., each bonded loan is usually divided into as many equal parts of $100 each, or of some multiple or aliquot thereof, as will equal the amount of the loan. Those who take one or more of these equal parts of a loan are called bondholders.

(*d*) Computations in stocks or bonds are made in accordance with the general principles of percentage, the *par or face value* of the stock or bond being very naturally regarded as the *basis* of such calculations [**207**]. Thus, if a corporation prospers and in consequence is paying better returns upon the par value of its stock than upon that value invested in other equally safe ways its stock will sell at such a % of premium (increase) on its par value as will approximately equal this difference; or if the returns on its par or face value are not so good as those of the same sum of money invested in other equally safe ways, its stock is not so desirable, and will only sell at such a % of discount (decrease) on its par value as will approximate the deficiency; etc.; etc.

266. The **par value** is the value specified upon the *face* of stocks or of bonds.

267. The **market value** is the value of stocks or of bonds upon the stock exchange or stock market.

NOTE. If the market value *exceeds* the par value, stocks or bonds are said to be *above par* or *at a premium;* and if the market value is *less than* the par value, stocks or bonds are said to be *below par* or *at a discount.* The amount of premium or discount per share is the difference between its par value and its market value.

268. **A dividend** is that part of the earnings of a corporation which is divided among its stockholders.

NOTE. Occasionally, though rarely, special dividends have been declared by corporations from other sources than their earnings, as from loans negotiated for that purpose, or from the sale of an extra issue of stock, etc.

269. An assessment is a sum levied upon the stockholders of a corporation to meet its losses, or its extraordinary expenditures for betterments, etc.

270. A quotation is the market value of $100 par value of stocks or of bonds.

NOTE. Mining stocks are mostly quoted at so much per share, without reference to their par value. In some stock exchanges, *all* stocks, the par value per share of which is less than $100, are thus quoted. *Bonds of all denominations* are quoted at the market value of each $100 of their face value.

271. A stock-broker is a person whose business it is to buy or sell stocks or bonds for others.

NOTE. The charge of a broker for buying or selling stocks or bonds, is called *brokerage*. Brokerage is estimated at a certain % of the par value, or at so much per share.

272. Identification of terms. 1. *The par value of stocks or of bonds is the base or multiplicand understood when no other base is specifically mentioned.*

2. *The specific rate of premium, of discount, of dividend, of assessment, of market value, of prime or gross cost when buying, or of net or gross proceeds when selling, is the rate or multiplier.*

3. *The premium, the discount, the dividend, the assessment, the market value, the prime or gross cost of a purchase, the net or gross proceeds of a sale, is a percentage or product.*

NOTE 1. A quotation is the specific rate % of market value. Hence, as stocks and bonds are bought and sold at their market value, the quotation should be regarded as the specific rate % of prime cost when buying, or the specific rate % of gross proceeds when selling. The usual symbol of a rate (%) being omitted from the quotation, the learner is cautioned against the common error of confounding a quotation which is a rate or multiplier in its relationship to other terms, with a percentage which is a product in its relationship.

NOTE 2. If there is no brokerage, the quotation or specific rate of prime cost when buying (Note 1) will also be the specific rate of gross cost or of investment; but if brokerage is included in the transaction, the rate of brokerage must be added to the quotation or specific rate of prime cost to obtain the specific rate of gross cost. Reversely, the rate of brokerage must be subtracted from any obtained rate of gross cost to find a required quotation or rate of prime cost.

NOTE 3. If there is no brokerage, the quotation or specific rate of gross proceeds when selling (Note 1) will also be the specific rate of net proceeds,

but if brokerage is included in the transaction, the rate of brokerage should be deducted from the quotation or specific rate of gross proceeds to find the specific rate of net proceeds. Reversely, the rate of brokerage should be added to any obtained rate of net proceeds to find a required quotation or rate of gross proceeds.

NOTE 4. The product of two factors being the same irrespective of the order of their arrangement, a favorite practise among brokers is to place the specific rate of the required percentage in the usual position of the multiplicand and to regard it as the required dollars *per share;* and to place one-hundredth of the par value or base in the usual position of the multiplier, and to regard it as *the number of shares.*

EXAMPLES FOR PRACTISE

[In the following examples, the par value per share is understood to be $100, when not otherwise expressed.]

Find the

1. Dividend on $4700 par of stock, rate of dividend 5%.

Par value or base × specific rate of dividend = dividend.

2. Assessment on $6200 par of stock, rate of assessment $3\frac{1}{2}$%.
3. Dividend on 163 shares of stock, rate of dividend 7%.
4. Premium on $1900 par of stock, rate of premium $4\frac{3}{8}$%.
5. Discount on 72 shares of stock, rate of discount $12\frac{5}{8}$%.

Find the market value of

6. $2300 par of bonds if the rate of premium is $6\frac{1}{4}$%.

Par value or base × rate of market value [Note 1] = market value.

7. 59 shares of stock if the rate of discount is $16\frac{3}{8}$%.
8. $2700 par of bonds if the rate of premium is $12\frac{5}{8}$%.
9. 148 shares of stock if the quotation is $87\frac{1}{8}$.
10. $3700 par of stock if quoted at $18\frac{3}{4}$% discount.
11. $8600 par of bonds if quoted at $29\frac{7}{8}$% premium.

Including $\frac{1}{8}$% brokerage, find the cost of

12. $6800 par of stock if bought at $9\frac{5}{8}$% premium.

Par value or base × rate of gross cost [Note 2] = gross cost.

13. 43 shares of stock if bought at $118\frac{3}{8}$.
14. $9300 par of bonds if bought at $23\frac{7}{8}$% premium.
15. $8500 par of stock if bought at $2\frac{1}{4}$% discount.

Including $\frac{1}{8}$ % brokerage, find the net proceeds of

16. $1800 par of bonds if sold at $105\frac{1}{8}$.

Par value × rate of net proceeds [Note 3] = net proceeds.

17. 37 shares of stock if sold at $7\frac{3}{8}$ % discount.

18. $2500 par of bonds if sold at $109\frac{1}{2}$.

19. $1200 par of stock if sold at $8\frac{3}{4}$ % premium.

If the par value of stock is

20. $7300, the market value $7592, find the % of premium.

Percentage of market value ÷ par value or base = rate of market value. Rate of market value or quotation − rate of par (100 %) = specific rate of premium.

21. $2800, market value $3433.50, find quotation [**205**, *c*].

22. $5400, the market value $6277.50, find the % of premium.

23. $8200, market value $6211.50, find the % of discount.

24. $3900, premium $755.63, find % of premium [**205**, *a*].

25. $3600, the discount $265.50, find the % of discount.

If, allowing $\frac{1}{8}$ % brokerage, the cost of

26. 73 sh. stock is $7637.63, at what % prem. were they bought?

Gross cost ÷ par value or base = rate of gross cost.
Rate of gross cost − rate of brok. = rate of prime cost, or quo. [Note. 2].
Quotation − rate of par (100 %) = specific rate of required premium.

27. 163 shares is $14996, at what quotation were they bought?

28. 52 shares is $4329, at what % discount were they bought?

29. $8400 par of stock is $9544.50, at what % premium were they bought?

If, allowing $\frac{1}{4}$ % brokerage, the net proceeds of

30. 63 shares are $5213.25, at what % discount were they sold?

Net proceeds ÷ par value or base = rate of net proceeds.
Rate of net pro. + rate of brok. = rate of gross pro., or quotation [Note 3].
Rate of par (100 %) − rate of quo. = specific rate of required discount.

31. $3400 par of stock are $3574.25, at what quo. was it sold?

32. 92 sh. stock are $7854.50, at what % disc. were they sold?

33. 126 shares of stock are $15576.75, at what % premium were they sold?

Find the par value, if the

34. Premium is \$191.25, and the rate of premium $12\frac{3}{4}$ %.

Percentage of prem. ÷ specific rate of prem. = par value or base.

35. Discount is \$954.75, and the rate of discount $8\frac{3}{8}$ %.

36. Dividend is \$829.50, and the rate of dividend $3\frac{1}{2}$ %.

37. Assessment is \$49.50, and the rate of assessment $5\frac{1}{2}$ %.

38. Market value is \$5875.25, and the quotation $82\frac{3}{4}$.

39. Market value is \$9634.25, and the rate of premium $8\frac{1}{4}$ %.

40. Market value is \$5372.25, and the rate of discount $7\frac{3}{8}$ %.

Allowing $\frac{1}{8}$ % brokerage, what par value of stock can be

41. Bought for \$6424.75, if quoted at $103\frac{1}{2}$?

Quo. or rate of prime cost + rate of brok. = specific rate of gross cost.
Percentage of gross cost ÷ its specific rate = its par value or base.

42. Bought for \$2259.75, if the quotation is $98\frac{1}{8}$?

43. Bought for \$6243.75, if the rate of premium is $38\frac{5}{8}$ %?

44. Bought for \$2932, if the rate of discount is $8\frac{1}{2}$ %?

45. Sold for \$10631.25, if the quotation is $78\frac{7}{8}$?

Quo. or rate of gross proceeds − rate of brok. = rate of net proceeds.
Percentage of net proceeds ÷ its specific rate = its par value or base.

46. Sold for \$1384.50, if the quotation is $115\frac{1}{2}$?

47. Sold for \$5577, if the premium is $26\frac{7}{8}$ %.

48. Sold for \$8066.75, if the discount is $1\frac{1}{2}$ %?

49. How many shares of stock can be bought at $2\frac{3}{4}$ % discount and sold at $1\frac{5}{8}$ % premium, to net a profit of \$297, allowing $\frac{1}{8}$ % brokerage for buying and the same for selling?

50. What are the net proceeds of 126 shares of stock sold at $18\frac{5}{8}$ % discount, brokerage $\frac{1}{8}$ %?

51. A gentleman bought stock at $15\frac{1}{4}$ % premium, brokerage $\frac{1}{8}$ %, and afterwards drew a dividend of \$480 upon the stock which he purchased. If the rate of dividend was 5 %, what was the total cost of the stock?

52. A man bought 80 shares of stock at $98\frac{3}{4}$; and subsequently sold 36 shares at $99\frac{1}{4}$, and the remainder at $99\frac{5}{8}$. Allowing $\frac{1}{8}$ % brokerage for buying and for selling, what was his net gain?

53. I receive an annual income of \$280 from 5 % bonds bought at $92\frac{5}{8}$, brokerage $\frac{1}{8}$ %. What was the cost of the bonds?

54. A man invested \$19857.50 in 4% bonds purchased at $84\frac{3}{8}$, brokerage $\frac{1}{8}$%. What was his annual income from the purchase?

55. An investor derives a quarterly income of \$525 from 6% bonds bought at $28\frac{3}{4}$% discount, brokerage $\frac{1}{8}$%. What did he pay for the bonds?

56. If \$18716.25 is realized as net proceeds from the sale of \$13800 par of bonds, at what % premium were they sold, allowing $\frac{1}{8}$% for brokerage?

RELATION OF INCOME TO INVESTMENT

273. INTRODUCTION. (*a*) As viewed by a corporation when it proposes to distribute a dividend or to levy an assessment, etc., the par or face value of the stock would be the only unchanging base that could be used as a standard in prorating such a dividend or assessment among its stockholders which would apply generally to all its shares, irrespective of their constantly fluctuating market value on the different days of their purchase by the present holders of the stock. From the standpoint of the investor, however, an entirely different standard would suggest itself in considering the expediency of an investment in a particular class of securities. The thought uppermost in his mind would very naturally be the % of income which he would realize *upon his investment*, rather than the % of income he would realize upon the par value. Other things being equal, a high rate of dividend on stocks or of interest on bonds, the market value of which is at a high premium, may not be so desirable as a lower rate of dividend on stocks or of interest on bonds the market value of which is at a lower premium, or possibly at a discount. Hence, in considering the expediency of a particular purchase, *investors regard the sum invested* (gross cost) *as the base.*

(*b*) If the investment is the contemplated base, it should be expressed by placing the phrase "on investment" immediately after the rate % which refers to such a base. Otherwise, in many instances, the par value may be *misunderstood* as the intended base. Thus, in the expressions "5% stocks," "6% bonds," etc., no base being expressed, they are respectively understood to refer to stocks paying 5% dividends on *par value*, bonds paying 6% interest on *par value;* but when reference is made to "stock paying 5% income *on investment*," it is meant that the stock referred to pays a dividend on its par value which is equivalent to 5% on the necessary investment to purchase it.

(*c*) If, in the same relation, two rates % of income are mentioned, one of which is understood to be on par value and the other is expressed on investment, the rate % of income on par value may be regarded as that many dollars of income per share (a percentage or product), and the rate % of income on investment may be retained as the specific rate of that income, or multiplier. (*d*) Similarly, *any* rate % on par value is convertible into the same number of dollars per share of the same specific name as the rate. (*e*) Conversely, any result obtained in dollars per share may be accepted as so much % on par value of the same specific name.

Identification of terms. 1. *The indicated investment is the base or multiplicand.* 2. *The % of income on investment is the rate or multiplier.* 3. *The income is the percentage or product.*

EXAMPLES FOR PRACTISE

At what price per share must

1. 7 % bonds be bought to realize 8 % income on investment?

7 % bonds are bonds paying \$7 interest on \$100 [Intro., *c*]. Hence, Percentage of income (\$7) ÷ its specific rate on inv. (8 %) = \$87½, inv.

2. 6 % stock be bought to realize 10 % income on investment?
3. 5 % bonds be bought to produce 4 % income on investment?
4. 8 % stock be bought to yield 5 % income on investment?
5. 9 % bonds be bought to produce 8 % income on investment?

Including ⅛ % brokerage, at what quotation must

6. 5 % stock be bought to yield 8 % income on investment?

5 % stock means stock paying \$5 dividend per \$100 share. Hence, inc. per sh. (\$5) ÷ rate of inc. on inv. (8 %) = inv. per sh. (\$62.50). Investment or gross cost per share (\$62½) − brokerage per share (\$⅛) = prime cost per sh. (\$62⅜), or rate of quotation (62⅜) [See Introduction, *e*].

7. 4 % bonds be bought to realize 5 % income on investment?
8. 6 % stock be bought to yield 8% income on investment?
9. 7 % bonds be bought to yield 10 % inc. on investment?
10. 3 % stock be bought to yield 4 % income on investment?

What % of income on investment will be realized if

11. 4 % stock be bought at 80.

4 % stock means stock paying \$4 dividend on each \$100 share. As there is no brokerage, the quotation or prime cost per share also expresses the

investment or gross cost per share ($80). Hence, income per share ($4) ÷ investment per share ($80) = specific rate of inc. on inv. (5 %).

12. 6 % stock is bought at 75?

13. 5 % bonds are bought at 25 % premium?

14. 4½ % stock is bought at 10 % discount?

15. 4 % bonds are bought at 20 % discount?

16. 8 % stock be bought at 119⅞ %, brokerage ⅛?

Income per sh. ($8) ÷ inv. per sh. ($119⅞ + $⅛) = specific rate of income on the investment base (6⅔%).

17. 3 % bonds be bought at 74¾, brokerage ¼ %?

18. 4 % stock be bought at 89⅞, brokerage ⅛ %?

19. 5 % bonds be bought at 79¼, brokerage ¼ %?

20. 4½ % stock be bought at 112⅜, brokerage ⅛ %?

What % stock can be bought

21. At 75, and yield 4 % income on investment?

Inv. per sh. or base ($75) × rate % of inc. on that inv. (4 %) = income per sh. from that investment ($3), or 3 % stock [See Introduction, *e*].

22. At 120, and produce 5 % income on investment?

23. At 25 % premium and yield 8 % income on investment?

24. At 20 % discount and realize 10 % income on investment?

25. At 124¾ and yield 4 % on investment, brokerage ¼ %?

Investment per share or base ($124¾ + $¼ = $125) × rate % of income on that investment (4 %) = income per share ($5), or 5 % stock.

26. At 74⅞ and yield 8 % income on investment, brokerage ⅛ %?

27. At 79¾ and produce 5 % income on investment, brok. ¼ %?

28. At 112⅜ and realize 4 % income on investment, brok. ⅛ %?

29. What % on investment is realized in purchasing a house and lot for $4000 which can be rented at $50 per month, allowing $100 per annum for deterioration in value and $180 per annum for taxes, repairs and other expenses?

30. What % on investment is realized in buying 9 % stock at 74⅞, allowing ⅛ % for brokerage?

31. What % bonds can be bought at 87⅜ to yield an income of 8 % on investment, brokerage ⅛ %?

32. At what quotation must 3 % bonds be purchased to produce an income of 5 % on investment, brokerage ⅛ %?

33. An investor paid \$6580 for stock from which he derived an annual income of \$329. What % of income upon his investment did he realize?

34. Both being equally secure, which is the more profitable investment, 5% stock bought at 20% premium, or 4% stock bought at 10% discount, and how much better?

35. A man bought 74 shares of stock at $85\frac{3}{8}$, and afterwards sold the same at $86\frac{1}{4}$. What was his gain, allowing $\frac{1}{8}$% brokerage for buying and selling?

36. An investor bought 50 shares of stock at $1\frac{3}{8}$% premium, drew a dividend thereon of 4%, and sold it at $98\frac{7}{8}$. What was his gain, allowing $\frac{1}{8}$% brokerage each way?

37. A man bought 300 shares of stock at $109\frac{5}{8}$, sold 90 shares at $110\frac{1}{4}$, 150 shares at $110\frac{7}{8}$, and the remainder at $111\frac{1}{8}$. Allowing $\frac{1}{8}$% for brokerage in each transaction, what was his net gain?

EXCHANGE

274. INTRODUCTION. (*a*) *Exchange* is the giving of one thing as an equivalent for another. Therefore, when a credit in one city where it is not needed is bartered for a credit in another city where it is required, at a less cost than the expressage for safely transmitting the credit money from the one city to the other, the transaction is very appropriately termed "exchange." Thus, if A of Baltimore owes \$5000 to B of New Orleans; and C of New Orleans owes \$5000 to D of Baltimore, and this was known to the parties concerned, the simplest way to dispose of both debts would evidently be for A in Baltimore to pay \$5000 to D of the same city, and for C of New Orleans to pay \$5000 to B of that city; for this *exchange* of credits would involve no expressage for transmission of money from either place to the other. Any such arrangement, however, is rendered impracticable by the unlikelihood of the persons named possessing the requisite knowledge to make such an exchange feasible. To make such a convenient disposition of credits effective, "exchange banks" are established in the principal commercial cities of the world, each exchange bank in one city having as its correspondent a similar bank in other cities, through which it can transact its outside business. Thus, an exchange bank in Baltimore can cash D's draft on C, as well as the drafts of other Baltimore firms on New Orleans, and

mail them for collection to its correspondent bank in New Orleans, the proceeds to be held in New Orleans subject to the order of the Baltimore bank; and A of Baltimore would very naturally go to his local bank to purchase a draft of $5000 upon its New Orleans correspondent and mail it to B, discharging in this manner his indebtedness without the cost of transmitting the actual money, while the Baltimore bank is also saved the expense of expressing from New Orleans to Baltimore the proceeds of D's draft on C by utilizing it in paying A's debt in that city to B, besides receiving a fee for cashing D's draft on C, and a fee for selling to A a draft on its New Orleans correspondent in favor of B.

(*b*) If New Orleans owes Baltimore about as much as Baltimore owes New Orleans the *course of exchange* between the two cities is said to be *at par* (that is, on an equality); but if New Orleans owes Baltimore more than Baltimore owes New Orleans, the course of exchange in New Orleans will be at a premium on Baltimore, the amount of the premium varying with the amount of the preponderance of indebtedness and the consequent cost of transmitting the actual money to overcome the preponderance; and at the same time the course of exchange in Baltimore on New Orleans will be at a discount to stimulate the buying of exchange on New Orleans and thus to diminish the preponderance. The highest premium or lowest discount can be but little more than the cost of safely transmitting the sum of money mentioned on the face of the bill of exchange from the debtor city to the creditor city.

275. A banker's draft or bill of exchange is an order written by one bank (called the *drawer* of the exchange), directing another bank (called the *drawee* of the exchange), to pay a specified sum of money to a designated person (called the *payee*) or to whomsoever the payee orders it to be paid.

NOTE. When the *drawer* and *drawee* reside in the same country, the draft is called a *domestic bill of exchange;* and when they reside in different countries, it is named a *foreign bill of exchange.*

276. The face or par of a bill of exchange is the sum specified therein to be paid to the payee or to his order.

NOTE. If, as is usually the case, a debt is payable where the payee resides, the *face* of a bill of exchange in payment of that debt should be the *sum due to the payee* by the purchaser of the bill, the purchaser defraying the necessary premium charged, or receiving the benefit of any discount allowed.

DOMESTIC EXCHANGE

277. Domestic exchange is exchange effected between cities of the same country.

NOTE. The method of computing domestic exchange is applicable to all countries which use the same coinage, and which must therefore express the rate of exchange *directly* by premium or discount on the one common coinage.

278. Identification of terms. 1. *The face of a bill of exchange is the base or multiplicand.* 2. *The % of any required premium, discount, or market value, is a rate or multiplier.* 3. *The premium, the discount, or the market value, is a percentage or product.*

NOTE 1. The rate of market value of a sight draft equals the rate of its face value (100 %) increased by the rate % of premium, or diminished by the rate % of discount.

NOTE 2. The rate of market value of a time draft equals the rate of market value of a sight draft [Note 1] diminished by the specific rate of interest for the given time [Note, **227**]. Hence, if both are of the same face value and bought at the same rate of exchange, the cost of a time draft is as much less than that of a sight draft, as will equal the interest on its face (the bank discount) for the given time.

NOTE 3. As drafts are bought and sold at their market value, their market value must also be their prime cost if bought, or their gross proceeds if sold. Hence, if brokerage is included, it should be computed upon the market value or prime cost, and added to the prime cost to find the gross cost, *if purchased;* or computed upon the market value or gross proceeds, and subtracted from the gross proceeds to find the net proceeds, *if sold.*

NOTE 4. (*a*) If a debt is payable at the place in which the debtor resides, the amount of the debt should be regarded as the *gross cost* of the draft, that is, the creditor should assume the expense of remittance; (*b*) but if the debt is payable at the place in which the creditor resides, the amount of the debt should be taken as the face of the draft, that is, the purchaser of the draft (the debtor) is properly chargeable with the expense of remittance.

EXAMPLES FOR PRACTISE

1. Find the cost of a sight draft of \$720 at $\frac{3}{8}$ % premium.

Face of draft or base (\$720) × rate or prem. ($\frac{3}{8}$ %) = premium (\$2.70).
Face of draft (\$720) + prem. (\$2.70) = market value or cost (\$722.70).

2. Find cost of a sight draft of \$630 at $\frac{1}{4}$ % premium.

3. Find market value of a sight draft of \$960 at $\frac{5}{8}$ % discount.

4. Find proceeds of a sight draft of \$568.30 at $\frac{1}{8}$ % premium.
5. Find cost of a sight draft of \$275.85 at $\frac{1}{2}$ % discount.
6. Find cost of a 30-day draft of \$328; prem. $\frac{5}{8}$ %, interest 6 %.

Face of draft or base (\$328) × rate or prem. ($\frac{5}{8}$ %) = premium (\$2.05).
Interest of \$328 for 30 days at 6 % = \$1.64, discount for interest.
Face (\$328) + prem. (\$2.05) − int. (\$1.64) = cost, \$328.41.

7. Find cost of a 60-day draft of \$843.75; prem. $\frac{1}{4}$ %, int. 6 %.
8. Find cost of a 90-day draft of \$476.28; prem. $\frac{3}{8}$ %, int. 5 %.
9. Find pro. of a 30-day draft of \$624; prem. $\frac{1}{8}$ %, int. 4 %.
10. Find cost of a 60-day draft of \$460; disc. $\frac{1}{4}$ %, int. 6 %.

Face of draft (\$460) × rate of disc. ($\frac{1}{4}$ %) = discount (\$1.15).
Interest of \$460 for 60 days at 6 % = \$4.60, discount for interest.
Face of draft (\$460) − disc. (\$1.15) − int. (\$4.60) = cost (\$454.25).

11. Find cost of 90-day draft of \$728.16; disc. $\frac{1}{2}$ %, int. 5 %.
12. Find pro. of 30-day draft of \$640; disc. $\frac{3}{8}$ %, int. 6 %.
13. Find market value of 60-day draft of \$160; discount $\frac{1}{8}$ %, interest 6 %.

Find the face of a sight draft which can be bought

14. For \$1239.09, if the rate of exchange is $\frac{1}{4}$ % premium.

Percentage of market value or product (\$1239.09) ÷ the specific rate of market value or multiplier [100 % + $\frac{1}{4}$ %, Note 1] = face value or multiplicand, \$1236. [Note 4, *b*].

15. For \$875.27, if the rate of exchange is $\frac{3}{8}$ % premium.
16. For \$932.85, if the rate of exchange is $\frac{5}{8}$ % discount.
17. For \$1465.20, if the rate of exchange is $\frac{1}{2}$ % premium.
18. For \$716.50, if the rate of exchange is $\frac{3}{4}$ % discount.

Find the face of a time draft which can be bought for

19. \$824.25, payable in 90 days; exch. $\frac{3}{8}$ % disc., int. 6 %.

Rate of face (100 %) − rate of exchange discount ($\frac{3}{8}$ % = .00375) − rate of int. for 90 da. at 6 % per annum (.015) = rate of market value (.98125).
Percentage of market value or product (\$824.25) ÷ specific rate of market value or multiplier (.98125) = face value or multiplicand (\$840).

20. \$753.25, if payable in 30 days, exch. $\frac{1}{4}$ % prem., int. 5 %.
21. \$1581.70, if payable in 60 days, exch. $\frac{5}{8}$ % disc., int. 4$\frac{1}{2}$ %.
22. \$3500, if payable in 90 days, exch. $\frac{1}{4}$ % prem., int. 6 %.

23. \$875, if payable in 60 days, exch. $\frac{1}{8}$ % disc., int. 4 %.

24. The net proceeds of a sale were \$2785.90, which the commission merchant remitted to his principal by a bank draft, payable at sight, purchased at $\frac{3}{8}$ % discount. What was the face of the draft? [Note 4, *a*]

25. A St. Louis merchant bought \$3258.25 worth of goods from a firm in New York payable in 60 days, and when the bill became due he remitted a bank draft to New York in settlement. What was the cost of the draft, if the course of St. Louis exchange on New York was at $\frac{1}{4}$ % premium? [Note 4, *b*]

FOREIGN EXCHANGE

279. Foreign exchange is exchange between different countries which also have *different monetary units.*

280. The **rate of foreign exchange** is the market value of the monetary unit of the country in which the drawee resides when it is estimated in the currency of the country in which the bill of exchange is purchased.

NOTE 1. The *intrinsic par of exchange* is the inherent or true value of such foreign coins as constitute the monetary units of their respective countries, as shown by the weight and purity of the gold or silver which they contain. In **138** will be found a TABLE showing the *intrinsic par* of the principal foreign monetary units when expressed in U. S. money, as proclaimed by the Secretary of the Treasury of the United States on January 1, 1912.

NOTE 2. The *commercial par of exchange*, or the *rate of foreign exchange*, is the market value of foreign monetary units as determined by the balance of trade between the country of the drawer and that of the drawee [**274**, *b*]. If the balance of trade is *against* the country on which the bill of exchange is drawn, that is, if it has bought more from the country of the drawer than it has sold to it, the commercial par of exchange on the country of the drawee will be at a discount upon the intrinsic par; but if the balance of trade is in favor of the country on which the draft is drawn, the commercial par will be quoted at a premium upon the intrinsic par [**274**, *b*].

NOTE 3. In the United States, the *rate of exchange* on Great Britain is expressed by giving the market value of £1 in United States money; on France, Belgium and Switzerland by giving the market value of \$1 in francs; on Germany by giving the market value of 4 reichsmarks in United States money; and on Holland by giving the market value of 1 guilder in United States money. Hence,

IDENTIFICATION OF TERMS

281. If the rate of exchange expresses the market value of 1 foreign monetary unit in United States money. 1. *The exchange value in United States money of 1 foreign monetary unit is the multiplicand.* 2. *The number of foreign monetary units is the multiplier.* 3. *The exchange value in United States money of all the foreign monetary units is the product.*

282. If the rate of exchange expresses the market value of $1 in foreign monetary units. 1. *The number of foreign monetary units that can be bought for $1 is the multiplicand.* 2. *The number of dollars to be invested in exchange is the multiplier.* 3. *The number of foreign monetary units that can be bought with all the dollars to be invested in exchange is the product.*

NOTE 1. As American exchange upon Germany expresses the value in United States money of 4 German reichsmarks, the rate of exchange must be divided by 4 to obtain the multiplicand of **281**.

NOTE 2. Notes 2, 3 and 4, **278**, apply also to foreign exchange, except that in computing interest upon *time* bills of exchange on Great Britain, 3 days of grace are invariably added to the nominal time. Grace is also allowed in a *few* American cities upon domestic time bills of exchange.

TO FIND THE COST OF A FOREIGN BILL OF EXCHANGE

283. The following forms of solution are employed when the buyer of the exchange owes money which is payable where the payee resides. The face of the bill should then be the amount of the debt, and any expense incurred in buying the bill of exchange should be borne by the remitter.

ILLUSTRATIVE EXAMPLES

1. What is the cost of a bill of exchange on London for £673 2s. 6d., if the rate of exchange is 4.8725?

SOLUTION. £673 2s. 6d. = £673.125 [Ill. Ex. 2, **160**]. $4.8725 is the exchange value in United States money of £1, or the multiplicand [**281**, 1]; £673.125 is the number of £'s, or the multiplier [**281**, 2]; therefore, multiply these factors ($4.8725 × 673.125 = $3279.80) to find the exchange value in United States money of all the given £'s, or product [**281**, 3].

2. What is the cost of a bill of exchange on Berlin for 675.20 reichsmarks if the rate of exchange is 96¼?

SOLUTION. $\$.96\frac{1}{2} \div 4 = \$.24\frac{1}{8}$ [Note. 1, **282**], the exchange value of 1 reichsmark, or the multiplicand [**281**, 1]; 675.20 is the number of reichsmarks, or the multiplier [**281**, 2]. Therefore multiply these factors ($\$.24\frac{1}{8} \times 675.20 = \$162.89 +$) to find the exchange value of all the reichsmarks, or the product [**281**, 3].

3. What is the cost of a bill of exchange on Paris for 3245.80 francs, if the rate of exchange is $5.20\frac{1}{4}$?

SOLUTION. $5.20\frac{1}{4}$ is the number of francs that can be bought for $1, or the multiplicand [**282**, 1]; and 3245.80 is the number of francs to be bought with *all* the required dollars to be invested, or the product [**282**, 3]. Therefore divide the given product (3245.80 fr.) by its given multiplicand factor ($5.20\frac{1}{4}$ fr.) to find its required multiplier factor (623.89 times $1, or $623.89), the number of dollars to be invested.

EXAMPLES FOR PRACTISE

Find the cost of a bill of exchange for

1. £416 10s., if the rate of exchange is 4.8650.
2. £725 17s. 6d., the rate of exchange being 4.8715.
3. £926 12s. 6d., if the rate of exchange is 4.8620.
4. 827.60 reichsmarks, if rate of exch. is 96 [Note 1, **282**].
5. 1927.50 reichsmarks, if the rate of exchange is $95\frac{3}{8}$.
6. 3460.25 reichsmarks, if the rate of exchange is $96\frac{1}{8}$.
7. 4675.20 francs, if the course of exchange is $5.18\frac{3}{8}$.
8. 798.40 francs, the rate of exchange being $5.19\frac{1}{8}$.
9. 1425.80 francs, if the rate of exchange is $5.18\frac{7}{8}$.
10. 2300 guilders, if the course of exchange is $40\frac{3}{8}$.
11. £826 2s. 6d.; exch. 4.8665, brok. $\frac{1}{8}\%$ [Note 3, **278**].
12. 6350 francs, if exchange is $5.18\frac{1}{4}$, and brokerage $\frac{1}{8}\%$.

TO FIND THE FACE OF A BILL OF EXCHANGE

284. The following forms of solution are applicable when the buyer of the exchange owes money which is payable where he (the buyer) resides. The *total cost* of the bill of exchange should then be equal to the amount of the debt, thus causing all expenses incurred in buying the bill to be borne by the payee.

ILLUSTRATIVE EXAMPLES

1. I paid $925.70 for a bill of exchange on Liverpool. What was the face of the bill, the rate of exchange being 4.86?

SOLUTION. $925.70 is the exchange value of all the required £'s, or the product [281, 3]; $4.86 is the exchange value of £1, or the multiplicand [281, 1]. Therefore divide the given product ($925.70) by its given multiplicand factor ($4.86) to find the required number of £'s, or the multiplier factor (£190 9s. 5½ d.).

2. I bought through a broker a bill of exchange on Belgium for which I paid him $640.80. If the rate of exchange was 5.18, and the brokerage ⅛ %, what was the face of the bill?

SOLUTION. $640.80 ÷ 1.00⅛ = $640 [Note 3, 278], the number of dollars to be invested in exchange, or the multiplier [282, 2]; 5.18 is the number of francs that can be bought for $1, or the multiplicand [282, 1]. Therefore multiply these two factors (5.18 fr. × 640 = 3315.20 fr.) to find the required number of francs which can be bought with all the dollars to be invested in the bill of exchange, or the product [282, 3].

NOTE. If brokerage is included, regard the amount of the debt to be canceled by the remittance as the gross cost of the bill of exchange, and divide it by 100 % + the rate of brokerage to find the prime cost or market value of the bill [Note 3, 278].

EXAMPLES FOR PRACTISE

Find the face of a bill which can be bought for

1. $1825.36, if the rate of exchange is 4.8668 to the £.
2. $746.25, if the rate of exchange is 4.855 to the £.
3. $825.70, if the course of exchange is 5.19¼ francs to $1.
4. $2580.20, exchange being 5.18¾ francs to $1.
5. $942.65, if exchange is 94⅞¢ to 4 reichsmarks.
6. $6250.75, if exchange is 95¼¢ to 4 reichsmarks.
7. $675.80, if exchange is 41¼¢ to 1 guilder.
8. $1248.35, if exchange is 40⅞¢ to 1 guilder.
9. $3650.25, exchange 4.8575 to £1, brokerage ⅛ %.
10. $785.90, exchange 41⅜¢ to 1 guilder, brokerage ¼ %.

BANKRUPTCY

285. INTRODUCTION. A firm is said to be *bankrupt* or *insolvent* when its financial resources are not sufficient to pay its debts as they fall due. The *assets* of a bankrupt firm are its total available property, including the collectible debts due to it from others; and the *liabilities* of a bankrupt firm are the debts it owes to others. The *assignee* or *trustee* is one who is appointed to take charge of the business of a bankrupt firm for the purpose

of converting its assets into cash sufficient to discharge its financial obligations; or, if its total assets are insufficient to discharge its total liabilities, to distribute among its creditors *such* a percentage of their individual claims as the firm's total assets when converted into cash are of its total liabilities. This distribution of available assets by the assignee among the creditors is called a *dividend.*

Identification of terms. 1. *The total liability of the firm is the base or multiplicand.* 2. *The % of the total liability which is distributed among a bankrupt firm's creditors, is the rate of dividend or multiplier.* 3. *The sum distributed among the bankrupt firm's creditors is the percentage of dividend or product.*

NOTE 1. The total dividend to all the creditors will always be the *net available assets,* that is, the gross available assets diminished by the expenses of assignment.

NOTE 2. Whatever the rate of dividend to all the creditors collectively may be, must also be the rate of dividend to each individual creditor. Hence, the total liability to each individual creditor is a base or multiplicand; the % of dividend is the rate or multiplier; and the dividend received by each individual creditor is a percentage or product.

EXAMPLES

1. The following statement was rendered by the assignee of a bankrupt merchant: Assets — merchandise as per inventory, $8411.15; personal accounts due the firm, $1618.45; real estate, $4500. Liabilities — notes due to others, $12000; due to F. A. Sadler for merchandise, $6500; due to R. M. Browning for services rendered, $2700. If the expenses of assignment were $325.60, what dividend should F. A. Sadler and R. M. Browning receive?

$8411.15 + $1618.45 + $4500 = $14529.60, total assets. Total assets ($14529.60) − expenses of assignment ($325.60) = net assets ($14204). $12000 + $6500 + $2700 = $21200, total liabilities. Percentage of total net assets or product ($14204) ÷ total liability, the base or multiplicand ($21200) = 67 %, the rate of dividend or multiplier. Hence,

$6500 × .67 = $4355, dividend to F. A. Sadler [Note 2].

$2700 × .67 = $1809, dividend to R. M. Browning.

2. A firm failed with liabilities amounting to $33864.25 and assets amounting to $24293.09. What % of dividend will the assignee be able to declare; and what dividend should Geo. C.

Round & Co. receive whose claim for $837.50 was allowed, if the expenses of the assignment were $1265.40?

3. The available resources of a bankrupt firm were $10284.75, and its liabilities $12537.50. If the expenses of assignment were $756.25, how much should J. P. Smith & Co. receive whose claim for $1785.20 was accepted?

4. A bankrupt firm owed A $580, B $1275.50, and C $875.30. Its realizable assets were $12142.25, and its total liabilities $17856.25. If the expenses of settling the bankruptcy were 4% of the assets, how much should A, B, and C receive?

INSURANCE

286. INTRODUCTION. Insurance is a pecuniary indemnity guaranteed by one party (*the insurer*) to another party (*the insured*) in certain specified contingencies, such as the death of the insured (*life insurance*); or a disabling accident to the insured (*accident insurance*); or against loss to property insured caused by fire (*fire insurance*); etc. The written contract between the insurer and the insured is called a *policy*. The *premium* is the sum paid to the insurer for the risk which is assumed, and is estimated at a certain % of the amount of insurance, though frequently expressed by giving the cost per $100 or per $1000 of the risk assumed. Thus, $\frac{3}{4}$%, 75 cents per $100, or $7.50 per $1000 are equivalent rates expressed in different forms.

Identification of terms. 1. *The sum insured is the base or multiplicand.* 2. *The % of premium is the rate or multiplier.* 3. *The premium is the percentage or product.*

NOTE 1. The product of the base and the rate will be the same, irrespective of the order of their arrangement. Hence, for convenience, when the rate of insurance is expressed at so much per $100 or per $1000, the usual position of the base and rate is reversed, and the rate per $100 or per $1000 is taken as the multiplicand, and the *number* of hundreds or of thousands of dollars is taken as the multiplier. Thus, the premium for $8000 of risk at $6.25 per $1000, would be computed at 8 times $6.25, or $50.

NOTE 2. Fire and marine policies contain an "average clause" under which, if the property is only *partially* insured and is totally destroyed within the period of insurance, the insurer is held responsible for the total sum in-

sured; but if the property is only partially destroyed, the insurer is only liable for such a part of the loss, as the sum for which the property is insured is of the full value of the property.

EXAMPLES

1. A man insured his house for \$6800, the rate of insurance being $\frac{5}{8}$ %. What premium did he annually pay?

Sum insured or base (6800) × rate of prem. ($\frac{5}{8}$ %) = prem. (\$42.50).

2. A house is insured at $\frac{3}{4}$ %. What was the sum insured if the premium paid is \$26.25?

3. A house is insured for \$8200 and the premium paid is \$71.75. What is the rate of insurance?

4. A house is insured for \$7500. If the rate of insurance is $\frac{7}{8}$ %, what premium did he annually pay?

5. A ship was insured for \$75300, the rate of insurance being $\frac{3}{4}$ %. What was the premium paid?

6. A man had his life insured for \$9000 at the annual rate of \$35.50 per \$1000. What annual premium does he pay? [Note 1]

7. A building valued at \$12000 and insured for \$8000 was damaged by fire. If the appraisement of the damage was \$2400, what sum was received from the insurer in settlement? [Note 2]

8. A stock of merchandise valued at \$40000 was insured in one company for \$20000 and in a second company for \$10000. If the stock was damaged by fire and water at an appraisement of \$6000, what proportion of the loss should be paid by each company?

9. A steamboat valued at \$120000 was insured in one company for \$20000, in a second company for \$30000, and in a third company for \$40000. If a loss of \$15000 was sustained, how much insurance should the owners receive from each company.

TAXES

287. A tax is an assessment levied for public purposes upon the person (*poll tax* or *capitation tax*), upon the income (*income tax*), or upon the property (*property tax*), of the inhabitants of a country.

Identification of terms. 1. *The assessed value of the property,*

or the amount of income when over a prescribed minimum, is the base or multiplicand. 2. *The rate of taxation is the rate or multiplier.* 3. *The tax is a percentage or product.*

NOTE. If the rate of taxation is expressed at so much on \$100, the computation may be made as explained in Note 1, **286**; or it may be reduced to an equivalent rate % (cents on \$1) by moving the decimal point of the rate on \$100 *two* places to the left. Thus, \$1.45 on \$100 = \$.0145 on \$1, or 1.45 %.

EXAMPLES

What is the tax, if the assessed value of the property is

1. \$8500 and the rate of taxation \$1.32 on \$100?

\$1.32 on \$100 = 1.32 % [Note]. Hence, assessed value or multiplicand (\$8500) × specific rate of taxation or multiplier (.0132) = percentage of tax or product (\$112.20).

2. \$16200 and the rate of taxation $\frac{7}{8}$ %?
3. \$9500 and the rate of taxation \$1.65 on \$100?
4. \$12300 and the rate 15 mills on \$1?
5. \$8700 and the rate of taxation $1\frac{1}{4}$ %?
6. \$23600 and the rate \$1.72 on \$100?
7. \$17200 and the rate 8 mills on the dollar?
8. My real and personal property are assessed at \$5300. What tax will I be required to pay if the rate of property tax is 75 cents on \$100, and my poll tax is \$1?
9. If the assessed value of the taxable property of a county is \$13578200 and the sum required by taxation is \$204538, what should be the rate of taxation on \$100, if there are 1730 residents who are subject to a poll-tax of 50 cents?
10. If the assessed value of real estate in a town is \$758200, and of personal property \$215000, and the sum required by taxation is \$14469, and 4 % of the tax upon personal property is estimated to be uncollectible, what should be the rate of taxation expressed in mills on \$1?

DUTIES OR CUSTOMS

288. INTRODUCTION. **Duties or customs** are taxes levied by a nation upon goods imported from foreign countries. By the Payne-Aldrich Tariff Act of 1909, these duties on some articles are fixed at a certain per cent. of their market value at the place

of purchase, including the value of the container or covering in which the goods are shipped unless such container or covering is otherwise specially subject to duty under some other provision of that tariff act; and on other articles are fixed upon the weight or measure of imported goods without reference to their value. When levied at a certain per cent. of their value, duties are called *ad valorem;* and when levied at so much per yard, per pound, per gallon, etc., they are called *specific.* Sometimes, upon the same article, both an *ad valorem* and a *specific* duty may be levied; as jute matting 15 cents per square yard and 30% ad valorem.

The ports at which duties are collected are called *ports of entry;* the public building in which business relating to customs is transacted is termed a *custom house;* and the chief official in charge of the customs is named the *collector of customs.*

IDENTIFICATION OF TERMS

Ad valorem duties. 1. *The market value at the place of purchase, reduced to United States money, is the base or multiplicand.* 2. *The % of ad valorem duty is the rate or multiplier.* 3. *The ad valorem duty is the percentage or product.*

Specific duties. 1. *The specific duty levied on 1 unit of the quantity subject to such duty, is the multiplicand.* 2. *The number of units subject to such duty is the multiplier.* 3. *The total specific duty is the product.*

NOTE 1. In reducing foreign monetary units to United States money, use Table **138**; and when estimating the ad valorem duty, omit any fraction of a dollar from the base if less than 50 cents, and count it as an additional dollar if 50 cents or more.

NOTE 2. In reducing tons to pounds, or the reverse, estimate 2240 pounds to a ton.

EXAMPLES FOR PRACTISE

Find the ad valorem duty upon foreign goods invoiced at

1. £260 17s. 6d., allowing 5% for leakage, rate of duty 40%.

£260 17s. 6d. = £260.875 [**160**]. \$4.8665 × 260.875 = \$1269.548 + [**138**]. 5% of \$1269.548 = \$63.477, allowance for leakage. \$1269.548 − \$63.477 = \$1206.071, practically \$1206 [Note 1], dutiable value. \$1206 × .40 = \$482.40, ad valorem duty.

2. 12768.70 marks, breakage 10%, duty 30%.
3. 9586.20 francs, leakage 5%, duty 45%.
4. £948 2s. 6d., duty 60%.

5. 17294.40 milreis, duty 25%.

6. 23742 yen, duty 35%.

Find the specific duty upon an importation of

7. 213 tons white lead at $2\frac{1}{2}$ cents per pound.

8. 345 gallons cod-liver oil at 15 cents per gallon.

9. 600 yds. matting, 2 yds. wide, at $3\frac{1}{2}$ cents per sq. yd.

10. What is the duty upon 2400 yards carpet which is 30 inches wide, if invoiced at 12s. 6d. per yard, and subject to an ad valorem duty of 40%, and to a specific duty of 28 cents per square yard?

11. What is the duty on 350 gross lead pencils invoiced at 32 francs per gross, if the ad valorem duty is 25% and the specific duty 45 cents per gross?

12. What is the duty upon an importation of 3700 yards of 27-inch dress goods if invoiced at 7s. 6d. per yard, and subject to an ad valorem duty of 50% and a specific duty of 8 cents per square yard?

SIMPLE PROPORTION

289. INTRODUCTION. *Proportion* is the name of that process by which a quantity (the required answer) is found which will have the same relation to a *similar* given quantity (expressing similar units to the required answer), as the given quantity of which the answer is required has to the similar given quantity of which the same specific answer is known. Thus, if 40 bushels of corn cost \$30 and it is required to find the proportionate cost of 80 bushels of the same kind of corn, the required cost must have the same relation to the given cost (\$30) as the given quantity of which it is required to find the cost (80 bu.) has to the similar given quantity (40 bu.) of which the cost is known; and as 80 bushels of which the cost is required are 2 times 40 bushels of which the cost is known, so the required cost must be 2 times the known cost (\$30), or \$60. The preceding process is appropriately called a proportion because the incomplete *couplet* of similar quantities of which the required answer is one term (\$30 to ?) is *proportional* to (have the same relation to each other as) another *couplet* of similar quantities of which both terms are given (40 bu. to 80 bu.).

NOTE 1. The two terms of a proportion which are of the same name are called a *couplet* of that proportion, and every problem in proportion must con-

tain at least *two couplets* (one complete couplet and one incomplete couplet). The first or left-hand term of each couplet is distinguishably called its *antecedent* (going before); and the second or right-hand term, its *consequent* (following after). The relation which the antecedent of any couplet has to its consequent is called the *ratio* (the relation) of that couplet.

NOTE 2. A problem in simple proportion is one in which a complete couplet of similar terms is given which have a certain relation to each other; and in which one term of a second, incomplete couplet of other similar terms is given and it is required to find the remaining term of that couplet which shall have the same relation to its given term, as the corresponding terms of the complete couplet have to each other.

NOTE 3. A *direct proportion* is one in which the required answer will be proportionately increased as the term of which the answer is required, is increased; or decreased as that term is decreased. Thus, if the required result is wages of a given number of men, the greater the number of men employed, the more the wages, and the fewer the number of men employed, the less the wages; or if the cost of a given number of articles is required, the more the number of articles bought, the greater the cost, and the fewer the number of articles, the less the cost, etc.

NOTE 4. An *inverse proportion* is one in which the required answer will be proportionately *diminished*, as the term of which the answer is required is *increased;* or *increased* as the term of which the answer is required, is *decreased.* Thus, if the required answer is days in which a given task can be performed by a given number of men, it is evident that the *greater* the number of men employed upon the task, the *fewer* will be the number of days to finish it; and the *fewer* the number of men employed, the *greater* the number of days to complete it.

ILLUSTRATIVE EXAMPLES

1. If 56 pounds of butter cost $11.76, what will 32 pounds of the same kind of butter cost?

SOLUTION

$$\frac{\$1.68 \quad 4}{\$11.76 \times \frac{32}{56} = \$6.72}$$
$$7$$
$$1$$

EXPLANATION. The required cost of 32 lb. has the same relation to the given cost of 56 lb. ($11.76) as the number of pounds of which the cost is required (32) has to the number of pounds of which the cost is given (56). As 32 lb. are $\frac{32}{56}$ of 56 lb., so the required cost of 32 lb. must be $\frac{32}{56}$ of the given cost of 56 lb., and $\$11.76 \times \frac{32}{56} = \$6.72$.

2. If 9 masons can build a wall in 8 working days, how many days will be required for 12 masons to build the same wall?

SOLUTION

$$\frac{\overset{2}{\cancel{8}}\text{ days}\times\overset{3}{\cancel{9}}}{\underset{\cancel{3}}{\cancel{12}}_{1}}=6\text{ days}$$

EXPLANATION. More masons (than 9 masons) can build the wall in proportionately *fewer* days; that is, as the number of masons is increased, so proportionately more of the wall will be built in 1 day, and the fewer will be the days required to complete the wall. Hence, 12 masons ($\frac{1}{3}^2$ or $\frac{4}{3}$ of 9 masons) can *inversely* build the wall in the *inverse* of $\frac{4}{3}$ of 8 days, that is, in $\frac{3}{4}$ of 8 days, or 6 days.

RULE. 1. *Compare the given term of which the answer is required with the given term which is similar to it in name and concerning which an affirmation is made which is similar in name to the required answer; and express the result of the comparison in the form of a common fraction, inverting its terms when the proportion is seen to be inverse* [Note 4]. 2. *Multiply the given term which is of the same specific name as the required answer by the obtained fraction.*

EXAMPLES FOR PRACTISE

1. If 200 pounds of bacon cost $24, what will 475 pounds of the same kind of bacon cost?

2. If 575 yards of cloth cost $477.25, what will 340 yards of the same cloth cost?

3. If $1125 are required weekly to pay 75 operatives in a factory, how much will be required weekly if 40 additional operatives are employed at the same wages?

4. How many pounds of beef will feed 320 men if 90 pounds are sufficient to feed 80 men?

5. If 75 sheep of a flock were sold for $292.50, how many sheep from the same flock should be sold at the same rate for $1053?

6. What will 642 tons of coal cost if 49 tons of the same coal can be bought for $303.80?

7. If 52 pounds of ham are bought for $12.48, what should be the cost of 75 lb. 12 oz. of the same kind of ham?

SUGGESTION. If one term of a problem in proportion is a compound number, it should first be reduced to a simple denominate number by **156** or **160**; and both it and the term with which it is to be compared should express similar denominate units.

8. If $520 will produce $10.40 interest, how much interest will $820 produce if loaned for the same time and at the same rate?

SUGGESTION. All of the given terms of a proportion may be of the same *general* name without necessarily being of the same *specific* name. Thus, in Ex. 8, $520 and $820 are specifically *dollars of principal;* and $10.40 and the required answer are specifically *dollars of interest.*

9. What principal will produce $17.82 interest if $3784 will produce $104.06 interest at the same rate per annum and in the same time?

10. If a certain principal will produce $19.98 interest in 296 days, in what time will the same principal produce $10.26 interest if loaned at the same rate per annum?

11. A house 30 feet high casts a shadow 18 feet long. How high is a steeple which casts a shadow 75 feet long at the same time in the day?

12. If a tree 50 feet high casts a shadow of 10 feet, how long a shadow will be cast at the same moment by another tree which is 40 feet high?

13. If 30 men can complete a given task in 40 days, how many men are necessary to complete it in 50 days?

14. In how many days can 75 men build a bridge, if it has been estimated that 125 men can build it in 18 days?

15. If 175 pounds of beef are sufficient to feed a gang of laborers for 5 days, how much beef will be necessary to feed them for 15 days?

16. What should be the tax on a house assessed at $4500, if a house in the same town which is assessed at $6000 is taxed $75?

17. A and B are partners in business, the former investing $5000 and the latter $7000. The net gains during a certain interval were $3600, which were divided between them in proportion to their respective investments. How much of the gain should each receive?

18. A clothier sold a suit of clothes at a profit of $4.80 and gained 20%. What would have been his % of gain if he had sold the suit at a profit of $6?

19. A merchant sold goods for \$644 and gained 15%. What would have been his % of gain if he had sold them for \$756?

20. A dealer sold merchandise for \$45 and gained 25%. At what price must he have sold the merchandise to gain 30%?

COMPOUND PROPORTION

290. INTRODUCTION. A problem in *compound proportion* is one in which an answer is required which depends upon more than one couplet of similar terms, and which therefore requires as many separate comparisons as there are given couplets upon which the required answer depends. Each separate comparison is made in the same manner as in simple proportion; and the resultant of the several comparisons (a *compound* fraction) when reduced to a simple fraction, will express what proportion of the given term which is of the same specific name as the required answer, must be taken to obtain the required answer.

ILLUSTRATIVE EXAMPLE

If 72 men can dig a sewer which is 240 yards long, 12 feet wide, and 9 feet deep, in 18 days, how many men will be required to dig in similar soil a sewer which is 360 yards long, 15 feet wide, and 24 feet deep, in 12 days?

SOLUTION

$$\frac{\cancel{360}^{3}}{\cancel{240}_{2}} \text{ of } \frac{\cancel{15}^{5}}{\cancel{12}_{1}} \text{ of } \frac{\cancel{24}^{\cancel{2}^{1}}}{\cancel{9}_{1}} \text{ of } \frac{\cancel{18}^{\cancel{2}^{1}}}{\cancel{12}_{\cancel{2}_{2}}} = \frac{15}{2}$$

$$\frac{15}{\cancel{2}} \text{ of } \cancel{72}^{36} \text{ men} = 540 \text{ men}$$

EXPLANATION. If 72 men are required to dig a sewer 240 yards long, the other dimensions being equal, it will require $\frac{\cancel{240}}{\cancel{360}}$ of 72 men to dig a sewer 360 yards long [Note 3, **289**]. If 72 men are required to dig a sewer 12 ft. wide, the remaining dimensions being equal, it will require $\frac{\cancel{12}}{\cancel{15}}$ of 72 men to dig a sewer 15 ft. wide. If 72 men are required to dig a sewer 9 ft. deep, the other dimensions being equal, it will require $\frac{\cancel{9}}{\cancel{24}}$ of 72 men to dig a sewer 24 ft. deep. If 72 men are required to dig a sewer in 18 days, the dimensions of each sewer being the same, it will *inversely* require $\frac{\cancel{18}}{\cancel{12}}$ of 72 men to dig it in 12 days [Note 4, **289**].

Each of the above separate comparisons shows what proportion of 72 men would be required if there were no other element entering into the comparison; therefore if the results of the several separate comparisons are combined into

a single expression, the resultant compound fraction ($\frac{360}{240}$ of $\frac{15}{12}$ of $\frac{24}{9}$ of $\frac{18}{12}$ of 72 men), or simplified by reduction, $\frac{15}{2}$ of 72 men, or 540 men will be the required number of men to dig a sewer 360 yd. long, 15 ft. wide, 24 ft. deep, in 12 da., if all the elements which enter into the comparison are *jointly* considered.

RULE. 1. *Separately compare each given term upon which the required answer depends with the corresponding given term of the same name, of which an affirmation is made which is similar in name to the required answer; and express the result of each separate comparison in the form of a common fraction.*

2. *Combine each separate fraction which expresses a simple proportion when it is separately considered into a compound fraction which will express the compound proportion when all the terms are jointly considered; and reduce the resultant compound fraction to a simple fraction in lowest terms.*

3. *Multiply the given term which is of the same specific name as the required answer by the obtained simple fraction.*

EXAMPLES FOR PRACTISE

1. If 15 men can make 30000 shingles in 5 days, how many shingles can 25 men make in 9 days?

2. If 12 men can earn $270 in 9 working days, how much can 28 men earn in 5 working days?

3. If $864 loaned for 126 days at 5% per annum will produce $15.12 interest, what principal loaned at 8% for 54 days will produce $5.10 interest?

4. If a gain of $68 is made by selling merchandise which cost $340 at 20% profit, what will be the proportionate gain from selling merchandise which cost $720 at 25% profit?

5. If by walking 4 miles an hour for 9 hours per day a pedestrian can walk a certain distance in 10 days, in how many days can he walk the same distance at the rate of 3 miles an hour for 8 hours a day?

6. If a contractor charged $336 for digging a cellar 54 ft. long, 35 ft. wide, and 12 ft. deep, what should be his proportionate charge for digging another cellar 45 ft. long, 30 ft. wide, and 9 ft. deep?

EQUATION OF PAYMENTS

291. INTRODUCTION. (*a*) *Equation*, as a general term, means the averaging or *equalizing* of two or more unequal quantities [Ex. 13, 54]. (*b*) When applied to payments, it is the process of finding *one date* at which a single payment of the total of two or more debts can be made which will be equal to paying each separate debt at its own separate due date. (*c*) When applied to terms of credit it is the process of finding *one uniform term of credit* for all the items of a purchase or a sale which is the equivalent of the different terms of credit at which the several items were separately purchased or sold.

(*d*) The equivalent date at which two or more payments, due at different dates, can be simultaneously made is called the *average* or *equated date;* and (*e*) the uniform term of credit which is the equivalent of two or more different terms of credit is called the *average* or *equated term of credit.*

TO FIND THE EQUATED DATE OF PAYMENT WHEN THE DATES OF SALE ARE EQUAL AND THE TERMS OF CREDIT UNEQUAL

ILLUSTRATIVE EXAMPLE

292. A merchant sold goods on June 16, 1911, as follows: $600 at 90 days, $800 at 30 days, and $700 at 60 days. What is the equated date for the payment of the sum of the three items?

SOLUTION

1911	The use of				The use of		
June 16	$600	for 90	da.	=	$54000	for 1	da.
	800	" 30	"	=	24000	"	"
	700	" 60	"	=	42000	"	"
The use of	$2100	for (?)	days	=	$120000	"	"

$120000 ÷ $2100 = 57 times 1 da., or 57 days.

June 16, 1911 + 57 da. = Aug. 12, 1911.

EXPLANATION. If the purchaser had been required to pay the sum of the three items ($2100) on the day of purchase (June 16), he would have been deprived, as per terms of purchase, of the use of $600 for 90 da., or its equivalent of 90 times $600, or $54000 for 1 da.; and of the use of $800 for 30 da., or its equivalent of 30 times $800, or $24000 for 1 da.; and of the use of $700 for 60 da., or its equivalent of 60 times $700, or $42000 for 1 da.; or a total

deprivation of the use of \$54000 + \$24000 + \$42000, or \$120000 for 1 da. Hence the equitable time for the payment of \$600 + \$800 + \$700, or \$2100, must be as many days after June 16 as will offset this deprivation. As the total purchase (\$2100) kept 57 days will as nearly as possible equal this deprivation (for \$2100 × 57 = \$119700, which is as near a product to \$120000 as is possible with an integral multiplier), so 57 da. after June 16, 1911, or Aug. 12, 1911, must as nearly as possible be the equated date of payment.

RULE. *Multiply each item by its term of credit, and divide the sum of the resulting products by the sum of the items, to find the equated term of credit.*

Add the equated term of credit to the date of purchase to find the equated due date.

NOTE 1. As the day is the lowest unit of time considered in terms of credit, if the division is not exact, disregard the remainder if less than half the divisor, or regard the remainder as another day if one-half the divisor or more.

NOTE 2. If the terms of credit are expressed in months, and the division is not exact, multiply the remainder by 30 and divide the resulting product by the original divisor, to reduce the remainder to days or 30ths of a month [**86**], applying Note 1 to the final remainder.

NOTE 3. If *all* the items of sale are already *equal*, they may be disregarded; and the unequal *terms of sale* will then need only to be averaged [Ex. 13, **54**]; but if the items are also *unequal*, the products of the items by their respective terms of credit must be averaged.

EXAMPLES FOR PRACTISE

Find the equated term of credit of the following:

(1)	(2)	(3)
\$500 at 3 months	\$800 at 2 months	\$525 at 3 months
700 " 2 "	200 " 4 "	680 " 2 "
400 " 4 "	900 " 1 "	720 " 4 "
600 " 1 "	450 " 3 "	815 " 1 "

Find the equated date of the following:

	(4)		(5)
1912		1911	
Apr 16	\$650 at 30 days	Oct. 23	\$620 at 90 days
"	470 " 90 "	"	400 " 30 "
"	520 " 60 "	"	750 " 60 "
"	350 " 90 "	"	800 " 90 "

6. A house was sold for $6000, one-third payable in cash, one-third in 6 months, and the remainder in 12 months, without interest. What is the equated time for paying the entire purchase money? [Note 3]

7. On Mch. 23, 1911, goods were sold for $645 at 60 days, for $415 at 30 days, and for $500 cash. What is the equated date for the payment of the entire bill?

8. On June 18, 1910, goods were sold to a customer as follows: $650 for cash, $800 at 60 days, and $400 for cash. At what time can the three items be paid without loss to either buyer or seller?

TO FIND THE EQUATED DATE OF PAYMENT WHEN THE DATES OF SALE ARE UNEQUAL AND THE TERMS OF SALE ARE EQUAL

ILLUSTRATIVE EXAMPLE

293. William Garig bought of George W. Keller merchandise as follows: Aug. 24, 1912, $862; Sept. 16, 1912, $547; Oct. 28, 1912, $485. If no credit was allowed, what was the equated date for paying the sum of the three items?

FIRST SOLUTION

Assumed date, July 31, 1912

1912		Days from assumed date	Use for 1 day of
Aug. 24	$862	× 24 =	3448 = 862 × 4 units 1724 = 862 × 2 tens
Sept. 16	547	× 47 =	3829 = 547 × 7 units 2188 = 547 × 4 tens
Oct. 28	485	× 89 =	4365 = 485 × 9 units 3880 = 485 × 8 tens
Use of	$1894	for (?) da. =	$89562 used for 1 da.

$89562 ÷ $1894 = 47 + times 1 da. after assumed date.

Hence, 47 da. after July 31, or Sept. 16 = equated date.

EXPLANATION. For convenience, assume as the required equated date, the last day of the month immediately preceding the earliest month in the

account (July 31); note the extent of the error resulting from this assumption, and make the proper correction.

Thus, if the first item ($862) were paid on this assumed date (July 31), though not due until Aug. 24, or 24 days later, the purchaser would be deprived of the use of $862 for 24 da., or the equivalent of the use of 24 times $862 for 1 da.

If the second item ($547) were paid on this assumed date (July 31), though not due until Sept. 16, or 47 da. later, the purchaser would be deprived of the use of $547 for 47 days, or of the equivalent use of 47 times $547 for 1 da.

If the third item ($485) were paid on this assumed date (July 31), though not due until Oct. 28, or 89 da. later, the purchaser would be deprived of the use of $485 for 89 da., or of the equivalent use of 89 times $485 for 1 da.

Hence, to pay the sum of the three items ($1894) on the assumed date (July 31) would cause a total deprivation to the purchaser equivalent to the use of $89562 for 1 da.; and to prevent this inequitable deprivation, the sum of the three items should be paid as many days after July 31 as will offset this loss, that is, as the total purchase ($1894) if kept 47 da. will equal the total deprivation ($89562 used for 1 da.), for $1894 × 47 = 89018, which is as near that deprivation as is possible to obtain with an integral multiplier, so 47 da. after July 31, or Sept. 16, must as nearly as possible be the equated date.

ABRIDGED SOLUTION

Assumed date, July 31, 1912

1912		Days from assumed date		Hundreds of dollars for 1 da.
Aug. 24	$862	× 24	=	34 = (8 hundred × 4 units) + 2 hundred to carry 172 = (86 tens × 2 tens) and nothing to carry
Sept. 16	547	× 47	=	38 = (5 hundred × 7 units) + 3 hundred to carry 219 = (54 tens × 4 tens) + 3 hundred to carry
Oct. 28	485	× 89	=	44 = (4 hundred × 9 units) + 8 hundred to carry 388 = (48 tens × 8 tens) + 4 hundred to carry

Use of $1894 for (?) da. = $89500 used for 1 day.

$89500 ÷ $1894 = 47 + times 1 day after July 31, or Sept. 16.

EXPLANATION. Both of the above solutions are similar in principle; but in the abridged solution much unnecessary labor is avoided by omitting the multiplication of such orders in the items as will not affect the final result; that is, by actually multiplying only such orders in the items as will produce hundreds of dollars for 1 day (mentally multiplying one or two of the next adjoining rejected orders of each item to obtain the correct carrying figure in hundreds, counting half a hundred or more as an additional hundred). The use of (interest of) less than half a hundred dollars for 1 day is less than our lowest unit of coinage (1¢), and does not therefore require consideration.

NOTE 1. The orders which, when multiplied, will produce results in hundreds of dollars for 1 day, are

Hundreds' order of the items × units' order of the days.

Tens' order of the items × tens' order of the days.

Units' order of the items × hundreds' order of the days.

After obtaining the sum of the products, annex two 0's to make that sum express its true value in even hundreds of dollars, and then proceed as in the first solution.

RULE. *Assume as the equated date the last day of the month immediately preceding the earliest month in the account. Multiply each item by the interval in days between the assumed date and the date of such item. Divide the sum of the obtained products by the sum of the items to find the number of days to count forward from the assumed equated date to obtain the correct equated date.*

EXAMPLES FOR PRACTISE

Find the equated due date of the following cash sales:

(1)		(2)		(3)	
1910, Apr. 20	$800	1911, Mch. 6	$416	1912, July 3	$582
May 13	750	May 19	300	Aug. 21	430
June 28	675	June 6	520	Oct. 12	618

(4)		(5)		(6)	
1911, May 23	$275	1910, Oct. 16	$275	1912, Feb. 18	$468
July 14	356	Nov. 28	350	Apr. 7	500
Aug. 10	400	Dec. 2	196	May 23	260
Sept. 21	540	Dec. 23	248	June 19	592

NOTE 2. If *all* the items have a uniform term of credit, and it is expressed in days, first find by the preceding process the equated *time* in days *exclusive* of the uniform term of credit; then add the uniform term of credit to the result to find the equated time *inclusive* of the term of credit; lastly find the equated *date* in the usual manner.

(7)			(8)		
1910, May 23	$563	at 90 da.	1911, July 14	$680	at 60 da.
June 10	625	"	Aug. 25	560	"
Aug. 2	718	"	Oct. 13	725	"

NOTE 3. If the uniform term of credit is expressed in months, first find the equated *date* exclusive of the months of uniform credit; then count forward from this obtained date, as many calendar months as are contained in the uniform term of credit.

(9)			(10)		
1912, Jan. 17	$368	at 2 mo.	1910, June 26	$680	at 3 mo.
Mch. 8	415	"	Aug. 10	753	"
May 23	298	"	Sept. 28	560	"

(11)			(12)		
1911, Apr. 24	$250	at 30 da.	1912, Sept. 29	$175	at 1 mo.
June 11	625	"	Oct. 18	256	"
July 21	700	"	Nov. 10	340	"
Aug. 14	290	"	Dec. 6	294	"

TO FIND THE EQUATED DATE OF PAYMENT WHEN THE DATES OF SALE AND THE TERMS OF CREDIT ARE BOTH UNEQUAL

ILLUSTRATIVE EXAMPLE

294. Find the equated date for paying the total of the following sales: 1912 — July 28, $652.35 at 60 days; Sept. 10, $471.20 at 30 days; Oct. 13, $537.63 at 90 days.

FIRST SOLUTION

Assumed date, June 30, 1912

1912		Days to due date		Use for 1 day of
July 28 at 60 da.	$652.35	× 88	=	5219 = (652 × 8 units) + 3 to carry 52188 = (652.3 × 8 tens) + 4 to carry
Sept. 10 at 30 da.	471.20	× 102	=	942 = (471 × 2 units) + 0 to carry 47120 = (471.20 × 1 hundred)
Oct. 13 at 90 da.	537.63	× 195	=	2688 = (537 × 5 units) + 3 to carry 48387 = (537.6 × 9 tens) × 3 to carry 53763 = (537.63 × 1 hundred)

Use of $1661.18 for (?) da. = $210307 used for 1 day.
$210307 ÷ $1661 = 127 — times 1 day after assumed date.
127 days after June 30, 1911 = Nov. 4, 1912, equated date.

EXPLANATION. Assume as the equated date the last day of the month which immediately precedes the earliest month found in the account to be averaged, calculate the extent of the error if the assumption proves wrong, and make the proper correction, as follows:

If the purchaser had paid the first item ($652.35) on June 30, though not due until 60 da. after July 28, or 88 da. after June 30, he would have been de-

prived, as per terms of sale, of the use of \$652.35 for 88 da., which would have been equivalent to an error of the interest on 88 times \$652.35 for 1 da.

If the second item (\$471.20) had been paid on June 30, though not due until 30 da. after Sept. 10, or 102 da. after June 30, the error would have been equivalent to the interest on 102 times \$471.20 for 1 da.

If the third item (\$537.63) had been paid on June 30, though not due until 90 da. after Oct. 13, or 195 da. after June 30, the error would have been equal to the interest on 195 times \$537.63 for 1 da.

Therefore, if the sum of the three items (\$1661.18) had been paid on the assumed date (June 30) instead of on their respective due dates, the total error would have been the sum of the three ascertained errors, or the interest on \$210307 for 1 da.; and to correct this error the sum of the three items (\$1661.18) should be kept by the purchaser as many days after June 30 as would be equivalent to the use of \$210307 for 1 da. As the sum of the products (\$210307) divided by the sum of the items (\$1661.18) equals 127−, so \$1661.18 kept 127 da. after the assumed date will offset this loss; and 127 days after June 30, or Nov. 4, 1912, must be the required date upon which the sum of the three purchases (\$1661.18) should be paid without involving the loss of the use of money by either purchaser or seller.

ABRIDGED SOLUTION

Assumed date, June 30, 1912

1912		Days to due date	Hundreds of dollars for 1 day
July 28 at 60 da.	\$652.35 ×	88	= { 52 (6 hundreds × 8) +4 522 (65 tens × 8) + 2
Sept. 10 at 30 da.	471.20 ×	102	= { 9 (4 hund. × 2) +1 471 (471 units × 1) + 0
Oct. 13 at 90 da.	537.63 ×	195	= { 27 (5 hund. × 5) + 2 484 (53 tens × 9) + 7 538 (537 units × 1) + 1

Use of \$1661.18 for ? da. = \$210300 used for 1 day.
\$210300 ÷ 1661 = 127 times 1 da. after assumed date.
June 30, 1912 + 127 da. = Nov. 4, 1912, equated date.

THE EXPLANATION of the abridged solution is similar to that of the first solution preceding it. They differ only in the method of performing the multiplications. The contracted method, involving no products of less value than hundreds of dollars for 1 day, is fully explained in the abridged solution of Ill. Ex., 293 and Note 1 following it.

RULE. *Add the term of credit of each item to the multiplier for that item as obtained by Rule 293; and continue in accordance with that Rule.*

NOTE 1. When dividing the sum of the products by the sum of the items, omit the cents from the divisor if less than 50, or increase the dollars by $1 if the cents are 50 or more.

EXAMPLES FOR PRACTISE

Find the equated date of payment of the following:

(1)

1910, Apr.	6	$600 at 90 da.
May	18	725 " 30 "
June	23	480 " 60 "

(2)

1911, Mch.	14	$750 at 60 da.
Apr.	25	825 " 90 "
June	13	690 " 30 "

(3)

1912, Feb.	17	$482.35 at 60 da.
Apr.	29	527.60 " 30 "
June	2	368.45 " 90 "
July	13	416.25 " 30 "

(4)

1911, Sept.	3	$248.15 at 90 da.
Oct.	10	361.78 " 60 "
Nov.	25	419.95 " 30 "
Dec.	20	195.80 " 60 "

(5)

1909, July	21	$682.40 at 30 da.
Sept.	12	546.15 " 90 "
Nov.	25	725.87 " 60 "
Dec.	4	498.78 " 90 "

(6)

1912, May	31	$425.89 at 60 da.
June	14	516.75 " 90 "
Aug.	3	398.78 " 60 "
Sept.	20	465.85 " 30 "

NOTE 2. If a term of credit is expressed in months, it has been found most convenient in practise to proceed as follows: — If goods are bought on May 16 at 3 months, it is readily seen that 3 months after May 16 will include the end of May (31st), the end of June, and the end of July (31st), or 2 31sts in all; hence write a small 2 over that term of credit (^{2}3 mo.), and when it is reached in the equation, it will be identified as 92 da., that is, as 3 months of 30 da. each, plus the increase for the *two* 31sts. Or, if an item at 2 months is dated Jan. 4, 1910, the term of credit will include the end of Jan. (31st), the end of Feb. (28th), a net minusage of 1, hence write $^{-1}$2 mo. over that term of credit, and count it as 60 da. less 1. Or, if the item is at 4 months, and is dated Nov. 24, 1911, the term of credit will include the end of Nov., the end of Dec. (31st) the end of Jan. (31st), and the end of Feb. (29th), or a net plusage of 1, expressed ^{1}4 months, and counted 120 da. plus 1; etc.

(7)			(8)		
1912, Apr. 16	$468.75	at 3 mo.	1911, July 12	$728.15	at 2 mo.
June 12	529.60	" 1 "	Sept. 3	465.70	" 4 "
July 15	387.90	" 2 "	Oct. 18	598.67	" 1 "
Aug. 28	625.75	" 4 "	Nov. 23	615.80	" 3 "

(9)			(10)		
1911, May 25	$568.35	at 3 mo.	1912, Jan. 18	$396.75	at 2 mo.
June 2	275.50	" 60 da.	Feb. 12	285.67	" 3 "
Aug. 17	348.25	" 2 mo.	Apr. 18	2968.52	" 60 da.
Sept. 13	465.72	" 30 da.	May 10	98.76	" 30 "

AVERAGING ACCOUNTS

295. INTRODUCTION. The process of *averaging an account is* an application of the principles of equation of payments to an account which has both debit and credit items. The method common to both is *to assume* a date of payment, to ascertain the resulting error by comparison of the due date of each item with the assumed common due date for all the items, and to make the proper correction. When all the items were debits, it was seen that the assumption of a date of payment earlier than that of the several items would uniformly result in a loss to the debtor. In averaging an account, however, which has both debit and credit entries, the resulting error from assuming a date of settlement earlier than that of the several items will only indicate a loss to the purchaser with respect to the debit entries; and will show a gain to him with regard to all credit entries. The *net* gain or *net* loss of the entire account must therefore be the determining factor in finding the proper correction of the assumed date.

296. Averaging an account is finding the proper time for paying the balance of an account which includes both debit and credit entries.

NOTE. The average date is that date at which the balance of an account should be paid to avoid the loss of the use of money by either party concerned in the account. Hence, if the balance of an account is paid *before* the average date, it is usual to *diminish* the balance by the compensatory discount thereon for the days of prepayment; and if paid *after* the average date, to *increase* the balance by the compensatory interest thereon for the days of postpayment.

TO AVERAGE AN ACCOUNT WITH ITEMS ON BOTH SIDES

ILLUSTRATIVE EXAMPLE

Find the equated date for paying the balance of the following account:

Dr. J. P. SMITH Cr.

1911				1911			
May 23	To Mdse.	$768	42	July 12	By Cash	$694	40
July 16	"	537	80	Aug. 16	"	478	75
Aug. 27	"	659	35	Sept. 25	"	354	80

FIRST SOLUTION

Assumed date, April 30, 1911

May 23	$768.42 × 23 =	2305 15368	July 12	$694.40 × 73 =	2083 48608
July 16	537.80 × 77 =	3765 37646	Aug. 16	478.75 × 108 =	3830 47875
Aug. 27	659.35 × 119 =	5934 72529	Sept. 25	354.80 × 148 =	2838 14192 35480
	$1965.57	$137547		$1527.95	$154906
	1527.95				137547
	$437.62, balance of items			bal. of products,	$17359

$17359 ÷ $438 = 40 − times 1 da. before Apr. 30, or Mch. 21.

EXPLANATION. Assuming April 30, 1911, as the date for paying all the debit items, it is found by Ill. Solution, **293**, that if Smith had paid the three debit items on the assumed date instead of their respective due dates, he would have lost the use of (the interest on) $137547 for 1 da.; and considering the credit items upon the same assumption that the debits were due (and should therefore have been paid) on April 30, it is found that Smith would have gained the use of $694.40 for 73 da., of $478.75 for 108 da., and of $354.80 for 148 da., or a total gain equivalent to the use of $154906 for 1 da. Hence, the net gain to Smith upon this assumption must be equivalent to the use of

$154906 (gain) − $137547 (loss), or to the net use of $17359 for 1 da. Smith should therefore pay the balance of his account ($437.62) as many days *before* the assumed date (April 30) as will offset this gain. As Smith's net gain ($17359) divided by Smith's balance ($437.62) equals 40 −, so 40 da. before April 30, or March 21, 1911, must be the equated date upon which Smith should pay this balance without loss of the use of money (of interest) by his creditor or himself.

ABRIDGED SOLUTION

Assumed date, April 30, 1911

May 23	$768.42 ×	23 =	23 154	July 12	$694.40 ×	73 =	21 486
July 16	537.80 ×	77 =	38 376	Aug. 16	478.75 ×	108 =	38 479
Aug. 27	659.35 ×	119 =	59 725	Sept. 25	354.80 ×	148 =	28 142 355
	$1965.57		$1375		$1527.95		$1549
	1527.95						1375
	$437.62						$174

$17400 ÷ $438 = 40 times 1 da. *before* Apr. 30, or Mch. 21.

RULE. *Assume as the equated date, the last day of the month which immediately precedes the earliest month in the account. Multiply each item by the interval in days between the assumed date and the due date of such item; and divide the balance of the resulting products by the balance of the items to find the error in the assumed date. Count the error in days forward from the assumed date if both balances are on the same side, or backward from the assumed date if the balances are on opposite sides.*

NOTE 1. The equated date for paying the balance of an account does not necessarily mean the date at which it *must* be paid, for it may be impossible for the debtor at that date to know that it would be so payable, as in the preceding Ill. Ex., in which the equated date is prior to the date of any of the purchases, perhaps even prior to the debtor's intention to make any purchase whatever. In other instances, the debtor may have the requisite knowledge of the equated date but may not possess the means on the equated date with which to pay his indebtedness. In either case, the equated date is the proper time, if practicable, for paying the balance of the account without loss of in-

terest to either debtor or creditor. If *for any reason*, the balance is paid *before* the equated date, the debtor is usually allowed discount on said balance for the days of prepayment, and if paid after the equated date, the creditor will usually claim interest upon said balance for the days of postpayment.

EXAMPLES FOR PRACTISE

Dr. (No. 1) WARREN H. SADLER Cr.

1910			1910		
Oct. 25	To Mdse.	$675 00	Nov. 2	By Cash	$249 00

Dr. (No. 2) GEO. C. ROUND Cr.

1911			1911		
Mch. 18	To Mdse.	$385 90	Apr. 8	By Cash	$246 75
May 24	"	472 75	June 16	"	187 35

Dr. (No. 3) E. BENSEL & Co. Cr.

1910			1910		
Apr. 17	To Mdse.	$678 35	May 6	By Cash	$263 50
June 12	"	594 20	July 18	"	345 75
July 24	"	725 45	Aug. 10	"	416 80

Dr. (No. 4) HENRY ALEXANDER Cr.

1912			1912		
July 6	To Mdse.	$416 72	Aug. 8	By Cash	$327 90
Aug. 29	"	532 50	Sept. 20	"	615 25
Oct. 14	"	374 65	Nov. 12	"	285 45

Dr. (No. 5) H. A. GRIESEMER Cr.

1911			1911		
May 13	To Mdse.	$256 15	June 21	By Cash	$243 60
July 16	"	476 35	Aug. 17	"	185 75
Aug. 30	"	348 50	Sept. 12	"	372 50

Dr. (No. 6) A. H. Atwood Cr.

1910					1910				
June 3	To Mdse.,	60 da.	$392	70	July 13	By Cash		$243	60
July 28	"	30 "	527	98	Aug. 25	" Note, 60 da.		185	75
Sept. 19	"	net	476	87	Oct. 10	" Cash		372	50

Dr. (No. 7) Fairman A. Sadler Cr.

1912					1912				
Feb. 23	To Mdse.,	90 da.	$485	90	Mch. 4	By Cash		$275	80
Apr. 17	"	60 "	372	65	May 16	" Note, 60 da.		300	00
May 26	"	30 "	427	25	July 8	" " 30 "		250	00

Dr. (No. 8) R. M. Browning Cr.

1911					1911				
Oct. 24	To Mdse., net		$628	50	Nov. 12	By Note, 90 da.		$352	75
					1912				
Dec. 18	"	90 da.	473	85	Jan. 21	" " 60 "		450	00
1912									
Jan. 20	"	60 "	532	65	Feb. 18	" Cash		275	30

Dr. (No. 9) H. C. Reitz Cr.

1910					1910				
Mch. 16	To Mdse., 3 mo.		$575	80	Apr. 28	By Note, 1 mo.		$384	45
May 19	"	net	618	35	June 6	" Cash		256	85
June 24	"	2 mo.	487	15	July 22	" Note, 3 mo.		416	75

Dr. (No. 10) W. M. Cutchin Cr.

1911					1911				
Jan. 18	To Mdse., 2 mo.		$394	67	Feb. 23	By Note, 4 mo.		$284	62
Feb. 20	"	1 "	472	50	Mch. 29	" " 2 "		316	25
Apr. 7	"	3 "	528	75	May 10	" " 3 "		472	38

Dr.	(No. 11)		C. L. REINDOLLAR				Cr.	
1912					1912			
Feb. 29	To Mdse., 60 da.		$826	45	Mch. 27	By Cash	$375	90
Mch. 18	"	Net	416	20	Apr. 12	" Note, 90 da.	462	75
May 20	"	3 mo.	752	68	June 30	" Cash	400	00
June 13	"	90 da.	528	34	July 18	" Note, 2 mo.	275	15

12. On June 24, 1911, H. M. Lee bought merchandise for $753.25 and 90 days' credit. If Lee made a cash payment of $500 on July 3, 1911, what is the equated date for paying the remainder?

NOTE 2. If part of a debt is paid before it is due, many merchants permit the debtor to postpone payment of the remainder for a proportionate time after it is due. The date of payment of the remainder will be the equated date, found in the usual manner. Such an extension, however, is not enforcible by law if the creditor objects.

13. On July 18, 1912, G. M. Hett bought merchandise for $900 and 60 days' credit; and on Aug. 3, 1912, paid $300 on account. How much did he owe on Dec. 13, 1912, at 6%?

NOTE 3. It is customary to charge interest upon the balance of an account for the number of days it is paid after its equated date.

14. A has two of B's notes; one of $580, dated Mch. 24, 1911, payable 4 months after date; and the other of $624, dated Apr. 16, 1911, payable 60 days after date. On May 6, 1911, B paid $700 to A, canceled the two notes, and gave a new 30-day non-interest-bearing note for the remainder. What was the date of the new note?

NOTE 4. The note must be so dated as to fall due on the equated date, at which time the unpaid balance for which it was given would have fallen due.

15. Thomas G. Hayes bought goods for $650 on Aug. 18, 1911, and 90 days' credit. On Sept. 23, 1911, he paid $175 on account; and on Oct. 5, 1911, he paid $225 and gave a 60-day interest-bearing note for the remainder. What should have been the date of the note?

NOTE 5. If a note bearing interest from its date is given in settlement of the balance of an account, its date should be the equated date (Note 3).

AVERAGING ACCOUNTS SALES

ILLUSTRATIVE EXAMPLE

297. Find the equated date for paying the net proceeds of the following sales on commission: Sales, 1912 — June 19, $285.90; July 17, $328.70; Aug. 22, $264.16. Charges, 1912 — July 2, cash advanced consignor, $350; July 16, freight, $28.70; July 30, drayage, $8.25; commission, 3%.

SOLUTION

Assumed date, May 31, 1912

June 19	$285.90	× 19 =	{ 26	
			{ 29	
July 17	328.70	× 47 =	{ 23	
			{ 131	
Aug. 22	264.16	× 83 =	{ 8	
			{ 211	
	$878.76		$428	

$42800 ÷ $879 = 49 times 1 da.
May 31 + 49 da. = July 19, 1912.
$878.76 × .03 = $26.36 +, com. on sales.

EXPLANATION. First find the equated date of the three sales by **293**, obtaining July 19, 1912, as the date for the total sales ($878.76) and for the total commission ($878.76 × .03 = $26.36).

Next, average the account by **296**, placing all the charges including the commission and its equated date, on the debit side; and placing the total sales ($878.76) and its equated date on the credit side.

Assumed date, June 30, 1912

1912					1912			
July 2	$350.	× 2 =	$7		July 19	$878.76	× 19 =	{ $79
" 16	28.70	× 16 =	{ 2					{ 88
			{ 3					$167
" 30	8.25	× 30 =	2			413.31		19
" 19 (com.)	26.36	× 19 =	{ 2		Bal.	$465.45		$148 Bal.
			{ 3					
	$413.31		19		$14800 ÷ $465 = 32 times 1 da.			
					June 30 + 32 da. = Aug. 1, 1911.			

EXAMPLES FOR PRACTISE

1. An agent sold a consignment as follows: 1910 — May 18, $7695.75; June 2, $5677.20; June 25, $4108.80. His charges

were: 1910 — May 15, freight, $315; May 20, cash advanced on consignment, $3500; commission, 1½%. Find the equated date for remitting the net proceeds to his principal.

2. Find the equated date for paying the net proceeds of the following commission sales: Sales, 1911 — Aug. 5, $287.35; Sept. 2, $328.49; Oct. 20, $289.30. Charges, 1911 — July 28, freight, $18.25; Aug. 3, cash, $500; commission, 2%.

3. Find the net proceeds of the following commission sales, and the equated date for remitting the proceeds to the consignor: Sales, 1912 — Apr. 17, $583.75; May 9, $872.65; June 12, $1075.80. Charges, 1912 — Apr. 23, drayage, $12.50; May 14, cash advanced, $1200; commission, 2%.

CASH BALANCE

298. INTRODUCTION. To take the balance of any account is to find the difference between its total debits and its total credits. To prefix "cash" to the word balance adds nothing to its significance; for all balances are payable in cash or its generally accepted equivalent. The term "cash balance," in business usage, has special reference to the difference, on the date of taking the cash balance, or date of adjustment, between the present worth of all the debit items and the present worth of all the credit items. This includes consideration of the debit interest on each debit item that falls due before the date of adjustment for the number of days between its due date at which it is legally payable to the date of adjustment when it is assumed to be paid; and the credit interest (discount) on each debit item that falls due after the date of adjustment for the number of days between the date of adjustment at which it is assumed as being payable, to its subsequent due date at which it is legally payable. It also includes consideration of the credit interest on each credit item, if cash, for the number of days between its date at which it is actually paid to the date of adjustment at which it is assumed to be paid; or if a time note or draft maturing before the date of adjustment, of credit interest for the number of days between its legal due date at which it is actually paid to the date of adjustment at which it is assumed to be paid; or if a time note or draft maturing after the date of adjustment, of debit interest (discount) for the number of days between the date of adjustment at which it is assumed to be paid and its legal due date at which it is actually paid.

299. The **cash balance** of an account is the difference, on the day of adjustment, between the present worth of all its debit items and the present worth of all its credit items.

ILLUSTRATIVE EXAMPLE

Find the cash balance of the following account on Dec. 13, 1911, at 6%.

Dr. JOSHUA LEVERING in account with J. H. TYLER Cr.

1911		Days	Int.	Amount	1911		Days	Int.	Amount
July 28	To Mdse., 3 mo.			$268 75	July 20	By Note, 2 mo.			$175 83
Aug. 12	" 30 da.			327 13	Sept. 2	" " 60 da.			207 27
Oct. 19	" Net			295 84	Oct. 28	" Cash			148 75

Dr. SOLUTION Cr.

1911		Days	Int.	Am'nt	1911		Days	Int.	Amount
July 28	To Mdse., 3 mo.	46	$2 06	$268 75	July 20	By Note, 2 mo.	84	$2 46	$175 33
Aug. 12	" 30 da.	93	5 07	327 13	Sept. 2	" " 60 da.	42	1 45	207 27
Oct. 19	" Net	55	2 71	295 84	Oct. 28	" Cash	46	1 14	148 75
Dec. 13	Bal. of Int.			4 79	Dec. 13	" Bal. of Int.		4 79	
					" "	" " " %			364 66
			$9 84	$896 51				$9 84	$896 51

EXPLANATION. Write in the days' column opposite each item, the number of days intervening between the due date of that item and the date of taking the cash balance (Dec. 13). Next write in the interest column, the interest on each item for the days specified in the days' column.

Then write the difference between the total debit interest and the total credit interest ($4.79) on the less side, thus balancing the interest columns; and as this interest balance shows that Levering owes Tyler $4.79 more interest than Tyler owes Levering, enter it as an additional item in the amount column on the debit side of Levering's account.

Lastly, write the difference between the total of the amount column on

the debit side and the total of the amount column on the credit side ($364.66), on the less side, as the required "cash balance" on Dec. 13, 1911.

RULE. 1. *Find the interest of each item of the account from its due date to the date of taking the cash balance.* 2. *Balance the interest columns.* 3. *Carry the interest balance to the main money column on the side opposite to that in which it is written in the interest column.* 4. *Balance the main money columns.*

NOTE 1. The interest of items which fall due *after* the date of adjusting the cash balance should be computed for the number of days between the date of adjustment and the due date of such items; and this interest should be written with *red ink* in the interest column on the same side as the item; but when the totals are taken, this red interest should be excluded from the total interest of the side on which it is written, and included with the total interest of the opposite side where it properly belongs.

NOTE 2. The face of an interest-bearing note should be regarded as the equivalent of so much cash paid on the day it commenced to draw interest; and the face of a non-interest-bearing note should be treated as the equivalent of so much cash *paid on its due date.*

EXAMPLES FOR PRACTISE

1. Find the cash balance of the following on Dec. 16, 1911, at 6 %.

Dr. EDMUND BERKELEY in account with J. B. T. THORNTON Cr.

1911		Days	Int.	Amount	1911		Days	Int.	Amount
July 18	To Balance	***	** **	$416 70	Aug. 20	By Cash	***	* **	$416 70
Sept. 24	" Mdse.	**	* **	286 45	Sept. 28	" "	**	* **	157 80
Oct. 2	" "	**	* **	341 60	Oct. 18	" "	**	* **	275 50
Nov. 20	" "	**	* **	298 40	Dec. 1	" "	**	**	198 20
Dec. 16	" Bal. of Int.		* **		Dec. 16	" Bal. of Int.		* **	
					" "	" " " %			*** **
			** **	**** **				** **	**** **

2. Find the cash balance of the following on Oct. 16, 1912, at 5 %.

Dr. Jos. J. Janney in account with N. Brawner Cr.

1912		Days	Int.	Amount	1912		Days	Int.	Amount
Mch. 16	To Mdse., 3 mo.	***	* **	$298 75	Apr. 7	By Note, 90 da.	**	* **	$178 25
May 28	" " 60 da.	**	* **	328 76	July 3	" " 2 mo.	**	* **	210 38
July 16	" " Net	**	* **	265 83	Aug. 8	" Cash	**	* **	158 16
Oct. 16	" Bal. of Int.			* **	Oct. 16	" Bal. of Int.		* **	*** **
					Oct. 16	" " " %			*** **
		**	**	*** **				** **	*** **

3. Find the cash balance on Nov. 18, 1911, at 6 %.

Dr. T. C. Will in account with W. P. Jordan Cr.

1911		Days	Int.	Amount	1911		Days	Int.	Amount
Mch. 21	To Mdse., 30 da.	***	** **	$385 92	Apr. 7	By Note, 30 da.	***	* **	$193 75
May 10	" " 3 mo.	***	* **	278 50	May 20	" " 3 mo.	**	* **	$216 25
July 28	" " Net	***	* **	328 65	July 7	" Cash	***	* **	265 18
Nov. 18	" Bal. of Int.			* **	Nov. 18	" Bal. of Int.		* **	
					Nov. 18	" " " %			*** **
		**	**	**** **				** **	**** **

4. Find the cash balance on Nov. 8, 1912, at 6 %.

Dr. Walter Watt in account with Neal Farr Cr.

1912		Days	Int.	Amount	1912		Days	Int.	Amount
July 13	To Mdse., Net	***	* **	$285 96	Aug. 20	By Cash	**	* **	$150 35
Sept. 23	" " 60 da.	**	**	346 75	Sept. 16	" Note, 30 da.	**	**	210 50
Oct. 3	" " 3 mo.	**	* **	417 30	Oct. 18	" " 4 mo.	***	* **	168 73
Nov. 8	" Bal. of Int.			* **	Nov. 8	" Bal. of Int.		* **	
					Nov. 8	" " " %			*** **
			* **	**** **				* **	**** **

5. Arrange the following items in the form of an account current, ruled as in the preceding examples, and find the cash balance on Sept. 5, 1912, at 5%: John King bought of J. Malone Gaulden, as follows: May 17, 1912, mdse. at 30 da., $378.25; June 28, 1912, mdse. at 4 mo., $296.52; July 10, 1912, mdse. net, $419.75; and King was credited on June 28, 1912, for his note of $198.63, at 90 da.; on July 5, 1912, for his note of $128.46 at 2 mo., and on July 20, 1912, for a cash payment of $293.75.

PARTNERSHIP

300. INTRODUCTION. (*a*) A *business partnership* is a combination of the resources of two or more persons for the establishment or the maintenance of a business enterprise. The amount of money, or property, or time, or the character of the duties to be assumed by each partner, or the manner of distributing the gains or losses, or any other necessary detail, is stipulated in an instrument of writing called *a partnership agreement.* Such a combination of persons is known as *a firm, a house,* or *a company;* and each person in the firm is called a *partner.* (*b*) The aggregate of what each partner contributes to the partnership is termed *his investment;* and the difference between his total investments and his total withdrawals is called *his net investment.* (*c*) The sum of what *all* the partners contribute is known as *the capital* of the firm; (*d*) and the difference between what all the partners have invested and what all the partners have withdrawn plus the net gain or minus the net loss since the last partnership adjustment, is called the *firm's net capital.*

(*e*) The *assets* or *resources* of a firm are its entire possessions, including obligations due to it from others; and (*f*) its *liabilities* are the obligations due by the firm to others. (*g*) If a firm's resources exceed its liabilities, it is said to be *solvent;* and the excess is known as the firm's *net solvency* or *net capital;* but (*h*) if a firm's liabilities exceed its resources, it is said to be *insolvent,* and the excess is known as the firm's *net insolvency* at that time.

(*i*) Due allowance being made for possible investments or withdrawals, if any, within the settlement-period, the *net gain* of a firm is the excess of its net capital at the end of that period over its net capital at the beginning of that period; (*j*) or the decrease of its net insolvency at the end of that period below its net insolvency at the beginning; (*k*) or the sum of its net capital at the *end of that* period and its net insolvency at the beginning.

(*l*) Due allowance being made for investments or withdrawals, if any, within the settlement-period, the *net loss* of a firm is the deficiency of its net capital at the end of that period below its net capital at the beginning of that period; (*m*) or the excess of its net insolvency at the end of that period above its net insolvency at the beginning; (*n*) or the sum of its net insolvency at the end of that period and its net capital at the beginning.

(*o*) It is thus seen that partnership adjustments involve consideration of *three terms:* first, the financial condition at the beginning; second, the financial condition at closing; third, the net gain or net loss during the interval. (*p*) These three terms have a fixed relation to each other, additive or subtractive, so that if any two of them are given, the third can be readily found. (*r*) Of these three terms, the *financial condition at the beginning* of the settlement-period, or that condition as modified by subsequent investments or withdrawals, if any, should uniformly be regarded as the natural standard of comparison, as follows:

301. To find the gain or loss of a firm. (*a*) Compare its financial condition (solvency or insolvency) at the end of a settlement-period with its financial condition at the beginning of that period, or with that condition as modified by subsequent investments or withdrawals, if any, *within* that settlement-period; and (*b*) *if its financial condition at the end of the period is found to be better than its condition at the beginning of the period, there has been a gain; if worse, a loss; and the amount of gain or loss will be the difference between these two conditions.*

NOTE 1. As the gain or loss is the difference between two financial conditions, and any difference added to the less term will equal the greater, so, if a gain or loss is given, and the financial condition at the close of the settlement-period is required, it will be the gain better, or the loss worse than the net financial condition at the beginning; or if the net financial condition at the beginning is required, it will be the gain worse, or the loss better than that at the end.

NOTE 2. The *difference between similar conditions*, that is, between one net capital and another net capital, or between one net insolvency and another net insolvency, is found by subtracting the less from the greater; but *the difference between opposite conditions*, that is, between a net capital and a net insolvency, or *vice versa*, is found by addition.

NOTE 3. A capital at the end of a period is better than at the beginning when it is greater, and it is worse when it is less; while an insolvency at the end of a period is better than at the beginning, when it is less, and it is worse when it is greater.

NOTE 4. A financial condition may be given *directly* as so much capital or insolvency; or it may be given *indirectly* by enumerating the different resources and liabilities, or by access to the books of account from which the several resources and liabilities may be obtained.

EXAMPLES FOR PRACTISE

Find the gain or loss:

1. Capital at beginning \$9285; capital at closing \$12345.
2. Capital at beginning \$15275.80; capital at closing \$12685.92.
3. Insolvency at beginning \$9685.75; insolvency at closing \$13472.65.
4. Insolvency at beginning \$18271.36; insolvency at closing \$14275.36.
5. Capital at beginning \$7286.50; insolvency at closing \$2615.25.
6. Insolvency at beginning \$4586.25; capital at closing \$6718.60.

Find the capital or insolvency at closing:

7. Capital at beginning \$12856.25; gain during interval \$4687.50.
8. Capital at beginning \$7268.42; loss during interval \$2178.65.
9. Insolvency at beginning \$5829.65; gain during interval \$6783.75.
10. Insolvency at beginning \$9685.30; gain during interval \$2785.90.
11. Capital at beginning \$3568.20; loss during interval \$6580.65.
12. Insolvency at beginning \$4689.75; loss during interval \$768.50.

Find the capital or insolvency at beginning:

13. Capital at closing \$7856.35; gain during interval \$2563.75.
14. Capital at closing \$3568.25; gain during interval \$6528.15.
15. Capital at closing \$9275.80; loss during interval \$1845.60.
16. Capital at closing \$1653.40; loss during interval \$8215.25.

17. Insolvency at closing \$3720.65; gain during interval \$1285.80.

18. Insolvency at closing \$675.80; gain during interval \$4718.10.

19. Insolvency at closing \$1925.25; loss during interval \$658.25.

20. Insolvency at closing \$2678.90; loss during interval \$5815.60.

21. A firm commenced business with the following resources: cash, \$2500; merchandise, \$12500; personal accounts due the firm, \$1846.25; bills receivable, \$956.75. Its liabilities at the beginning were: due to others for merchandise bought, \$3500; and on outstanding notes, \$1256.18. At closing they had resources as follows: merchandise as per inventory \$15265.70; cash, \$780; due the firm on personal accounts, \$4625.80; bills receivable, \$1475.50; and their liabilities were: due to others on personal accounts, \$1725.80; and on outstanding notes, \$865.70. What was the net gain or loss of the firm during the interval?

22. A and B formed a copartnership, the former investing $\frac{2}{3}$ of the capital and the latter $\frac{1}{3}$, the agreement between them requiring the gains or losses to be divided equally. At settlement the net gains were found to be \$6000, their resources at that time being merchandise on hand \$12500, cash \$3500, personal accounts \$4200, bills receivable \$1826; and their liabilities, outstanding notes against the firm \$4500, unsettled bills for merchandise bought \$2700. How much did each partner invest at the beginning?

23. A, B, and C engage in business as partners, with a capital of \$18000, A having invested $\frac{5}{12}$ of the capital, B $\frac{4}{12}$, and C $\frac{3}{12}$. At closing, the firm's books showed a net gain of \$6420, which according to agreement was to be shared equally. What was each partner's interest in the firm at settlement?

DIVISION OF GAINS OR LOSSES AMONG PARTNERS IN PROPORTION TO THEIR ORIGINAL INVESTMENTS

302. INTRODUCTION. Unless otherwise specified in the partnership agreement, the courts presume that the gains or losses are, by implication, to be divided *equally* among all the partners,

irrespective of the money they may have individually invested, that not being regarded as the only source of possible gains. What a partner may lack pecuniarily may be offset by his possession of greater skill in business, greater influence over patronage in the community, or by assuming greater responsibility of management, etc. In the examples under this article, it is assumed that the partners' investments were made at commencing business and were left unchanged by subsequent investments or withdrawals, and that inequalities among the partners in other respects than money, if any, have been provided for by special allowances (sometimes improperly called salaries) to be taken from the gross gains before general distribution of the net gain is made among all the partners in proportion to their respective pecuniary investments alone.

ILLUSTRATIVE EXAMPLE

A and B are partners, the former investing $6000 and the latter $8000, and agreed to share the gains or losses in proportion to their respective investments. If the net gain during the first year was $5600, what was the share of each?

SOLUTION

$6000 + $8000 = $14000, total capital

$$\frac{\$5600 \times 6000}{14000} = \$2400, \text{ A's share}$$

$$\frac{\$5600 \times 8000}{14000} = \$3200, \text{ B's share}$$

EXPLANATION. Applying 289, A's required share of the gain has the same relation to the total gain ($5600) as A's investment ($6000) which produced his share of the gain has to the total investment ($6000 + $8000 = $14000) which produced the total gain ($5600). As A's investment ($6000) is $\frac{6000}{14000}$, or $\frac{3}{7}$ of the total investment, so A's share of the gain should be $\frac{3}{7}$ of the total gain ($5600), and $\frac{3}{7}$ of $5600 = $2400.

Proceed in a similar manner to find B's share of the firm's gain. In verification, add A's share of gain to B's, to find if the sum equals the total gain.

RULE. *Write one partner's investment and the total gain as a compound numerator, and the firm's capital as the denominator. Extend the resulting fraction if necessary, to express dollars and cents.*

NOTE. To find each partner's investment, if his share of the gain is given, write one partner's gain and the firm's capital as a compound numerator, and the total gain as the denominator.

EXAMPLES FOR PRACTISE

1. A and B united in conducting a grocery business, A investing \$6480 and B \$5670, and agreed to divide the gains or losses between them in proportion to investment. If the net gains were \$3600, how much should each partner have received?

2. A, B, and C have formed a partnership, A investing \$8000, B \$10000 and C \$12000. The net gain during the first year was \$9000 which was divided among them in proportion to their respective investments. How much of the gain should each partner have received?

3. A and B engaged in business with a joint capital of \$23275; and when the gains were adjusted, A received \$2400 and B \$3600. How much did each invest? [Note]

4. A, B, and C are partners, A investing \$12000, B \$15000 and C \$18000; and agreed to share gains or losses in proportion to their investments. When the books were adjusted, it was found that the total resources of the firm were \$78216 and their total liabilities \$24816. What was each partner's share of the gain?

DIVISION OF GAINS OR LOSSES AMONG PARTNERS IN PROPORTION TO THEIR AVERAGE INVESTMENTS

303. INTRODUCTION. (*a*) To divide the gains or losses among partners in proportion to their original investments as in **302** is proper only when those investments remain unchanged during the entire settlement-period. If a change is made in the original investment of one or more partners by subsequent investments or withdrawals, some modification of **302** will be necessary to effect an equitable distribution of gains or losses. If, for instance, a partner were to make an investment subsequent to his original investment and thus increase his contribution to the gain-producing capital of the firm, it is evident that he should receive a corresponding inerease of the firm's gain in proportion to his increased investment, and also that it would be as improper to proportion his gains for the entire year to his subsequent greater investment when his investment for the preceding portion of the year was less, as it would have been to proportion his gains to his original less investment when his investment for a subsequent portion of the year was greater.

(*b*) This modification of any partner's share of the gain in

proportion to his modification of his original contribution to the profit producing capital of the firm is equitably effected by reducing the several investments of each partner for dissimilar periods of a year to equivalent investments for similar periods of a lower unit of time than a year, as *for one month*, or *for one day*. Hence,

(*c*) If *all* the partners' investments were not made for the entire settlement period, they should be reduced, by average, to equivalent investments for the longest obtainable *equal period of time*, as *for one month* or *for one day;* and the gains or losses should be divided among the partners in proportion to their equivalent investments for that common lower unit of time.

ILLUSTRATIVE EXAMPLE

A and B formed a copartnership, and agreed to share gains or losses in proportion to investment. A invested $9000 on Jan. 1, 1910; withdrew $2000 on May 1, 1910; and invested $5000 on Aug. 1, 1910. B invested $8000 on Jan. 1, 1910; invested $4000 on Apr. 1, 1910; and withdrew $3000 on Sept. 1, 1910. If the net gains during the year were $6000, what was each partner's share?

SOLUTION

$9000 invested for 1 yr.	= 12 times	$9000, or	$108000 for 1 mo.
5000 invested for 5 mo.	= 5 times	5000, or	25000 for 1 mo.
A's total investment,			$133000 for 1 mo.
$2000 withdrawn for 8 mo.	= 8 times	$2000, or	16000 for 1 mo.
A's net investment,			$117000 for 1 mo.
$8000 invested for 1 yr.	= 12 times	$8000, or	$ 96000 for 1 mo.
4000 invested for 9 mo.	= 9 times	4000, or	36000 for 1 mo.
B's total investment,			$132000 for 1 mo.
$3000 withdrawn for 4 mo.	= 4 times	$3000, or	12000 for 1 mo.
B's net investment,			$120000 for 1 mo.

$117000 + $120000 = $237000, firm's net capital for 1 mo.

$$\$6000 \times \frac{\overset{39}{\not 117\not 000}}{\underset{79}{\not 237\not 000}} = \frac{\$234000}{79} = \$2962.03 -, \text{ A's share of the gain}$$

$$\$6000 \times \frac{\overset{40}{\not 120\not 000}}{\underset{79}{\not 237\not 000}} = \frac{\$240000}{79} = \$3037.97 +, \text{ B's share of the gain}$$

EXPLANATION. As the investments of the partners were not uniform throughout the year, recourse must be had to a shorter common unit of time to which the net investments of the several partners can be reduced, and in proportion to which the net gain of the firm can be divided. As the original investments and subsequent modifications were *all* made on the *first* day of some calendar month, the investments and withdrawals of each partner are capable of being reduced by average to equivalent investments and withdrawals for the similar period of one month, that is, in A's case to a net investment of $117000 for 1 mo., in B's case to a net investment of $120000 for 1 mo., and with reference to the firm to an equivalent net capital of $117000 + $120000, or $237000 for 1 mo. Hence, divide the net gain of the firm ($6000) between A and B in proportion to these equivalent investments per month, as in **302** for investments per year.

RULE. 1. *If all the investments and withdrawals of all the partners have been made on the first day of some calendar month, divide the gain or loss of the firm among the partners as in* **302**, *but in proportion to each partner's average net investment and to the firm's average net capital for one month.*

2. *If one or more investments or withdrawals of one or more partners have been made on different dates of a calendar month, divide the gain or loss of the firm among the partners as in* **302**, *but in proportion to each partner's average net investment and to the firm's average net capital for one day.*

NOTE 1. To find a partner's average net investment for one month or for one day, multiply each investment or withdrawal by the number of months or days from its date to the end of the settlement-period, and subtract the sum of the resulting withdrawal products from the sum of the resulting investment products.

NOTE 2. If an allowance is made to a partner for special labor, or risk, or skill, or influence, or responsibility, it should be considered as an extra share of the gain allotted to that partner for that special investment, in addition to his proportionate share of the remainder of the gain which is divided among *all* the partners in proportion to their respective investments in cash or its equivalent. This special share of the gain should therefore be deducted from the given net gain (except in the *unusual* case of the special allowance having been previously drawn and entered in the *expense account*), before such gain is proportioned among all the partners; and this special allowance to a partner should be added to his *proportional share* to find that partner's actual share of the total gain.

NOTE 3. In applying Note 1, much labor will be saved by obtaining the equivalent net investments in *hundreds* of dollars for one month or for one day, as shown and explained in the abridged solution of **293**.

EXAMPLES FOR PRACTISE

1. A and B form a partnership on Jan. 1, 1910, A investing $9000 and B $6000. On June 1, A invested $3000 additional, and on Sept. 1, he withdrew $2000. On May 1, B withdrew $2000, and on Aug. 1, he invested $5000. On Jan. 1, 1911, the net gains were $4000, which were divided between the partners in proportion to their respective investments. What was each partner's share?

2. A, B, and C engaged in partnership, and agreed to share gains or losses in proportion to average investment per month. A invested $12500 on Jan. 1, 1911; and withdrew $2500 on June 1, 1911. B invested $9000 on Jan. 1, 1911; invested $2000 on Apr. 1, 1911; and withdrew $3000 on Aug. 1, 1911. C invested $15000 on Jan. 1, 1911, and permitted it to remain unchanged throughout the year. If the net gains for the year were $7000, what was each partner's share? What was each partner's interest in the firm after the adjustment?

3. A and B engage in business as partners on Jan. 1, 1912, A investing $15000 and B $18000; and agreed to share gains or losses in proportion to average investment per month. A made a further investment of $5000 on Aug. 1, 1912; and B withdrew $2000 on Mch. 1, 1912. On May 1, 1912, the firm took in C as a third partner who then invested $8000, and on July 1, 1912, invested an additional $6000. On Jan. 1, 1913, the capital of the firm was $65000. What was each partner's interest in the firm on the latter date?

4. A and B are partners, having agreed to share gains or losses in proportion to average investment per day. A invested $18000 on Jan. 1, 1911; withdrew $2000 on May 16, 1911; and withdrew $1000 on Sept. 12, 1911. B invested $12000 on Jan. 1, 1911; and invested $9000 on July 23, 1911. If the net gains during the year were $6000, what was each partner's share? What was each partner's interest in the firm on Jan. 1, 1912?

5. A and B united in partnership and agreed to share gains or losses in proportion to average investment per day. A invested $8000 on Jan. 1, 1911; invested $4000 on May 19, 1911; and withdrew $2000 on Oct. 8, 1911. B invested $12000 on Jan. 1,

1911; withdrew $2000 on Apr. 23, 1911; and withdrew $1000 on Nov. 2, 1911. What was each partner's interest in the firm on Jan. 1, 1912, if its capital at that time was $25000?

6. A and B engaged in business as partners and agreed to share gains or losses in proportion to average investment per day. A invested $12000 on Jan. 1, 1911; invested $7000 on June 20, 1911; and withdrew $4000 on Oct. 12, 1911. B invested $15000 on Jan. 1, 1911; withdrew $3000 on Apr. 29, 1911; and withdrew $3000 on Aug. 24, 1911. On Jan. 1, 1912, the books were adjusted, when it was found that the total resources of the firm were $40000, and their total liabilities $8000. What was each partner's interest in the firm on Jan. 1, 1912?

WHEN EACH PARTNER'S PROPORTIONATE PART OF THE GAIN OR LOSS IS ARBITRARILY FIXED IN THE PARTNERSHIP AGREEMENT, AND HIS INVESTMENT IS REQUIRED TO BE IN PROPORTION TO HIS STIPULATED SHARE OF THE GAINS OR LOSSES

304. INTRODUCTION. (*a*) A third method of distributing gains or losses among partners, and in many respects the most equitable, is to assign to each partner a fixed part of the gain, irrespective of his pecuniary investment. There are many possible forms of investment besides that of money. Any contribution of a partner to a business which will add materially to its prosperity, such as superior business talent, greater social influence, excess of time or labor, disproportionate responsibility in management, etc., are properly investments, and frequently the most important elements in acquiring gains. If one partner invests twice as much *money* as another partner, that of itself should not be a sufficient reason for giving him twice as much of the gain, except he also invest twice as much time, twice as much responsibility of management, and, generally, twice as much of all the contributory elements in the acquisition of the firm's gains.

(*b*) A mere difference in pecuniary investment can readily be adjusted between two partners if they are equal in *all other* respects, by giving to each partner one-half the firm's net gains, and requiring the partner who invested less than one-half the capital to pay interest upon his deficiency at the market rate per annum, to the other partner who must have invested correspondingly more than one-half the capital, since both together must have contributed two-halves or the entire capital.

(*c*) If, however, the partners are unequal in other respects

than the money which they have invested, that is, if A of the firm of A and B, is, in all those qualities which put together constitute a successful business man, twice as capable as B, but financially is deficient, then A should equitably receive twice as much of the firm's net gains as B, or two-thirds to B's one-third; and as the proper measure of the use of money is the interest thereon at the current rate per annum for the time it is used, A should pay to B interest on any deficiency of his investment below two-thirds of the entire capital. Conversely B should receive one-third of the firm's net gain and receive interest on the excess of his financial investment above one-third of the entire capital; for B's excess of investment above one-third of the firm's capital must exactly balance A's deficiency below two-thirds of the firm's capital, as the firm's capital, or what both have invested, must equal three-thirds of itself.

(*d*) The allowance of interest by one partner who is deficient in his required average investment to another partner who is in excess, is most conveniently accomplished by crediting each partner with the interest on his net average investment; and debiting the firm's interest account, or its practical equivalent of *subtracting the total interest on the several partners' net average investments from the firm's net gains; and proportioning the remainder of the net gain among the partners in accordance with the partnership agreement.*

ILLUSTRATIVE EXAMPLE

A and B engaged in business as partners on Jan. 1, 1911, the agreement between them specifying that A, by reason of his superior business ability, should receive $\frac{4}{7}$ of the net gain of the business, and B the remaining $\frac{3}{7}$; and that each should be accountable for investing as many sevenths of the net capital as he was to receive sevenths of the net gain, and be required to pay interest at 6 % upon as much of his required number of sevenths of the net capital as he failed to invest, and be entitled to receive interest at the same rate per annum on his excess if he invested more than his stipulated number of sevenths. At the end of the year the net gain was found to be $4291. A invested $12000 on Jan. 1, 1911; withdrew $4000 on Apr. 16, 1911; invested $7000 on Aug. 25, 1911; and withdrew $3000 on Oct. 8, 1911. B invested $15000 on Jan. 1, 1911; withdrew $2000 on Mch. 17, 1911; withdrew $1500 on July 5, 1911; and invested $5000 on Sept. 3, 1911. What was each partner's interest in the firm at the end of the year?

SOLUTION

Int. of \$12000 from Jan. 1 to Dec. 31, inclusive (1 yr.)	\$720.
Int. of \$7000 from Aug. 25 to Dec. 31, inclusive (128 da.)	149.33+
Interest on A's total investments	\$869.33
Int. of \$4000 from Apr. 16 to Dec. 31, inclusive (259 da.)	\$172.67−
Int. of \$3000 from Oct. 8 to Dec. 31, inclusive (84 da.)	42.
Interest on A's total withdrawals	\$214.67

\$869.33 − \$214.67 = \$654.66, int. on A's net average investment.

Int. of \$15000 from Jan. 1 to Dec. 31, inclusive (1 yr.)	\$900.
Int. of \$5000 from Sept. 3 to Dec. 31, inclusive (119 da.)	99.17−
Interest on B's total investments	\$999.17
Int. of \$2000 from Mch. 17 to Dec. 31, inclusive (289 da.)	\$ 96.33+
Int. of \$1500 from July 5 to Dec. 31, inclusive (179 da.)	44.75
Interest on B's total withdrawals	\$141.08

\$999.17 − \$141.08 = \$858.09, int. on B's net average investment.

\$654.66, obtained int. on A's net average investment.
858.09, obtained int. on B's net average investment.
\$1512.75, interest on firm's net average capital.

Firm's net gain (\$4291) − interest on firm's net average capital (\$1512.75) = firm's gain to be proportioned among the partners as per agreement (\$2778.25). [See Introduction, *d.*]

$\frac{4}{7}$ of \$2778.25 = \$1587.57 (+ A's int. on net av. inv.) = A's gain
$\frac{3}{7}$ of \$2778.25 = \$1190.68 (+ B's int. on net av. inv.) = B's gain

(\$12000 + \$7000) − (\$4000 + \$3000), or A's net investment	= \$12000.
A's stipulated proportion of the firm's net gain	= 1587.57
A's interest on net average investment	= 654.66
A's interest in the firm at end of year	= \$14242.23
(\$15000 + \$5000) − (\$2000 + \$1500), or B's net investment	= \$16500.
B's stipulated proportion of the firm's net gain	= 1190.68
B's interest on net average investment	= 858.09
B's interest in the firm at end of year	= \$18548.77

EXPLANATION. Compute the interest upon each of A's investments and withdrawals from its date to the end of the settlement period (Dec. 31, 1911, inclusive), and subtract the total interest on his withdrawals from the total interest on his investments to find the total interest on A's net average investment ($654.66), with which A should be credited.

Similarly find the interest on B's net average investment, obtaining $858.09, with which B should be credited.

Deduct A's and B's special allowance for interest ($654.66 + $858.09 = $1512.75) from the net gain of the firm ($4291) to find the remainder of the net gain ($2778.25) which is to be divided between the partners as per agreement, $\frac{4}{7}$ of $2778.25 ($1587.57) being credited to A, and $\frac{3}{7}$ of $2778.25 ($1190.68) being credited to B.

The sum of either partner's net investment, the interest on his net average investment, and his stipulated proportion of the net gain after the net gain had been diminished by the interest on the firm's net average capital, will be that partner's interest in the firm at the time of adjustment.

NOTE 1. If the interest on any partner's total investments is greater than the interest on his total withdrawals, the difference between these totals will be the interest on that partner's net average investment for which he should be credited; but if less, the difference between these totals will be the interest on that partner's net average insolvency for which he should be debited.

NOTE 2. If the average financial condition of each of the partners for the settlement period, without an exception, is solvent, the sum of the interest on their respective net average investments will be the interest on the firm's net average capital for the same period; and if insolvent, the sum of the interest on their respective net average insolvencies will be the interest on the firm's net average insolvency. If, however, the average financial condition of one or more of the partners is solvent and that of others insolvent, the difference between the sum of all the solvent partners' interest on their net average investments and the sum of all the insolvent partners' interest on their net average insolvencies, will be the interest on the firm's net average capital if the sum of the interest on the net average solvencies is the greater, or it will be the interest on the firm's net average insolvency if the sum of the interest on the net average insolvencies is the greater.

NOTE 3. Subtract the interest on the firm's net average capital from the net gain, or add it to the net loss, to find the gain or loss which is to be apportioned among the partners in accordance with the partnership agreement, for if the firm's "interest account" had been debited for the interest on each solvent partner's net average investment for which that partner had been credited and that "interest account" had been credited with the interest on each insolvent partner's net average insolvency for which that partner had been debited, and the firm's "interest account" had been balanced, and the obtained balance had been included in finding the firm's gain or loss, it would have correspondingly diminished the gain or increased the loss.

NOTE 4. Add the interest on the firm's net average insolvency to the firm's net gain, or subtract it from the firm's net loss, to find the gain or loss which is to be apportioned among its several partners; for this interest expresses the net debit interest of the several partners for which the firm's "interest account" should exhibit a corresponding "credit balance." If this "credit balance" had been included in finding the firm's gain or loss, it would have correspondingly increased the firm's net gain or diminished the firm's net loss.

EXAMPLES FOR PRACTISE

1. A and B engaged in business as partners on Jan. 1, 1911, A agreeing to invest $\frac{3}{5}$ of the capital and to receive $\frac{3}{5}$ of the net gain of the firm, and B to invest $\frac{2}{5}$ of the capital and to receive $\frac{2}{5}$ of the net gain. It was further agreed that interest at 8 % annum would be allowed to either partner for investing more than his stipulated number of fifths of the capital, and to be charged against any partner for any deficiency of stipulated investment. A invested $9000 on Jan. 1, 1911; invested $3000 on Apr. 12, 1911; withdrew $1500 on June 9, 1911; invested $2500 on Aug. 13, 1911; and withdrew $900 on Nov. 8, 1911. B's net investment was $18000, and the interest on his net average investment was $758.90. If the net gain of the firm during the year was $7500, what was each partner's interest in the business on Jan. 1, 1912 (that is, on Dec. 31, 1911, inclusive)?

2. A and B formed a partnership on Jan. 1, 1912, and agreed to invest equally and to share equally in gains or losses, and interest at 9 % per annum was to be allowed to either partner for any excess over stipulated investment, or to be charged against either partner for any deficiency of investment. The net gain of the firm on Jan. 1, 1913, was found to be $7600. A invested $15200 on Jan. 1, 1912; withdrew $2600 on May 23, 1912; and invested $3800 on Sept. 18, 1912. B invested $18000 on Jan. 1, 1912; invested $1600 on Mch. 17, 1912; withdrew $2000 on June 8, 1912; and invested $3000 on July 16, 1912. What was each partner's interest in the firm on Jan. 1, 1913?

3. A, B and C entered into a copartnership agreement to commence business on Jan. 1, 1911, and agreed to invest equally and to share equally in gains or losses, and to allow interest at 6 % per annum for any excess over stipulated investment, and to

charge interest at the same rate for any deficiency. A invested \$16000 on Jan. 1, 1911; invested \$4000 on May 26, 1911; and withdrew \$2400 on July 12, 1911. B invested \$18600 on Jan. 1, 1911; and withdrew \$1500 on Oct. 19, 1911. C invested \$14000 on Jan. 1, 1911; withdrew \$2000 on May 4, 1911; and invested \$5000 on Aug. 26, 1911. If the net gain of the firm at the end of the year was \$12600, what was each partner's interest in the business at that time?

4. A, B and C enter into partnership to engage in business, A agreeing to invest $\frac{4}{9}$ of the capital, B to invest $\frac{3}{9}$, and C $\frac{2}{9}$, and to receive the same proportion of the net gain of the business. It was also agreed that each partner was to be credited with interest at 10% per annum for any excess, and to be debited with interest at the same rate for any deficiency, of stipulated investment. A invested \$9000 on Jan. 1, 1912; and withdrew \$500 on June 25, 1912. B invested \$14000 on Jan. 1, 1912; and invested \$3000 on May 14, 1912. C invested \$7500 on Jan. 1, 1912; invested \$3500 on July 2, 1912; and withdrew \$2000 on Sept. 20, 1912. On Jan. 1, 1913, the books were adjusted to the end of the preceding day, and showed a net gain of \$8730. What was each partner's interest in the firm at settlement?

5. A and B entered into partnership, A agreeing to invest $\frac{6}{11}$ of the capital and to receive $\frac{6}{11}$ of the total gain, and B to invest $\frac{5}{11}$ of the capital and to receive $\frac{5}{11}$ of the total gain, and interest at a fixed rate per annum was to be credited to either partner for any excess, or debited against him for any deficiency, of required investment. At settlement it was discovered that the net capital of the firm was \$26200; that A's total interest on investments was \$518.65, his total interest on withdrawals was \$168.15, and his net investment \$15300; and that B's total interest on investments was \$927.83; his total interest on withdrawals \$216.48; and his net investment \$8700. Find each partner's interest in the firm at settlement.

APPENDIX

USEFUL COMMERCIAL CALCULATIONS

THE ALIQUOT METHOD OF COMPUTATION

305. An aliquot of a number is any number which will exactly divide it.

NOTE. An *aliquot part* of a given number is the *fractional part* which expresses the relation of the aliquot to that number. Thus, 8 is an aliquot of 32, and $\frac{1}{4}$ is the aliquot *part* which 8 is of 32. The following are aliquots of one dollar (100 cents):

$50¢ = \$\frac{1}{2}$	$20¢ = \$\frac{1}{5}$	$10¢ = \$\frac{1}{10}$	$5¢ = \$\frac{1}{20}$
$33\frac{1}{3}¢ = \$\frac{1}{3}$	$16\frac{2}{3}¢ = \$\frac{1}{6}$	$8\frac{1}{3}¢ = \$\frac{1}{12}$	$2\frac{1}{2}¢ = \$\frac{1}{40}$
$25¢ = \$\frac{1}{4}$	$12\frac{1}{2}¢ = \$\frac{1}{8}$	$6\frac{1}{4}¢ = \$\frac{1}{16}$	$2¢ = \$\frac{1}{50}$

ILLUSTRATIVE EXAMPLE

306. What is the cost of 52 yd. muslin at $12\frac{1}{2}¢$ per yard?

SOLUTION. $12\frac{1}{2}¢ = \$\frac{1}{8}$. Therefore at $\$\frac{1}{8}$ per yd., 8 yd. will cost \$1, and 52 yd. will cost as many times \$1 as 8 yd. are contained times in 52 yd., or $6\frac{1}{2}$ times \$1 = \$6.50.

RULE. *Divide the given quantity by the quantity which \$1 can buy.*

EXAMPLES FOR PRACTISE

Find the cost of

1. 512 yd. at $12\frac{1}{2}¢$ per yd.
2. 748 lb. at $25¢$ per lb.
3. 465 gal. at $20¢$ per gal.
4. 236 bu. at $50¢$ per bu.
5. 726 lb. at $16\frac{2}{3}¢$ per lb.
6. 417 yd. at $33\frac{1}{3}¢$ per yd.
7. 292 gal. at $6\frac{1}{4}¢$ per gal.
8. 610 lb. at $5¢$ per lb.
9. 583 yd. at $25¢$ per yd.
10. 329 lb. at $12\frac{1}{2}¢$ per lb.
11. 837 gal. at $50¢$ per gal.
12. 913 doz. at $20¢$ per doz.

ILLUSTRATIVE EXAMPLE

307. At 25¢ per pound, how many pounds of cheese can be bought for $9?

SOLUTION. 25¢ = $\frac{1}{4}$. Therefore at $\frac{1}{4}$ per lb., $1 can buy 4 lb., and 9 times $1, or $9, can buy 9 times 4 lb., or 36 lb.

RULE. *Multiply the quantity which $1 can purchase by the number of dollars to be expended, expressing cents in the total cost as a fraction of a dollar.*

EXAMPLES FOR PRACTISE

How much can be bought for

1. $375 at 20¢ per yd.?
2. $291 at $16\frac{2}{3}$¢ per gal.?
3. $148 at 25¢ per lb.?
4. $86 at 50¢ per bu.?
5. $61.50 at 25¢ per gal.?
6. $41.25 at $12\frac{1}{2}$¢ per yd.?
7. $278.75 at $33\frac{1}{3}$¢ per lb.?
8. $109.20 at 45¢ per doz.?

308. A price which is not itself an aliquot of $1 may frequently be resolved into two or more components which are. Thus,

$7\frac{1}{2}$¢ = 5¢ + $2\frac{1}{2}$¢, or $\$\frac{1}{20}$ + $\frac{1}{2}$ of $\$\frac{1}{20}$.
15¢ = 10¢ + 5¢, or $\$\frac{1}{10}$ + $\frac{1}{2}$ of $\$\frac{1}{10}$.
$18\frac{3}{4}$¢ = $12\frac{1}{2}$¢ + $6\frac{1}{4}$¢, or $\$\frac{1}{8}$ + $\frac{1}{2}$ of $\$\frac{1}{8}$.
$22\frac{1}{2}$¢ = 20¢ + $2\frac{1}{2}$¢, or $\$\frac{1}{5}$ + $\frac{1}{8}$ of $\$\frac{1}{8}$.
$27\frac{1}{2}$¢ = 25¢ + $2\frac{1}{2}$¢, or $\$\frac{1}{4}$ + $\frac{1}{10}$ of $\$\frac{1}{4}$.
$37\frac{1}{2}$¢ = 25¢ + $12\frac{1}{2}$¢, or $\$\frac{1}{4}$ + $\frac{1}{2}$ of $\$\frac{1}{4}$.
55¢ = 50¢ + 5¢, or $\$\frac{1}{2}$ + $\frac{1}{10}$ of $\$\frac{1}{2}$.
$62\frac{1}{2}$¢ = 50¢ + $12\frac{1}{2}$¢, or $\$\frac{1}{2}$ + $\frac{1}{4}$ of $\$\frac{1}{2}$.
75¢ = 50¢ + 25¢ or $\$\frac{1}{2}$ + $\frac{1}{2}$ of $\$\frac{1}{2}$.

ILLUSTRATIVE EXAMPLE

309. Find the cost of 764 yd. cloth at $37\frac{1}{2}$¢ per yard.

SOLUTION

At 25¢ per yd., 764 yd. will cost $191 ($\frac{1}{4}$ of $764).
" $12\frac{1}{2}$¢ " " " " " " 95.50 ($\frac{1}{2}$ of $191).
" $37\frac{1}{2}$¢ " " " " " " $286.50

EXAMPLES FOR PRACTISE

Find the cost of

1. 618 yd. at $18\frac{3}{4}¢$ per yd.
2. 428 lb. at $15¢$ per lb.
3. 316 bu. at $62\frac{1}{2}¢$ per bu.
4. 520 gal. at $7\frac{1}{2}¢$ per gal.
5. 640 yd. at $55¢$ per yd.
6. 732 gal. at $22\frac{1}{2}¢$ per gal.
7. 684 lb. at $\$1.37\frac{1}{2}$ per lb.
8. 562 bu. at $\$1.15$ per bu.

NOTE 1. The above process can be advantageously applied to common fractions of such denominators as are usually employed in business.

Multiply (**9**) \$216 by $\frac{5}{16}$; (**10**) \$756 by $\frac{9}{32}$.

SOLUTION

(9)		(10)	
\$216 = 16 sixteenths		\$756 = 32 thirty-seconds	
$\frac{1}{4}$ of 216 = 54	= 4 "	$\frac{1}{4}$ of 756 = 189	= 8 " "
$\frac{1}{4}$ of 54 = $13\frac{1}{2}$	= 1 "	$\frac{1}{8}$ of 189 = $23\frac{5}{8}$	= 1 " "
\$67½ or \$67.50 = $\frac{5}{16}$.		\$212⅝ or \$212.625 = $\frac{9}{32}$.	

EXAMPLES FOR PRACTISE

Multiply		Multiply		Multiply	
11. \$648	by $\frac{5}{8}$	**14.** 824 lb.	by $\frac{9}{16}$	**17.** 372 gal.	by $\frac{9}{64}$
12. \$432	by $\frac{3}{16}$	**15.** 268 yd.	by $\frac{17}{32}$	**18.** 716 yd.	by $\frac{19}{32}$
13. \$596	by $\frac{5}{32}$	**16.** 562 bu.	by $\frac{11}{16}$	**19.** 384 bu.	by $1\frac{5}{8}$

310. A price which is not itself an aliquot of \$1, may lack an aliquot of being \$1. Thus,

$99¢ = \$1 - \frac{1}{100}$ of \$1.	$95¢ = \$1 - \frac{1}{20}$ of \$1
$98¢ = \$1 - \frac{1}{50}$ of \$1	$87\frac{1}{2}¢ = \$1 - \frac{1}{8}$ of \$1
$97\frac{1}{2}¢ = \$1 - \frac{1}{40}$ of \$1	$75¢ = \$1 - \frac{1}{4}$ of \$1

ILLUSTRATIVE EXAMPLE

Find the cost of 136 yd. cloth at $75¢$ per yd.

SOLUTION

136 yd. at \$1 per yd. = \$136.
" " " $\$\frac{1}{4}$ " " = 34 ($\frac{1}{4}$ of \$136).
" " " $\$\frac{3}{4}$ " " = \$102 (\$136 − \$34).

EXAMPLES FOR PRACTISE

Find the cost of

1. 260 yd. @ 95¢	**4.** 473 gal. @ 99¢	**7.** 152 bu. @ \$1.95
2. 512 lb. @ 87½¢	**5.** 390 bu. @ 98¢	**8.** 284 bbl. @ \$6.90
3. 432 gal. @ 75¢	**6.** 256 bbl. @ \$5.75	**9.** 520 bbl. @ \$7.80

NOTE 1. The above process can also be used when the multiplier is a common fraction and its numerator lacks an aliquot of being equal to its denominator.

Multiply (**10**) 1264 by $\frac{7}{8}$; (**11**) 928 by $\frac{3}{4}$.

SOLUTION

(**10**)		(**11**)	
1264	= 8 eighths	928	= 4 fourths
$\frac{1}{8}$ of 1264	= 158 = 1 "	$\frac{1}{4}$ of 928	= 232 = 1 "
(1264 − 158)	= 1106 = $\frac{7}{8}$	(928 − 232)	= 696 = $\frac{3}{4}$

EXAMPLES FOR PRACTISE

Multiply	Multiply	Multiply
12. 519 by $\frac{2}{3}$	**15.** 453 by $\frac{4}{5}$	**18.** 293 by $4\frac{7}{8}$
13. 673 by $\frac{3}{4}$	**16.** 369 by $\frac{5}{6}$	**19.** 467 by $2\frac{3}{4}$
14. 527 by $\frac{7}{8}$	**17.** 258 by $\frac{9}{10}$	**20.** 625 by $3\frac{4}{5}$

NOTE 2. The above process will also be found useful when a multiplier lacks 12 or less of being 100, 1000, 10000, etc.

21. Multiply 386 by 997.

SOLUTION

386 × 1000 = 386000
386 × 3 = 1158
386 × 997 = 384842

EXPLANATION. For convenience, first multiply by 1000, as explained in **30**, obtaining a result which is 3 times 386 too great. Hence, deduct 3 times 386 from the result, obtaining 384842 as the correct product.

EXAMPLES FOR PRACTISE

Multiply	Multiply	Multiply
22. 875 by 97	**25.** 52 by 995	**28.** 712 by 9998
23. 426 by 99	**26.** 628 by 994	**29.** 256 by 980
24. 642 by 96	**27.** 329 by 993	**30.** 7384 by 9200

TO FIND THE COST OF GRAIN WHEN THE PRICE IS PER BUSHEL AND THE QUANTITY OF GRAIN IS EXPRESSED IN POUNDS

ILLUSTRATIVE EXAMPLE

311. Find the cost of 5687 lb. oats at 49¢ per bushel of 32 lbs.

SOLUTION

5687
$.49
51183
22748
8)2786.63
4)348.329 −
$87.082

EXPLANATION. At 49¢ per pound, the cost would have been 5687 times 49¢, or $2786.63; therefore at 49¢ per bushel of 32 pounds, this obtained cost must be 32 times the correct cost. Hence, divide $2786.63 by 32, or by the factors of 32 (8 and 4) to find the correct cost.

RULE. *Multiply the weight in pounds by the price per bushel, and divide the product by the weight in pounds of 1 bushel of the given kind of grain.*

EXAMPLES FOR PRACTISE

Find the cost of

1. 15207 lb. wheat at $1.09 per bu. of 60 lb. (10 × 6).
2. 9873 " corn " 68¢ " " " 56 " (8 × 7).
3. 5928 " beans " $2.10 " " " 60 "
4. 1753 " timothy seed at $2.35 per bu. of 45 lb. (9 × 5).
5. 2874 " buckwheat at $1.25 per bu. of 48 lb. (8 × 6).

NOTE. The above process can be advantageously employed in finding the cost of *any* merchandise when the quantity is expressed in units of one denomination and the price is per unit of a different denomination, thus avoiding an awkward intermediate fraction.

6. 5368 lb. coal at $6.75 per ton of 2000 lb. [39].
7. 18249 " " " $7.25 " " " " "
8. 27235 " " " $6.50 " " " " "
9. 8739 " " " $7.10 " " " 2240 " (10 × 8 × 7 × 4).
10. 14296 " " " $7.50 " " " 2240 "
11. 3876 " ear corn at $3.25 per bbl. of 350 lb. (10 × 7 × 5).
12. 9285 " freight at 45¢ per cwt. [38].

13. 25462 ft. lumber at \$18.50 per M.

14. 12348 lb. dressed beef at \$10.15 per 100 lb.

15. 6285 lb. flour at \$5.20 per bbl. of 196 lb. ($7 \times 7 \times 4$).

ADDITION BY GROUPING FIGURES

312. Skill in addition is an invaluable accomplishment which should be acquired by every one who wishes to follow a business career. No method of addition has so many advantages and so few disadvantages as that of adding one column at a time and, while doing so, training the eye to take within the range of its vision as many successive figures in the column as the mind may be capable of simultaneously adding. At first, the learner should attempt to group only those figures which are within the power of his *present capacity*, and let actual experience demonstrate the limitations of his individual ability. Efficiency in grouping will increase by exercise. 10 is the usual maximum grouping recommended for beginners.

ILLUSTRATIVE EXAMPLE

4 } 9	7 }	2 }
5 }	2 } 10	3 } 7
2 }	1 }	2 }
3 } 9	3 }	4 }
4 }	5 } 9	6 } 10
9	1 }	2 }
2 }	8	7 } 10
6 } 9	3 }	1 }
1 }	5 } 10	3 }
4 }	2 }	6 } 9
1 } 10	6 }	2 }
5 }	2 } 9	3 } 6
8 } 10	1 }	1 }
2 }	4 }	9 }
6 }	3 }	1 } 10
1 } 10	2 } 10	2 }
3 }	1 }	3 } 9
1 }	6 }	4 }
4 } 8	3 } 9	5 }
3 }	9	3 } 8
82	0	9
	8	6

EXPLANATION. Successive figures in the several columns which admit of grouping into convenient combinations are here connected by a brace to enable the learner the better to comprehend the manner in which figures may be thus massed. The higher figures as 9's or 8's, which have no correspondingly low contiguous figures, as 1's or 2's, that can be combined with them, should be added singly.

Thus, add the groupings of the units' column (8, 17, 27, 33, 42, 52, 62, 69), obtaining 69. Write 9 units in the units' column, and carry 6 tens to the tens' column.

6 (to carry), 15, 24, 34, 43, 53, 61, 70, 80, obtaining 80 tens, or 8 hundreds and 0 tens. Write 0 tens in the tens' column, and carry 8 hundreds to the hundreds' column.

8 (to carry), 16, 26, 36, 46, 55, 64, 73, 82, obtaining 82 hundreds, making the sum of the three columns equal 8209.

EXAMPLES FOR PRACTISE

(1)	(2)	(3)	(4)	(5)	(6)	(7)
3	2	21	456	142	326	569
5	7	62	633	357	762	413
4	1	93	291	213	691	324
9	4	35	832	431	325	741
6	3	21	165	565	443	246
2	2	58	312	252	232	632
8	5	14	421	343	586	171
7	4	63	256	224	213	514
2	9	31	432	769	732	465
6	6	58	318	132	964	922
4	2	42	271	348	143	873
5	3	75	726	622	212	221
3	7	34	193	896	747	512

(8)	(9)	(10)	(11)	(12)
2674	64287	487639	528964	8492716
3425	13612	261324	272625	1637154
4283	32291	432152	313428	1231142
1716	48453	145464	293112	3536126
9134	21432	925515	654577	4172641
2942	32125	343123	132322	2629239
7123	76562	326682	721831	7314275
1358	23128	471307	276146	2163913
2542	94319	819731	819120	1342592
3291	62643	961252	125394	4289411
4876	24356	327218	363283	4526956
9212	25191	712892	142317	2133242
2794	43737	231429	654976	3641727
6243	12453	546561	211732	5324361
2922	56214	324326	732142	2472613
4636	31925	453656	129124	1413486
1256	93172	652146	476164	6235612
5126	59213	254916	323622	2249232
1716	29561	756926	342429	7264145
8226	39343	154976	248928	2295724

CROSS-MULTIPLICATION

313. In obtaining the necessary extensions of many of the separate items in making out bills, invoices, etc., much valuable time will be saved by the following process, when the price and the quantity consist of 2 or 3 orders, as is ordinarily the case.

ILLUSTRATIVE EXAMPLE

Find the cost of 23 yards cloth at 45 cents per yard.

SOLUTION

$$\begin{array}{r} \$.45 \\ 23 \\ \hline \$10.35 \end{array}$$

EXPLANATION. The product of two numbers is the product of each order of one number separately multiplied by each order of the other number [Prin. 2, **26**]. Hence,

First multiply the units' orders (3 times 5 units = 15 units, or 1 ten and 5 units). Write 5 units as the units' order of the product, and carry 1 ten.

Second, multiply each tens' order by its opposite units' order (3 times 4 tens + 5 times 2 tens + 1 ten to carry = 23 tens, or 2 hundreds and 3 tens). Write 3 tens as the tens' order of the product, and carry 2 hundreds.

Third, multiply the tens' orders (4 tens × 2 tens = 8 hundreds) + 2 carried hundreds = 10 hundreds; thus making the complete product 1035 cents, or $10.35.

EXAMPLES FOR PRACTISE

Multiply	Multiply	Multiply	Multiply
1. 48 by 37	**4.** 86 by 63	**7.** 27 by 35	**10.** 73 by 54
2. 53 by 74	**5.** 58 by 24	**8.** 65 by 23	**11.** 36 by 27
3. 29 by 82	**6.** 95 by 32	**9.** 42 by 67	**12.** 52 by 73

POWERS AND ROOTS

314. **A power** of a number is the product of that number if taken two or more times as a factor.

NOTE 1. The *second power* or *square* of 3 is the product of 3 taken *twice* as a factor (3 × 3), or 9; the *third power* or *cube* of 2 is the product of 2 taken *three times* as a factor (2 × 2 × 2), or 8; the *fourth power* of 5 is the product of 5 taken *four times* as a factor (5 × 5 × 5 × 5), or 625; etc.

NOTE 2. The *exponent* of a power is a small figure placed diagonally above a number to denote *how many times* it is to be taken as a factor. Thus 5^2 expresses the square of 5; 7^3 the cube of 7; 8^5 the fifth power of 8; etc.

315. The **root** of a power is *one* of the number of equal factors of which that power is the product.

NOTE 1. The *square root* of a power is *one* of its two equal factors; the *cube root* is *one* of its three equal factors; the *fourth root* is *one* of its four equal factors; etc.

NOTE 2. The *radical sign* $\sqrt{\ }$ is usually written at the left of a power to indicate that a root of it is to be taken; and a small figure called an *index* is written within the angle of the radical sign to show *what root* is to be taken. When no index is written, the index 2 is understood. Thus, $\sqrt{16}$ indicates the square root of 16; $\sqrt[3]{512}$ the cube root of 512; $\sqrt[4]{81}$ the fourth root of 81.

316. **Principles.** 1. *If, commencing at the right of any given power of an integral number, it is divided by a separatrix into sections of as many figures each as there are units in its exponent, the number of such sections, counting a possibly incomplete section at the extreme left as another section, will denote the number of orders in its root: and the enumeration of those sections will indicate the relative value of their corresponding root orders.*

2. (a) *The highest or left-hand section contains the given power of the highest root order:* (b) *any excess of the highest section over the given power of the highest root order, written at the left of the next following section, will approximately express the value of the given power of the second highest root order and its involved relations with the highest root order:* (c) *any excess of the highest two sections over the given power of the highest two root orders, written at the left of the next following section, will approximately express the value of the given power of the third highest root order and its involved relations with the highest two root orders; etc.*

ILLUSTRATIVE EXAMPLES

317. What is the square root of 4624?

SOLUTION

$$
\begin{array}{rl}
 & 46|24(6 \text{ tens} \\
6^2 = 36 & \quad 8 \\
\hline
130)1024(& 8 \text{ units} \\
 910 & \\
\hline
 114 &
\end{array}
$$

$70^2 = 4900$

$60^2 = 3600$

$10)\overline{1300}$

Average square per unit = 130

$\sqrt{4624}$ = 6 tens and 8 units, or 68.

EXPLANATION. (*a*) Commence at the right of the given power and divide it by a separatrix into sections of two figures each, thus finding that the required root contains two orders, tens and units (Prin. 1, **316**).

(*b*) The greatest number the square of which is contained in the left-hand or tens' section (46) is 6 tens. Write 6 tens as the highest order of the required square root, subtract its square ($6^2 = 36$) from its corresponding section (46), to the remainder (10) bring down the next following section (24) and the result (1024) will accurately express the square of the next following or units' order of the required root + its involved relations with the tens' order of the root (*b*, Prin. 2, **316**).

(*c*) The required square root is thus found to be more than 6 tens (or 60 units if expressed in terms of the next lower root order), and less than 7 tens (or 70 units if similarly expressed); and if the difference between the square of 60 units ($60^2 = 3600$) and the square of 70 units ($70^2 = 4900$), or 1300, be divided by the difference between their respective roots (70 − 60), or 10, the quotient (130) will be the average square of the ten consecutive numbers between 60 and 70; and as the difference is so slight as to be practically negligible, it (130) can be accepted tentatively as the average square of as many of these consecutive numbers as may be included in the next following or units' order of the required root.

(*d*) Divide the square of the units' order of the required root and its involved relations as found in *b* (1024) by the assumed average square per unit and its involved relations as found in *c* (130) to find tentatively the number of units in the units' order (7 units and a significant remainder, practically 8 units). Hence, 6 tens and 8 units, or 68, is the required square root.

NOTE. In the above solution, the average square of the *ten* consecutive squares between 60^2 and 70^2 (130) is slightly greater than the average square of the *eight* consecutive squares between 60^2 (3600) and the square of the true root ($68^2 = 4624$) which is 128 $(4624 - 3600 \div 8)$. Hence, the final remainder (114) from dividing by the assumed average square (130) lacks $(130 - 128) \times 8$, or 16 of containing the tentative divisor (130) *exactly* 8 times.

318. What is the cube root of 105823817?

SOLUTION

105|823|817(4 hundreds — $50^3 = 125000$

$4^3 = 64$ 7 — $40^3 = 64000$

6100)41 823(6 tens — $10)\overline{61000}$

36 600 — Average cube per unit = 6100

5 223

105 823 — $480^3 = 110592000$

$47^3 = 103\,823$ 3 — $470^3 = 103823000$

676900)2 000 817(2 units — $10)\overline{6769000}$

1 353 800 — Average cube per unit = 676900

647 017

Hence, $\sqrt[3]{105823817} = 4$ hundreds, 7 tens, 3 units, or 473.

EXPLANATION. (*a*) Commence at the right of the given cube and divide it by a separatrix into sections of three figures each, thus finding that the required root contains three orders, hundreds, tens, and units (Prin. 1, **316**).

(*b*) The greatest number the cube of which is contained in the left-hand or hundreds' section (105) is 4 hundred. Write it (4 hundred) as the highest order of the required root (*a*, Prin. 2, **316**), subtract its cube ($4^3 = 64$) from its corresponding section (105), to the remainder (41) bring down the next following section (823), and the result (41823) will approximately express the cube of the next following or tens' order of the required root + its involved relations with the hundreds' order of the root (*b*, Prin. 2, **316**).

(*c*) It is now found that the required cube root is greater than 4 hundred (or 40 tens if expressed in terms of the next lower root order) and less than 5 hundred (or 50 tens if similarly expressed). If therefore the difference between the cube of 40 tens ($40^3 = 64000$) and the cube of 50 tens ($50^3 = 125000$), or 61000, be divided by the difference between their respective roots (50 − 40), or 10, the quotient (6100) will be the average cube of the ten consecutive numbers between 40 and 50; and as the difference is so slight as to be practically negligible, this result (6100) can be accepted tentatively as the average cube of as many of these consecutive numbers as may be included in the next following or tens' order of the required root.

(*d*) Divide the cube of the tens' order of the required root and its involved relations, as found in *b* (41823) by the assumed average cube per unit and its involved relations as found in *c* (6100) to find tentatively the number of units in the tens' order (6 tens and a significant remainder, practically 7 tens).

(*e*) The required root is now found to be greater than 47 tens (or 470 units if expressed in terms of the next lower root order) and less than 48 tens (or 480 units if similarly expressed). If therefore the difference between 470^3 (103823000) and 480^3 (110592000), or 6769000, be divided by the difference between their respective roots (480 − 470), or 10, the quotient (676900) will be the average cube of the ten consecutive numbers between 470 and 480; which may also be accepted tentatively as the average cube of as many of these consecutive numbers as are included in the next following or units' order of the required root.

(*f*) Subtract the cube of the already obtained root orders ($47^3 = 103823$) from their corresponding sections (105823), to the remainder (2000) bring down the next following section (817), and the result (2000817) will accurately express the cube of the next following or units' order of the required root + its involved relations with the preceding root orders (*c*, Prin. 2, **316**).

(*g*) Divide the cube of the units' order of the required root and its involved relations as obtained in *f* (2000817) by the assumed average cube per unit in the units' order and its involved relations as obtained in *e* (676900) to find tentatively the number of units in the units' order (2 units and a significant remainder, or 3 units). Hence, 4 hundreds, 7 tens, and 3 units, or 473, is the required cube root.

319. In the following solutions, the average power per unit is more briefly obtained by finding the difference between the given power of the root orders already found and the given power of those root orders increased by 1 *without expressing them in terms of the next lower root order*, and annexing to this difference as many ciphers lacking 1 as there are units in the exponent of the given power. Much unnecessary labor may also be avoided in the successive divisions to find the next following root order by using only the highest two orders of the successive divisors, and by omitting from the right of their corresponding dividends as many figures as have been rejected from the right of their respective divisors. The processes being identical for all roots, no further explanations are considered necessary. If, however, the learner should fail to understand any step in the following solutions, he can refer to the corresponding step in the preceding explanations for the required information.

ILLUSTRATIVE EXAMPLES

1. What is the fifth root of 3796375994368?

SOLUTION

$$
\begin{array}{rl}
 & 379|63759|94368(3 \text{ hundred} \\
3^5 = 243 & \quad 2 \\
78|10000)136|63759(1 \text{ tens} & \\
\underline{78} & \\
58 &
\end{array}
$$

$4^5 = 1024$

$3^5 = \underline{243}$

Average power per unit = 7810000

$$
\begin{array}{rl}
 & 37963759 \\
32^5 = 33554432 & \quad 8 \\
55|809610000)440|932794368(7 \text{ units} & \\
\underline{390} & \\
50 &
\end{array}
$$

$33^5 = 39135393$

$32^5 = \underline{33554432}$

Average power per unit = 5580961000

$\sqrt[5]{3796375994368}$ = 3 hundred, 2 tens, 8 units, or 328.

2. What is the fourth root of 552114385936?

SOLUTION

5521|1438|5936(8 hundred
$8^4 = 4096$ 6
24|65000)142|51438(5 tens
123
19

$9^4 = 6561$
$8^4 = 4096$
Average power per unit = 2465000

55211438
$86^4 = 54700816$
25|88945000)51|06225936(2 units
51

$87^4 = 57289761$
$86^4 = 54700816$
Average power per unit = 2588945000

$\sqrt[4]{552114385936}$ = 8 hundred, 6 tens, 2 units, or 862.

3. What is the square root of .02795584?

SOLUTION

.02|79|55|84(1 tenth
$.1^2 = .01$ 6
30)179(5 hundredths
150
29

$2^2 = 4$
$1^2 = 1$
Average per unit = 30

.0279
$.16^2 = .0256$
330)2355(7 thousandths
2310
45, insignificant remainder

$17^2 = 289$
$16^2 = 256$
Average per unit = 330

.027955
$.167^2 = .027889$ 2
3350) 6684(1 ten-thousandths
3350
3334

$168^2 = 28224$
$167^2 = 27889$
Average per unit = 3350

$\sqrt{.02795584}$ = 1 tenth, 6 hundredth, 7 thousandth, 2 ten-thousandth, or .1672.

RULE. (a) *Commence at the right of the given power, if an integer, and separate it into sections of as many figures each as there are units in the exponent of that power. The number of sections thus found, counting a possibly incomplete section at the extreme left as another section, will denote the number of orders in the required root, and the enumeration of those sections will indicate the relative value of their corresponding root orders.*

(b) *Find the greatest number the given power of which is contained in the left-hand section, and write it as the highest order of the required root.*

(c) *Subtract the given power of the obtained highest root order from the left-hand section, and to the remainder bring down the next following section, to find the dividend of the next following root order.*

(d) *Find the difference between the given power of the obtained root order and the given power of that order increased by 1, and to this difference annex as many ciphers less 1 as there are units in the exponent of the given power, to find tentatively the average given power per unit of the next following root order.*

(e) *Divide the obtained dividend of the next following root order* (c) *by the average given power per unit of the next following root order* (d) *to find tentatively the number of units in the next following root order, increasing the quotient by 1 if the remainder be significantly great as compared with the divisor.*

(f) *Thus continue to subtract the given power of the obtained root orders from their corresponding sections, to the remainder bring down the next following section, and divide the result by the average power per unit of the next following root order to find tentatively the number of units in that root order, until all the root orders have been successively obtained.*

NOTE 1. In applying *e* of the above *Rule*, if the quotient from dividing the dividend of the next following root order by the divisor of that root order, should be 1 greater or 1 less than the true root order, as is sometimes the case, it will incidentally be made evident in performing the next following process. Thus, if the given power of the obtained root orders should prove to be greater than the sections from which they were evolved, it would show that the last obtained root order should be diminished by 1; or if the given power of the obtained root orders plus 1 should prove to be equal to or less than the sections from which the obtained root orders were evolved, it would show that the last *obtained* root order should be increased by 1.

NOTE 2. The roots of pure decimals are obtained in the same manner as those of integers, except that decimals should be separated into sections by commencing *at the left* (at the decimal point). Mixed decimals are divided into sections by commencing at the decimal point and separating the integral part into sections *from the right*, and the decimal part into sections *from the left*.

NOTE 3. The root of a common fraction is obtained by separately finding the root of its numerator and of its denominator. If the numerator and denominator are not perfect powers, first reduce to a decimal, and apply Note 2.

NOTE 4. If, after bringing down and considering the last section of an integral number, it is found to be an imperfect power, continue the process by successively annexing a section of ciphers as many times as there may be decimal places required in the root. If preferred, the obtained decimal part of the root can then be approximated to a common fraction of any required denominator by **125**.

EXAMPLES FOR PRACTISE

Applying **314**, find the indicated power of

1. 9^2	**5.** 8^3	**9.** 5^4	**13.** $(\frac{2}{3})^3$	**17.** $(.6)^5$
2. 28^2	**6.** 15^3	**10.** 13^5	**14.** $(\frac{4}{7})^2$	**18.** $(.27)^4$
3. 162^2	**7.** 246^3	**11.** 6^7	**15.** $(2\frac{1}{2})^4$	**19.** $(1.28)^3$
4. 345^2	**8.** 528^3	**12.** 8^6	**16.** $(5\frac{1}{4})^3$	**20.** $(13.72)^2$

Find the indicated root of

21. $\sqrt{5476}$	**29.** $\sqrt{2394829969}$	**37.** $\sqrt[3]{9340607016}$
22. $\sqrt{132496}$	**30.** $\sqrt{3910501156}$	**38.** $\sqrt[3]{823183846848}$
23. $\sqrt{19321}$	**31.** $\sqrt[3]{804357}$	**39.** $\sqrt[3]{1003003001}$
24. $\sqrt{609961}$	**32.** $\sqrt[3]{425259008}$	**40.** $\sqrt[3]{41962796875}$
25. $\sqrt{169744}$	**33.** $\sqrt[3]{14886936}$	**41.** $\sqrt[4]{28398241}$
26. $\sqrt{82864609}$	**34.** $\sqrt[3]{997002999}$	**42.** $\sqrt[4]{169879162896}$
27. $\sqrt{74701449}$	**35.** $\sqrt[3]{177504328}$	**43.** $\sqrt[5]{3707398432}$
28. $\sqrt{14561856}$	**36.** $\sqrt[3]{575930368}$	**44.** $\sqrt[5]{109914468611443}$

45. Allowing 2150.42 cu. in. to a bushel, what must be the dimensions of a cubic bin which will hold 75 bushels of wheat, carrying the dimensions to two decimal places?

46. Allowing 231 cu. in. to a gallon, what are the dimensions of a cubic cistern which will hold 30000 gallons of water — to three decimal places?

47. How many feet long is each side of a square field which contains 5 acres — to two decimal places?

SUGGESTION. Acres should be reduced to square units of the denomination in which the linear dimensions are required to be expressed [146].

48. If each edge of a cube is 23 inches long, what are its contents?

49. If each side of a square lot is 135 ft. 6 in. long, what is its area?

50. Allowing 34.844 cu. ft. to a ton, what must be the dimensions of a cubic bin which will hold 18 tons of coal — to two decimal places?

51. A rectangular lawn is three-fourths as wide as it is long. If its area is 768 sq. yd., what are its dimensions?

SUGGESTION. In Ex. 51, divide the given area by $\frac{3}{4}$ to find the area of a square in which the width will exactly equal the length, extract the square root of the result to find the length, etc.

52. A rectangular room is three times as long as it is wide. What are its dimensions if its area is 5547 sq. ft.?

53. A rectangular platform is two-thirds as long as it is wide. If its area is 1536 sq. ft., what are its dimensions?

54. A rectangular reservoir which is six times as long as it is high and wide has a capacity of 3951018 cu. ft. of water. What are its dimensions?

SUGGESTION. In Ex. 54, divide the given capacity by 6 to find the capacity of a cubic reservoir in which the length is exactly equal to the height and the width, extract the cube root of the result to find the height and the width, etc.

55. An excavation which is four times as long as it is wide and deep required the removal of 740772 cu. ft. of earth. What were its dimensions?

56. What are the dimensions of a rectangular volume the width of which is $\frac{3}{5}$ as much as its length and its height, if it contains 12288 cu. in.?

ANSWERS

Article 19 (Pages 8–9)

(1.) 712 (2.) 11601 (3.) 19671 (4.) 23845
(5.) 81711 (6.) 844804

Article 20 (Page 9)

(1.) 2957 (2.) 2700 (3.) 3130 (4.) 3481 (5.) 14112
(6.) 11952 (7.) 3629 (8.) 7989

Article 23 (Pages 10–11)

(1.) 4694 (2.) 3314 (3.) 55532 (4.) 54383
(5.) 49193 (6.) 91366 (7.) 26560827 (8.) 23080267
(9.) 53959942 (10.) 878259774 (11.) 678320279 (12.) 560974939
(13.) 458891 (14.) 23292554 (15.) $18214 (16.) $44282
(17.) 68 yr. (18.) $25436

Article 24 (Page 11)

(1.) 17471 (2.) 36222 (3.) 18027 (4.) 1456676
(5.) 3843018 (6.) 4370095

Article 27 (Page 13)

(1.) 343671290 (2.) 195254794 (3.) 652377231
(4.) 173914846 (5.) 567901224 (6.) 918274360
(7.) 672296224072 (8.) 840480082764 (9.) 161592011024
(10.) 5520042600048 (11.) 429178386456 (12.) 240129270228

Article 28 (Page 14)

(1.) 410078874 (2.) 7131997308 (3.) 4588885791
(4.) 2464319167 (5.) 36669055794 (6.) 57056110255
(7.) 20613772704 (8.) 5668586134672 (9.) 3064616671347
(10.) 2809026140045 (11.) 7637304099436 (12.) 65145273284448
(13.) 17553132769200 (14.) 10241538016248 (15.) 1757140 lb.
(16.) $13003232 (17.) $801 (18.) 50005 mi.

Article 29 (Page 15)

(1.) 488376 (2.) 2591904 (3.) 4230792 (4.) 38878854
(5.) 72810144 (6.) 564150708 (7.) 553358592 (8.) 22802256

Article 30 (Page 15)

(1.) 876300 (2.) 95780000 (3.) 756230 (4.) 62875000
(5.) 4132900000 (6.) 728650000

Article 31 (Page 16)

(1.) 625800 (2.) 245856000 (3.) 375706100
(4.) 387144000 (5.) 377131000 (6.) 1745530000

Article 32 (Page 17)

(1.) 3369600000 (2.) 604800000 (3.) 158580000
(4.) 25956000000 (5.) 38681000000 (6.) 8772000000
(7.) 306000000000 (8.) 41446000000

Article 35 (Page 19)

(1.) 17965624 (2.) 275786427 (3.) 158812276
(4.) 118524835 (5.) 264189593 (6.) 146965927
(7.) 4027429807 (8.) 2852436483 (9.) 6998433041
(10.) 8253482678 (11.) 6297587576 (12.) 12867144043

Article 36 (Page 20)

(1.) $38102155\frac{7}{23}$ (2.) $22541067\frac{15}{41}$ (3.) $11577549\frac{60}{83}$
(4.) $18452803\frac{14}{57}$ (5.) $8997200\frac{49}{58}$ (6.) $5538344\frac{7}{79}$
(7.) $12957056\frac{28}{45}$ (8.) $7902780\frac{52}{67}$ (9.) $916413\frac{88}{325}$
(10.) $9009655\frac{243}{456}$ (11.) $6279520\frac{58}{925}$ (12.) $1497692\frac{2189}{4365}$
(13.) $113820\frac{68271}{72349}$ (14.) $8752\frac{560812}{603035}$ (15.) $22881\frac{158285}{412078}$
(16.) $8994\frac{755782}{890065}$ (17.) $262950\frac{27204}{27884}$ (18.) $5658\frac{135751}{728163}$

Article 37 (Page 21)

(1.) $32019\frac{19}{28}$ (2.) $55260\frac{9}{32}$ (3.) $17073263\frac{19}{45}$
(4.) $1301747\frac{39}{72}$ (5.) $981785\frac{11}{64}$ (6.) $95606267\frac{11}{54}$
(7.) $38315124\frac{64}{96}$ (8.) $57461093\frac{41}{84}$ (9.) $14124\frac{21}{168}$
(10.) $14452\frac{187}{504}$ (11.) $45426\frac{75}{126}$ (12.) $30704\frac{87}{315}$

Article 38 (Page 22)

(1.) $6785\frac{9}{10}$ (2.) $826\frac{78}{100}$ (3.) $92\frac{567}{1000}$ (4.) $2\frac{6885}{10000}$
(5.) $1483\frac{762}{1000}$ (6.) $78341\frac{6}{100}$ (7.) $2871\frac{15}{10000}$ (8.) $62520\frac{3}{1000}$

Article 39 (Page 23)

(1.) $10920\frac{541}{800}$ (2.) $1530\frac{3762}{6000}$ (3.) $10689\frac{218}{400}$ (4.) $974\frac{5475}{7000}$
(5.) $1448\frac{1964}{4100}$ (6.) $336\frac{41653}{218000}$ (7.) $106\frac{418889}{493000}$ (8.) $679\frac{58267}{63000}$
(9.) $17557\frac{8234}{4800}$ (10.) $185\frac{87291}{350000}$

Article 43 (Page 26)

(1.) \$14159.56 (2.) \$2171.02 (3.) \$1350.24 (4.) \$458.481 (5.) \$26.705

Article 44 (Page 26)

(1.) \$319.88 (2.) \$18.837 (3.) \$1013.45 (4.) \$378.32 (5.) \$141.95
(6.) \$218.80 (7.) \$6.963 (8.) \$78.05

Article 45 (Page 27)

(1.) $1739375 (2.) $36876 (3.) $1018.61 (4.) $387270
(5.) $4824.59 (6.) $25715.52 (7.) $54974.64 (8.) $13488.816
(9.) $58508.10 (10.) $261024

Article 46 (Page 28)

(1.) $1356.65 (2.) $98.37+ (3.) $18.62+ (4.) $10.62−
(5.) $40.16− (6.) $6.96+ (7.) $8.51+ (8.) $18.66−

Article 47 (Page 29)

(1.) 5944 (2.) $966\frac{137}{243}$ (3.) $392\frac{1246}{1715}$ (4.) $922\frac{1626}{3267}$ (5.) $695\frac{9550}{12850}$
(6.) $514361\frac{15}{18}$ (7.) $695366\frac{6}{9}$ (8.) $1625\frac{2388}{2700}$ (9.) $72\frac{2154}{8240}$ (10.) $126\frac{3}{145}$

Article 54 (Pages 34–38)

(1.) $1578.05 (2.) $507 (3.) $5.59 (4.) $603.13 (5.) $4932
(6.) $213.75 (7.) $2897 (8.) 15¢ (9.) $542 (10.) $6.89 (11.) 68¢
(12.) $1324.64 (13.) $417.22 (14.) 12 yr. (15.) 65° (16.) $.14$\frac{101}{135}$
(17.) 54 da. (18.) 455 da. (19.) 4 da. (20.) 30 da. (21.) 71 yr.
(22.) 1732 (23.) 72 yr. (24.) 52 yr. (25.) A $5000; B $10000; C $15000; D $25000 (26.) A $65; B $195 (27.) A $40; B $160; C $320
(28.) $3000; $3275 (29.) A $6300; B $6868 (30.) A $5000; B $7387
(31.) A $5000; B $6200; C $5800 (32.) wife $12450; son $11825; daughter $12750 (33.) 5625 mi. (34.) 9814 votes (35.) $532.96 (36.) 1360 mi. (37.) 19 mi. (38.) $25 (39.) 40248 books (40.) 80 coins

Article 58 (Page 41)

(1.) 2, 3, 3, 3 (2.) 2, 2, 3, 3 (3.) 2, 2, 3, 7 (4.) 2, 2, 2, 3, 3, 5
(5.) 2, 2, 3, 3, 7 (6.) 3, 3, 3, 5, 7 (7.) 2, 2, 3, 3, 5, 7 (8.) 2, 2, 3, 3, 3, 7, 7 (9.) 3, 3, 3, 5, 5, 5 (10.) 3, 3, 3, 2, 2, 2, 5, 5, 5 (11.) 2, 2, 2, 3, 3, 5, 7, 7 (12.) 2, 2, 2, 2, 2, 3, 3, 3, 3, 11 (13.) 2, 2, 2, 3, 3, 3, 7, 11 (14.) 2, 2, 2, 3, 3, 5, 5, 5, 5 (15.) 2, 2, 3, 3, 5, 7, 13

Article 63 (Page 42)

(1.) 360 (2.) 3528 (3.) 1080 (4.) 576 (5.) 324 (6.) 450
(7.) 37800 (8.) 9000 (9.) 41580 (10.) 450 (11.) 288 (12.) 6006

Article 67 (Page 45)

(1.) 37 (2.) 45 (3.) 204 (4.) 125 (5.) 49 (6.) 625
(7.) 192 (8.) 243 (9.) 729 (10.) 2401 (11.) 1728 (12.) 16

Article 69 (Page 46)

(1.) 438 (2.) 29 (3.) 80 (4.) 35 (5.) 15 bu. (6.) $50

Article 82 (Page 49)

(1.) $\frac{15}{25}$ (2.) $\frac{35}{56}$ (3.) $\frac{24}{32}$ (4.) $\frac{32}{64}$ (5.) $\frac{144}{216}$ (6.) $\frac{192}{512}$
(7.) $\frac{4}{16}$ (8.) $\frac{112}{128}$ (9.) $\frac{A^2}{256}$ (10.) $\frac{12}{64}$ (11.) $\frac{112}{4}$ (12.) $\frac{2816}{16}$

Article 83 (Page 50)

(1.) $\frac{5}{8}$ (2.) $\frac{5}{16}$ (3.) $\frac{1}{4}$ (4.) $\frac{17}{24}$ (5.) $\frac{1}{2}$ (6.) $\frac{3}{4}$ (7.) $\frac{13}{64}$ (8.) 19 (9.) 8

Article 84 (Page 51)

(1.) $\frac{1}{2}$ (2.) $\frac{3}{4}$ (3.) $\frac{7}{8}$ (4.) $\frac{3}{8}$ (5.) $\frac{5}{8}$ (6.) $\frac{5}{8}$ (7.) $\frac{3}{5}$ (8.) $\frac{3}{4}$
(9.) $\frac{5}{16}$ (10.) $\frac{11}{32}$ (11.) $\frac{13}{16}$ (12.) $\frac{19}{32}$ (13.) $\frac{5}{8}$ (14.) $\frac{9}{32}$ (15.) $\frac{7}{8}$ (16.) $\frac{3}{4}$

Article 86 (Page 52)

(1.) $\frac{1}{2}$ (2.) $\frac{1}{2}$ (3.) $\frac{3}{4}$ (4.) $\frac{3}{4}$ (5.) $\frac{3}{8}$ (6.) $\frac{3}{4}$ (7.) $\frac{13}{16}$ (8.) $\frac{3}{4}$

Article 87 (Page 53)

(1.) $\frac{144}{16}$ (2.) $\frac{200}{8}$ (3.) $\frac{85}{8}$ (4.) $\frac{51}{4}$ (5.) $\frac{57}{2}$ (6.) $\frac{213}{4}$ (7.) $\frac{2819}{16}$
(8.) $\frac{16201}{64}$ (9.) $\frac{1485}{32}$ (10.) $\frac{2285}{25}$ (11.) $\frac{2270}{5}$ (12.) $\frac{6541}{8}$ (13.) $\frac{4524}{128}$
(14.) $\frac{6993}{32}$ (15.) $\frac{1866}{10}$ (16.) $\frac{2064}{5}$

Article 88 (Page 53)

(1.) 16 (2.) 32 (3.) 8 (4.) 14 (5.) $45\frac{3}{16}$ (6.) $26\frac{25}{32}$ (7.) $6\frac{45}{64}$
(8.) $108\frac{3}{16}$ (9.) $1085\frac{1}{3}$ (10.) $489\frac{7}{16}$ (11.) $839\frac{2}{5}$ (12.) $66\frac{13}{125}$ (13.) $127\frac{19}{128}$ (14.) $854\frac{23}{32}$ (15.) $837\frac{5}{64}$

Article 91 (Page 55)

(1.) $\frac{20}{24}$, $\frac{9}{24}$ (2.) $\frac{12}{24}$, $\frac{16}{24}$, $\frac{21}{24}$ (3.) $\frac{51}{288}$, $\frac{264}{288}$, $\frac{128}{288}$ (4.) $\frac{36}{48}$, $\frac{40}{48}$, $\frac{39}{48}$ (5.) $\frac{8}{12}$, $\frac{8}{12}$, $\frac{6}{12}$ (6.) $\frac{27}{72}$, $\frac{12}{72}$, $\frac{16}{72}$ (7.) $\frac{12}{30}$, $\frac{27}{30}$, $\frac{24}{30}$ (8.) $\frac{30}{48}$, $\frac{28}{48}$, $\frac{27}{48}$ (9.) $\frac{27}{144}$, $\frac{70}{144}$, $\frac{24}{144}$ (10.) $\frac{105}{15}$, $\frac{9}{15}$, $\frac{10}{15}$ (11.) $\frac{87}{192}$, $\frac{168}{192}$, $\frac{56}{192}$ (12.) $\frac{40}{80}$, $\frac{45}{80}$, $\frac{48}{80}$ (13.) $\frac{14}{24}$, $\frac{16}{24}$, $\frac{14}{24}$, $\frac{11}{24}$ (14.) $\frac{12}{24}$, $\frac{18}{24}$, $\frac{13}{24}$, $\frac{22}{24}$ (15.) $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$ (16.) $\frac{21}{24}$, $\frac{216}{24}$, $\frac{120}{24}$, $\frac{22}{24}$

Article 92 (Pages 56–57)

(1.) $1\frac{7}{12}$ (2.) $1\frac{19}{24}$ (3.) $1\frac{7}{12}$ (4.) $1\frac{27}{40}$ (5.) $1\frac{3}{4}$ (6.) $1\frac{19}{24}$ (7.) $1\frac{43}{48}$
(8.) $1\frac{7}{15}$ (9.) $2\frac{103}{210}$ (10.) $2\frac{887}{960}$ (11.) $2\frac{43}{60}$ (12.) $2\frac{11}{32}$ (13.) $99\frac{101}{120}$
(14.) $502\frac{3}{8}$ (15.) $255\frac{4}{15}$ (16.) $162\frac{1}{24}$ (17.) $730\frac{47}{144}$ (18.) $98\frac{1}{12}$
(19.) 278^{1} yd. (20.) 303^{3} yd. (21.) 180^{15} lb. (22.) 2511^{4} lb. (23.) 2103^{12} bu. (24.) 3106^{10} bu. (25.) 10308^{24} bu.

Article 94 (Page 58)

(1.) $\frac{7}{16}$ (2.) $\frac{1}{12}$ (3.) $\frac{17}{35}$ (4.) $\frac{1}{12}$ (5.) $\frac{17}{24}$ (6.) $\frac{1}{6}$ (7) $\frac{13}{48}$ (8.) $\frac{5}{12}$
(9.) $\frac{4}{45}$ (10.) $\frac{1}{75}$ (11.) $\frac{3}{8}$ (12.) $\frac{3}{8}$ (13.) $1\frac{1}{12}$ (14.) $\frac{11}{24}$ (15.) $\frac{25}{32}$ (16.) $\frac{5}{12}$

Article 95 (Page 60)

(1.) $311\frac{1}{3}$ (2.) $313\frac{1}{3}$ (3.) $390\frac{5}{8}$ (4.) $143\frac{5}{9}$ (5.) $57\frac{7}{24}$ (6.) $17\frac{13}{16}$
(7.) $20\frac{3}{4}$ (8.) $35\frac{23}{48}$ (9.) $18\frac{25}{72}$ (10.) $28\frac{13}{48}$ (11.) $28\frac{11}{48}$ (12.) $16\frac{7}{12}$
(13.) $22\frac{13}{24}$ (14.) $29\frac{13}{48}$ (15.) $22\frac{3}{8}$ (16.) $28\frac{1}{3}$

Article 97 (Page 62)

(1.) $\frac{1}{2}$ (2.) $\frac{5}{12}$ (3.) $\frac{1}{12}$ (4.) $\frac{1}{12}$ (5.) $2\frac{2}{3}$ (6.) 3 (7.) 4 (8.) $2\frac{1}{2}$ (9.) $1\frac{1}{2}$ (10.) 35 (11.) $504\frac{3}{4}$ (12.) 184 (13.) $1761\frac{3}{4}$ (14.) $\frac{1}{3}$ (15.) $\frac{1}{4}$

Article 98 (Page 62)

(1.) $4751\frac{1}{4}$ (2.) $2660\frac{5}{8}$ (3.) $3421\frac{1}{3}$ (4.) $2073\frac{1}{2}$ (5.) 7140
(6.) $1966\frac{2}{3}$ (7.) $19986\frac{3}{4}$ (8.) $12437\frac{1}{4}$ (9.) $11846\frac{1}{6}$ (10.) $20008\frac{1}{3}$
(11.) $34728\frac{3}{4}$ (12.) 32538

Article 99 (Page 63)

(1.) $2075\frac{5}{12}$ (2.) $2757\frac{3}{8}$ (3) $4804\frac{59}{72}$ (4.) $893\frac{79}{80}$ (5.) $941\frac{7}{8}$
(6.) $8200\frac{1}{35}$ (7.) $27422\frac{7}{10}$ (8) $19895\frac{5}{24}$ (9.) $9062\frac{8}{27}$ (10.) $27554\frac{23}{32}$
(11.) $160938\frac{11}{128}$ (12.) $864966\frac{5}{18}$ (13.) $241458\frac{7}{16}$ (14.) $1258300\frac{13}{32}$
(15.) $645883\frac{35}{54}$

Article 101 (Page 65)

(1.) $\frac{5}{24}$ (2.) $\frac{1}{8}$ (3.) $\frac{3}{32}$ (4.) $\frac{1}{18}$ (5.) $\frac{1}{45}$ (6.) $\frac{3}{32}$ (7.) $\frac{3}{8}$
(8.) $\frac{7}{9}$ (9.) $\frac{1}{4}$ (10.) $\frac{3}{56}$ (11.) $\frac{23}{35}$ (12.) $1\frac{1}{3}$ (13.) $\frac{1}{64}$ (14.) $\frac{1}{32}$
(15.) $\frac{7}{12}$

Article 102 (Page 66)

(1.) 8 (2.) 24 (3.) 27 (4.) 10 (5.) $19\frac{1}{5}$ (6.) $2\frac{1}{3}$ (7.) $44\frac{1}{3}$
(8) 32 (9.) 6 (10.) $5192\frac{1}{4}$ (11.) $1766\frac{4}{7}$ (12.) 904

Article 103 (Page 67)

(1.) $\frac{5}{6}$ (2.) $1\frac{1}{6}$ (3.) $3\frac{3}{5}$ (4.) $4\frac{1}{6}$ (5.) 6 (6.) $11\frac{1}{3}$ (7.) $\frac{5}{42}$ (8.) $\frac{1}{3}$
(9.) $1\frac{5}{27}$ (10.) $\frac{3}{4}$ (11.) $\frac{1}{2}$ (12.) $1\frac{175}{248}$

Article 104 (Page 67)

(1.) $558\frac{17}{45}$ (2.) $146\frac{101}{288}$ (3.) $240\frac{79}{112}$ (4.) $138\frac{374}{405}$ (5.) $110\frac{247}{425}$
(6.) $155\frac{19}{512}$ (7.) $8\frac{483}{620}$ (8.) $336\frac{1985}{2076}$ (9.) $12\frac{8559}{4860}$

Article 105 (Page 68)

(1.) $163\frac{11}{208}$ (2.) $140\frac{148}{301}$ (3.) $110\frac{98}{103}$ (4.) $26\frac{551}{986}$ (5.) $17\frac{272}{2917}$
(6.) $10\frac{1890}{2337}$ (7.) $107\frac{1282}{2425}$ (8.) $49\frac{1549}{3569}$ (9.) $59\frac{949}{1895}$

Article 106 (Page 69)

(1.) $25\frac{1}{668}$ (2.) $23\frac{31}{188}$ (3.) $13\frac{529}{644}$ (4.) $184\frac{11}{278}$ (5.) $80\frac{2761}{3186}$
(6.) $378\frac{122}{485}$ (7.) $6\frac{6327}{12892}$ (8.) $11\frac{5939}{10746}$ (9.) $41\frac{1031}{1836}$ (10.) $6\frac{7}{16}$
(11.) $311\frac{1}{4}$ (12.) $371\frac{15}{16}$ (13.) 37 (14.) $151\frac{5}{16}$ (15.) $12\frac{1}{2}$

Article 108 (Pages 70–71)

(1.) $\$\frac{15}{32}$ (2.) 6 yd. (3.) 2 pk. (4.) 8 lb. (5.) $\frac{16}{27}$ hr. (6.) $\frac{9}{64}$ A.
(7.) $\frac{35}{36}$ hr. (8.) 3 da. (9.) 13 da. (10.) 95 da. (11.) $7\frac{1}{32}$ mi.
(12.) $\$8\frac{1}{1920}$ (13.) $23\frac{7}{15}$ yd. (14.) $\frac{2}{15}$ (15.) $7\frac{1}{2}$ gal. (16.) $108\frac{5}{96}$ yd.

Article 110 (Pages 74–75)

(1.) $6300 (2.) 60 lb. (3.) $\frac{9}{16}$ (4.) $5628 (5.) $2316 (6.) $\frac{6}{7}$
(7.) $\frac{53}{54}$ (8) $\frac{5}{8}$ (9.) 171 sheep (10.) $\frac{1}{3}$ (11.) $7800 (12.) $18275

Article 111 (Pages 75–77)

(1.) $2990\frac{47}{48}$ (2.) $26\frac{4}{15}$ acres (3.) $\$21.83\frac{25}{32}$ (4.) $\frac{195}{461}$ (5.) $211\frac{191}{240}$ mi. (6.) $\$1.83\frac{1}{4}$ (7.) $\$12793\frac{2}{5}$ (8.) 1900 mi. (9.) $19\frac{7}{272}$ hr. (10.) $\$3943.78\frac{1}{3}$ (11.) $\frac{7}{18}$ da. (12.) $\frac{69}{92}$ (13.) grammar \$.69; speller \$.23 (15.) $2\frac{11}{12}$ da. (16.) $2\frac{10}{13}$ da. (17.) 30 da. (18.) $81\frac{31}{96}$ (19.) $\frac{31}{48}$ (20.) $884\frac{1}{2}$ (21.) $212\frac{7}{16}$ lb. (22.) 400 lb. (23.) 10 min. (24.) \$450 (25.) $\frac{5}{6}$ (26.) \$13450; \$10760 (27.) \$12800 (28.) $568\frac{2}{3}$ (29.) $454\frac{11}{24}$ (30.) $\frac{2}{5}$ (31.) A \$4280; B \$3210; C \$2568 (32.) \$2125

Article 123 (Page 83)

(1.) .5 (2.) .75 (3.) .625 (4.) .25 (5) .875 (6.) .375 (7.) .3125 (8.) .28125 (9.) .125 (10.) .140625 (11.) .6875 (12.) .8 (13.) .36 (14.) .136 (15.) .203125 (16.) 18.1875 (17.) 465.21875 (18.) 67.49609375 (19.) .71875 (20.) .328125 (21.) .9375 (22.) .34375 (23.) .109375 (24.) .8125 (25.) .546875 (26.) .03125 (27.) 58.4375 (28.) 63.53125 (29.) 37.046875 (30.) 78.0625 (31.) 168.44− (32.) 34.06− (33.) 72.72− (34.) 91.688− (35.) 26.094− (36.) 49.016− (37.) 23.4688− (38.) 71.7656+ (39.) 37.8125 (40.) 8.26563− (41.) 7.34867− (42.) 9.52722+ (43.) 1.276431+ (44.) 3.296429− (45.) 2.834538−

Article 124 (Page 84)

(1.) $\frac{3}{4}$ (2.) $\frac{3}{8}$ (3.) $\frac{1}{200}$ (4.) $\frac{1}{8}$ (5.) $\frac{7}{80}$ (6.) $\frac{11}{16}$ (7.) $\frac{15}{16}$ (8.) $\frac{1}{400}$ (9.) $\frac{5}{32}$ (10.) $\frac{11}{32}$ (11.) $\frac{19}{32}$ (12.) $\frac{1}{160}$ (13.) $5\frac{3}{80}$ (14.) $17\frac{3}{400}$ (15.) $8\frac{9}{32}$ (16.) $2\frac{7}{800}$ (17.) $19\frac{2}{3}$ (18.) $6\frac{1}{3}$ (19.) $9\frac{5}{7}$ (20.) $16\frac{3}{11}$

Article 125 (Page 85)

(1.) $\frac{11}{16}$ (2.) $\frac{1}{2}$ (3.) $\frac{3}{4}$ (4.) $\frac{5}{8}$ (5.) $\frac{4}{5}$ (6.) $\frac{1}{4}$ (7.) $\frac{17}{32}$ (8.) $\frac{2}{3}$ (9.) $\frac{5}{6}$

Article 126 (Pages 86–87)

(1.) 923.6129 (2.) 80.9235 (3.) 816.19108 (4.) 656.82651 (5.) 5.65395 (6.) 626.109375 (7.) 76.250375 (8.) 92.603845 (9.) 123.079− (10.) 12.5576+ (11.) 20.676− (12.) 1128.037265 (13.) 136137.1064684 (14.) 9341.9448348

Article 127 (Page 87)

(1.) 46.786 (2.) 580.8655 (3.) 38.4322 (4.) 34.858 (5.) 75.383 (6.) 24.747 (7.) 257.1286 (8.) 67.6871 (9.) 14.175 (10.) .53133 (11.) 26.0675 (12.) 142.50125 (13.) 6.7535625 (14.) 28.103+ (15.) 67.2320− (16.) 5.83+ (17.) 231.9944 (18.) .77427

Article 128 (Pages 88–89)

(1.) 22221.54 (2.) 590.7568 (3.) 33.20697 (4.) .070924 (5.) .54528 (6.) 8.58426 (7.) .003973 (8.) 39.845 (9.) 4.774225 (10.) 20.660234375 (11.) 24.768 (12.) 15.181809375 (13.) 6.15470625 (14.) 4.02636 (15.) $10.138\frac{1}{3}$ (16.) $1.80111\frac{1}{6}$ (17.) $2.7500\frac{16}{27}$ (18.) $.3847\frac{1}{42}$ (19.) 71.83 (20.) 620 (21.) 7.183

Article 129 (Page 90)

(1.) 5.447 (2.) 2.8 (3.) .572 (4.) 2.5837
(5.) 46250 (6.) .0682 (7.) .006 (8.) .0000032
(9.) 190000 (10.) .0004 (11.) 27000 (12.) 125
(13.) 4.0625 (14.) 78.36 (15.) 4.36 (16.) 726.8
(17.) 1.1353− (18.) 12.82− (19.) .051− (20.) 22.83−
(21.) 12.782− (22.) .526+ (23.) 946.92+ (24.) 75.035+
(25.) 90.0671+ (26.) .55 (27.) 2.8345 (28.) 416.78
(29.) .052384 (30.) .034$\frac{5}{8}$ (31.) .067− (32.) .017833+
(33.) .09685 (34.) .1091−

Article 130 (Page 92)

(1.) \$18.80− (2.) \$13.33+ (3.) \$41.17+ (4.) \$151.30+
(5.) \$1394.54 (6.) \$4.36+ (7.) \$5.40− (8.) \$35.67+
(9.) \$14.86− (10.) \$15.11+

Article 131 (Pages 92–94)

(1.) 46.2394842 (2.) 2416.07421875 bu. (3.) 361.3953 (4.) $\frac{11}{12}$
(5.) \$240.885+ (6.) 73 (7.) .75 yd. (8.) 637.5 bu. (9.) .375
(10.) 432 brls. (11.) \$9600 (12.) .74 (13.) \$1560 (14.) wife \$13566; daughter \$8996.40; son \$5997.60 (15.) house \$2000; lot \$1500
(16.) \$16400 (17.) \$82.40 (18.) \$34 (19.) \$468 (20.) 5400 inhabitants (21.) A .46; B .54 (22.) \$710.10 (23.) .28 (24.) \$95 (25.) \$396

Article 156 (Page 100)

(1.) 9583 d. (2.) 147600 lb. (3.) 1082 pt. (4.) 19080 m. (5.) 113271120 sec. (6.) 103944 gr. (7.) 47234 d. (8.) 5628 in. (9.) 3248 cu. ft. (10.) 181190$\frac{1}{4}$ sq. ft. (11.) 1714356 in. (12.) 179456 oz.
(13.) 2221200 sec. (14.) 1139 qt. (15.) 2108 gi. (16.) 460858572 sq. in. (17.) 1188864 cu. in. (18.) \$6.72 (19.) \$412.65 (20.) \$14.80

Article 157 (Page 101)

(1.) 96 bu. 1 pk. 6 qt. 1 pt. (2.) 41 da. 6 hr. 54 min. 42 sec. (3.) 4 mi. 101 rd. 2 ft. 3 in. (4.) 18 A. 158 sq. rd. 28 sq. yd. 8 sq. ft. 72 sq. in.
(5.) £24 8s. 11d. (6.) 26 T. 13 cwt. 60 lb. 4 oz. (7.) 11 rm. 15 sh.
(8.) 61 cd. 1 cd. ft. 11 cu. ft. (9.) 9 lb. 9 oz. 6 pwt. (10.) 437 cd.
(11.) £32 3d. (12.) \$53 86¢ 7 m. (13.) 196 gal. 1 qt. 1 pt. 3 gi.
(14.) 33 da. 3 hr. 24 min. 12 sec. (15.) 9 mi. 290 rd. 5 yd. 1 ft. 1 in.
(16.) 233 bu. (17.) 165 mi. 311 rd. 4 yd. 1 ft. 6 in. (18.) 19 cu. yd. 17 cu. ft. 1505 cu. in. (19.) £74 6s. (20.) 43 gal. 2 gi.

Article 158 (Page 102)

(1.) 17s. 6d. (2.) 273 da. 18 hr. (3.) 1 pt. 2 gi. (4.) 2 pk. 2 qt. 1$\frac{1}{2}$ pt. (5.) 136 da. 21 hr. (6.) 142 rd. 1 yd. 8 in. (7.) 2 ft. 6 in.
(8.) 3 cwt. 75 lb. (9.) 13s. 9d. (10.) 2 ft. 9 in.

Article 159 (Page 103)

(1.) 1 pt. 2 gi. (2.) 13 da. 16 hr. 30 min. (3.) 2 pk. 1 qt. 1 pt.
(4.) 3 qt. 3 gi. (5.) 17 cwt. 52 lb. (6.) 4 oz. 10 pwt. (7.) 110 rd.
(8.) 10s. 5.7d. (9.) 3 pk. 5 qt. 1 pt.

Article 160 (Pages 103–104)

(1.) £$\frac{1}{8}$ (2.) \$$\frac{1}{2}$ (3.) $\frac{1}{8}$ gal. (4.) $\frac{1}{8}$ bu. (5.) $\frac{1}{8}$ da. (6.) .875 gal. (7.) £.625 (8.) .5 yd. (9.) .4844− bu. (10.) .222− da.

Article 161 (Page 104)

(1.) 44 da. 10 hr. 3 min. 18 sec. (2.) 106 T. 56 lb. 13 oz. (3.) £66 3s. 11d. 1 far. (4.) 16 bu. 2 pk. 4 qt. 1 pt. (5.) £199 3s. 2d. 2 far. (6.) 58 da. 5 hr. 49 min. 29 sec. (7.) 97 cu. yd. 2 cu. ft. 1639 cu. in. (8.) 2 da. 10 hr. 56 min. (9.) 3 cd. ft. 14 cu. ft.

Article 162 (Page 105)

(1.) 6 bu. 3 pk. 2 qt. 1 pt. (2.) 25 da. 17 hr. 15 min. (3.) 16 cd. 70 cu. ft. (4.) 90 gal. 3 qt. 2 gi. (5.) £216 15s. 3d. (6.) 7 A. 142 sq. rd. 12 sq. yd. 21 sq. ft. 36 sq. in. (7.) 11 gal. 1 qt. 1 pt. (8.) £187 9s. 9d.

Article 163 (Page 106)

(1.) 19 yr. 8 mo. 17 da. (2.) 25 yr. 6 mo. 6 da. (3.) 36 yr. 7 mo. 10 da. (4.) 15 yr. 7 mo. 21 da. (5.) 64 yr. 2 mo. 8 da. (6.) 7 yr. 10 mo. 12 da. (7.) 13 yr. 10 mo. (8.) 33 yr. 8 mo. (9.) 18 yr. (10.) 25 yr. 1 da.

Article 164 (Page 107)

(1.) 300 da. (2.) 228 da. (3.) 270 da. (4.) 232 da. (5.) 225 da. (6.) 79 da. (7.) 269 da. (8.) 259 da. (9.) 297 da. (10.) 9 da. (11.) 191 da. (12.) 290 da.

Article 165 (Page 108)

(1.) 45 gal. (2.) £ 169 3s. 11d. (3.) 151 da. 23 hr. 15 min. (4.) 280 lb. 3 oz. 12 pwt. (5.) 643 bu. 3 pk. 4 qt. (6.) £26561 14s. 7d. 3far. (7.) 1252 mi. 45 rd. 5 yd. 6 in. (8.) 4560 yr. 5 da. (9.) 11587 bu. 2 pk. 7 qt. 1 pt. (10.) 25040 T. 2 cwt. 52 lb. (11.) 1615 da. 22 hr. 13 min. (12.) 2725 gal. 2 qt. (13.) 13 mi. 64 rd.

Article 166 (Page 109)

(1.) 7 gal. 3 qt. 1 pt. (2.) 9 yd. 2 ft. 7 in. (3.) £32 5s. 6d. (4.) 17 cd. 34 cu. ft. (5.) 23 bu. 3 pk. 7 qt. (6.) £3 7s. 8d. (7.) 3 T. 17 cwt. 94 lb. (8.) 7 gal. 3 qt. $3\frac{1}{17}$ gi. (9.) 3 yr. 9 mo. 18 da. (10.) 3° 27′ 35″

Article 167 (Page 110)

(1.) 28 times (2.) 128 times (3.) 59 times (4.) 73 times (5.) 48 times

Article 168 (Pages 110–111)

(1.) 2210 bu. 3 pk. 2 qt. (2.) 24 spoons (3.) \$787.50 (4.) 311 da. (5.) \$42.43+ (6.) 43 yd. (7.) \$11.90 (8.) 6 lb. 9 oz. 5 pwt. (9.) 1 bu. 3 pk. 5 qt. (10.) 10 sq. rd. 25 sq. yd. 1 sq. ft. 126 sq. in. (11.) £738 3s. 4d. (12.) 23 gal. 1 qt. 2 gi. (13.) 259 mi. 125 rd. 2 yd. (14.) \$10.17 (15.) 32 brl. (16.) \$207.88+ (17.) \$195.31− (18.) \$3275.76+ (19.) \$632.65+ (20.) \$173.43− (21.) \$146.36+ (22.) 3501.55+ fr. (23.) £108 11s. 5+d. (24.) 1771.84+ rubles (25.) *1288.86+ pesetas.*

Article 180 (Page 115)

(1.) 461348 sq. dm. (2.) 3468.3457 Dg. (3.) 46817.3 dg. (4.) 56.78 S. (5.) 19.653824 Ha. (6.) 28246000 mm. (7.) 341.7283 Dl. (8.) 166.52716 mi. (9.) 147.9352 gal. (10.) 1347.8125 bu. (11.) 177.15672 Dl. (12.) 56732.48 dg. (13.) 68.58015 M. (14.) 290.528028 Sq. M. (15.) 566.2035 M. (16.) 11.922348 Hl. (17.) $1028.34+ (18.) 3949.53− fr.

Article 185 (Page 117)

(1.) 85 sq. yd. 3 sq. ft. (2.) 336 sq. yd. (3.) 5 sq. yd. 7 sq. ft. 72 sq. in. (4.) 24 sq. yd. 6 sq. ft. (5.) 635 sq. yd. 5 sq. ft. (6.) 4940 sq. yd. 7 sq. ft. 72 sq. in. (7.) 3 A. 85 sq. rd. 10 sq. yd. 8 sq. ft. 108 sq. in. (8.) 1 A. 74 sq. rd. 25 sq. yd. 1 sq. ft. 72 sq. in. (9.) 8 sq. mi. 19 A. 10 sq. rd. (10.) 114 sq. mi. 240 A. (11.) 9 sq. yd. 7 sq. ft. 18 sq. in. (12.) 49 sq. rd. 13 sq. yd. 6 sq. ft. 108 sq. in. (13.) 65.486$\frac{1}{2}$ sq. yd. (14.) 94$\frac{4}{9}$ sq. yd. (15.) 17 A. 9 sq. rd. 29.75 sq. yd. (16.) 48 sq. ft. 30 sq. in. (17.) 38 sq. ft. (18.) $21 (19.) $54

Article 186 (Page 118)

(1.) 5 yd. 2 ft. (2.) 74 rd. (3.) 28 ft. (4.) 9 ft. (5.) 3 rd. 1 yd. 1 ft. 6 in.

Article 187 (Pages 119–121)

(1.) 69 yd. 1 ft. (2.) 37 yd. 1 ft. (3.) 72 yd. 1 ft. 6 in. (4.) 136 yd. (5.) $10.80 (6.) 972 bricks (7.) 1512 bricks (8.) 10725 bricks (9.) 607$\frac{1}{2}$ blocks (10.) 3569 pieces (12.) 61 yd. 9 in. (13.) 58 yd. (14.) 44 yd. 2 ft. (15.) 93 yd. 2 ft. 2 in. (16.) 70 yd. 1 ft. 10 in.

Article 189 (Page 122)

(1.) 12 bd. ft. (2.) 10$\frac{1}{2}$ bd. ft. (3.) 10$\frac{1}{2}$ bd. ft. (4.) 23$\frac{1}{2}$ bd. ft.
(5.) 18$\frac{1}{2}$ bd. ft. (6.) 20$\frac{1}{24}$ bd. ft. (7.) 11$\frac{1}{24}$ bd. ft. (8.) 60 bd. ft.
(9.) 86 bd. ft. (10.) 18$\frac{3}{4}$ bd. ft. (11.) $40.61+ (12.) $33.54

Article 191 (Pages 124–125)

(1.) 672 cu. ft. (2.) 71 cu. yd. 3 cu. ft. (3.) 10 cu. ft. 216 cu. in. (4.) 37 cu. yd. 1 cu. ft. (5.) $86.40 (6.) $35.19+ (8.) 15 ft. (9.) 21 ft. (10.) 8 ft. (11.) 1 ft. 6 in. (12.) 16 ft.

Article 192 (Pages 126–128)

(2.) 113.20− bu. (3.) 115.31+ bu. (4.) 65.74+ bu. (5.) 43.67+ bu. (6.) 2272.21− gal. (7.) 4578.08− gal. (8.) 325.82+ gal. (9.) 22.28− bu. (10.) 32315.84+ gal. (11.) 2 ft. 5.87− in. (12.) 4 ft. 2.57− in. (13.) 10 ft. 2.56− in. (14.) 128 bu. (15.) 5 T. 896 lb. (16.) 4 ft. 2 in. (17.) 241.92 bu. (18.) 617.14− bu. (19.) 151.2 bu. (21.) 2961.1008 gal. (22.) 564.0192 gal. (23.) 54.12− bu. (24.) 9 ft. 3.70− in. (25.) 24 cu. ft. 939.6 cu. in. (26.) 3694.5216 cu. in. (27.) 6768.2304 gal. (28.) 125.1351 gal. (29.) 106.624 gal. (30.) 80.6344 gal.

Article 201 (Page 134)

(1.) $56 (2.) $24 (3.) 182 lb. (4.) 714 men (5.) 314 sheep (6.) 730 gal. (7.) $8 (8.) 6 tons (9.) £173 1s. 6 d. (10.) 897 gal. (11.) 895

sheep (12.) 192 horses (13.) 252 da. (14.) 476 mi. (15.) 390 yd. (16.) 400 lb. (17.) 342 cattle (18.) 8970 yd. (19.) real est. $12255; pers. prop. $16245 (20.) 270 pupils (21.) 66924 inhab. (22.) 301 acres (23.) $11565

Article 202 (Pages 136–137)

(1.) 5 times (2.) .25 (3.) 33⅓% (4.) 34% (5.) 35% (6.) 85% (7.) 62% (8.) ½% (9.) 25% more (10.) 20% more (11.) 65% less (12.) 24% less (13.) 4% (14.) 42% (15.) 76% (16.) 85% (17.) 24% (18.) A 28%; B 35%; C 37% (19.) 64% (20.) 18% (21.) 23%

Article 203 (Pages 138–139)

(1.) $9 (2.) 800 gal. (3.) 700 yd. (4.) 560 tons (5.) 538 gal. (6.) $8496.25 (7.) $6352 (8.) $46764 (9.) $386.25 (10.) $748.75 (11.) $1283.75 (12.) £781 5s. (13.) 862.5 tons (14.) $327.75 (15.) 725 acres (16.) $3795 (17.) $6750 (18.) $18000 (19.) $28728 (20.) $649.50

Article 204 (Pages 140–141)

(1.) 367 bu. 12 lb. (2.) 56 T. 1074 lb. (3.) 195 bu. 43 lb.
(4.) 556½ yd. (5.) 481 bu. 4 lb. (6.) 125 mi. 1690 yd.
(7.) 67 cd. 2 cd. ft.

Article 205 (Page 143)

(1.) 42% (2.) 39% (3.) 28½% (4.) 97% (5.) 37½%
(6.) 41½% (7.) 63% (8.) 29½% (9.) 21.577% (10.) 99.643%

Article 206 (Page 144)

(1.) $720 (2.) 5775 natives; 2100 Germans; 1575 Irish; 1050 others (3.) $1175 (4.) $2700 (5.) 375 acres (6.) 1200 lb. (7.) $18095 (8.) 30%

Article 207 (Page 145)

(1.) 35% (2.) $1700 (3.) $8500 (4.) 16%
(5.) 1096518 yd. (6.) $45805.26 (7.) $248856.02 (8.) $65884.50

Article 208 (Pages 145–148)

(1.) $9113.84 (2.) A 45%; B 55% (3.) 975 gal. (4.) 56¼% (5.) 80% (6.) A 50% more than B; B 33⅓% less than A (7.) $22350 (8.) 32% (9.) $5520 (10.) 650 acres (11.) 15% (12.) 27% (13.) $10890 (14.) $1299.80 (15.) 76% (16.) $14.57 (17.) 40%; 60%; $54; $81 (18.) 25%; 31¼%; 43¾%; $19.86; $24.83; $34.76 (20.) adj. house $3400; corner house $4352 (21.) A $15720; B $13100; C $14148; D $10611 (22.) A $84; B $67.20; C $52.50; D $87.50; E $350 (23.) $72000 (24.) wages 34.606%; repairs 5.684%; fuel 4.068%; fixed charges 25.855% (25.) 4500 bu. (26.) $60000

Article 214 (Pages 149–154)

(1.) $14.45 (2.) $33.08 (3.) $77.22 (4.) $125.70 (5.) $57.03 (6.) $162.08 (7.) $344.05 (8.) $857.38 (9.) $897.99 (10.) $687.23 (11.) $299.88 (12.) $382.33 (13.) $352.46 (14.) $52.98 (15.) $92.44 (16.) $66.60 (17.) 25% (18.) 16% (19.) 46% (20.) 18% (21.) 8% (22.) 36% (23.) 24% (24.) 14% (25.) 18% (26.) 12½% (27.) 20%

(28.)18¾% (29.) 16⅔% (30.) 19% (31.) $194.60 (32.) $468.75 (33.) $769
(34.) $843.25 (35.) $418.26 (36.) $694.72 (37.) $368.45 (38.) $79.32
(39.) $253.60 (40.) $62.56 (41.) $78.50 (42.) $625.38 (43.) $47.26
(44.) $23.04 (45.) 25% (46.) 20% (47.) $4.56 (48.) $375 (49.) $300
(50.) $9120 (51.) net loss $200 (52.) $11795 (53.) 20% (54.) $82.50
(55.) 25¢ (56.) $28.13− (57.) 30% (58.) 18.7% (59.) $24000 (60.) 40%
(61.) 37½% (62.) $53.25 (63.) 20% (64.) total purchases $61965; gain
15% (65.) $32960.88 (66.) prod. mat. 33⅓%; prod. labor 50%; overhead
exp. 16⅔% (67.) prod. labor 47−%; prod. mat. 40+%; overhead exp.
13+% (68.) $13 (69.) $142.23 (70.) $21304.12 (71.) $8400 (72.) $55400

Article 220 (Page 156)

(1.) $241.01 (2.) $314.77 (3.) $636.87 (4.) $309.86 (5.) $521.98
(6.) $140.39 (7.) $414.82 (8.) $670.46 (9.) $354.12 (10.) $222.86
(11.) $455.38 (12.) $17.88 (13.) $551.34

Article 221 (Page 158)

(1.) .72 (2.) .64125 (3.) .82935 (4.) .576 (5.) .54 (6.) .729
(7.) .45448 (8.) .59976 (9.) .8210565 (10.) .325 (11.) .224 (12.) .2628
(13.) .2305 (14.) .352 (15.) .09693

Article 222 (Page 159)

(1.) 5% (2.) 20% (3.) 10% (4.) 12½% (5.) 30% (6.) 16⅔%
(7.) 15% (8.) 5%

Article 223 (Page 160)

(1.) $48.67 (2.) $24.51 (3.) $99.56 (4.) $386.38 (5.) $176.55
(6.) $39.57 (7.) $76.94 (8.) $19.90 (9.) $12.79 (10.) $96.08

Article 231 (Page 163)

(1.) $5.99 (2.) $23.60 (3.) $10.89 (4.) $18.87 (5.) $24.85
(6.) $33.12 (7.) $35.89 (8.) $11.52 (9.) $240.93 (10.) $177.44

Article 232 (Page 164)

(1.) $12.26 (2.) $55.85 (3.) $16.18 (4.) $4.54 (5.) $8.52
(6.) $25.87 (7.) $87.06 (8.) $13.04 (9.) $16.28 (10.) $37.39
(11.) $42.43 (12.) $9.57 (13.) $55.80 (14.) $93.42

Article 233 (Pages 167–169)

(1.) $25.93 (2.) $11.64 (3.) $30.61 (4.) $29.68 (5.) $8.01
(6.) $5.90 (7.) $19.81 (8.) $42.37 (9.) $4.33 (10.) $14.48
(11.) $1.34 (12.) $38.23 (13.) $2.70 (14.) $224.99 (15.) $232.61
(16.) $.08 (17.) $10 (18.) $41.17 (19.) $5.79 (20.) $187.44
(21.) $268.16 (22.) $130.87 (23.) $339.69 (24.) $2984.72 (25.) $22.24
(26.) $3 (27.) $13.79 (28.) $21714.51 (29.) $10.16 (30.) $224.87
(31.) $440.20 (32.) $126.45 (33.) $317.77 (34.) $33.76 (35.) $11.97
(36.) $32.01 (37.) $3.51 (38.) $.88 (39.) $44.55 (40.) $8.25
(41.) $103.68 (42.) $22.96 (43.) $4.56 (44.) $8.45 (45.) $5.70
(46.) $1.36 (47.) $.48 (48.) $502.58 (49.) $197.42 (50.) $139.32
(51.) $858.30 (52.) $110.89 (53.) $190.84

Article 234 (Page 170)

(1.) $18.55 (2.) $20.31 (3.) $15.52 (4.) $66.98 (5.) $10.80
(6.) $7.72 (7.) $15.42 (8.) $51.44 (9.) $46.11 (10.) $2.57
(11.) $.71 (12.) $472.67

Article 235 (Page 172)

(1.) $29.56 (2.) $10.80 (3.) $25.22 (4.) $50.57 (5.) $4.09
(6.) $53.03 (7.) $41.31 (8.) $40.65 (9.) $820.92 (10.) $683.70
(11.) $232.31 (12.) $26.23 (13.) $20.08 (14.) $303.58 (15.) $2132.85
(16.) $1022.37 (17.) $1359.58 (18.) $21.93

Article 236 (Page 174)

(1.) $1.99 (2.) $6.44 (3.) $1.69 (4.) $3.41 (5.) $10.90
(6.) $28.25 (7.) $327 (8.) $22.80 (9.) $240 (10.) $17.27
(11.) $28.36 (12.) $17 (13.) $119.88 (14.) $20.48 (15.) $12.68
(16.) $211.33 (17.) $79.50 (18.) $555.42 (19.) $1638 (20.) $130.05

Article 237 (Pages 175–176)

(1.) $8.66 (2.) $10.38 (3.) $9.02 (4.) $15.44 (5.) $25.94 (6.) $14.30
(7.) $5.29 (8.) $24.12 (9.) $20.72 (10.) $117.91 (11.) $51.57
(12.) $86.83 (13.) $1811.94 (14.) $55.64 (15.) $475.71 (16.) $705.75
(17.) $78.36 (18.) $526.75 (19.) $998.68 (20.) $84.64 (21.) $465.13
(22.) £39 13s. 2.88 far. (23.) £492 8s. 1.05 far. (24.) £108 7s. 6d. 2.09 far. (25.) £91 8s. 9d. 1.55 far.

Article 239 (Page 177)

(1.) 9% (2.) 5% (3.) 8% (4.) 4½% (5.) 7% (6.) 4½%

Article 240 (Page 178)

(1.) 153 da. (2.) 183 da. (3.) 94 da. (4.) 307 da. (5.) 3 yr. 346 da.

Article 241 (Page 179)

(1.) $582.86 (2.) $4105.28 (3.) $4126.40 (4.) $9500 (5.) $418.50
(6.) $7239.64 (7.) $3875.91

Article 242 (Page 181)

(1.) $596.52 (2.) $1913.70 (3.) $741.03 (4.) $3349.81 (5.) $485.94
(6.) $2472.28 (7.) $951.85 (8.) $1589.13

Article 243 (Pages 183–185)

(1.) $835.85 (2.) $165.69 (3.) $190.72 (4.) $767.59
(5.) $1053.69 (6.) $64.89 (7.) $48.32 (8.) $20.40
(9.) $193.95 (10.) $78.87 (11.) $106.32 (12.) $103.44
(13.) $684.78 (14.) $1878.87 (15.) $801.03 (16.) $698.95
(18.) $2065.42 (19.) $2906.29 (20.) $1006.97 (21.) $983.26
(22.) $360.08 (23.) $80.64 (24.) $1448.19 (25.) $593.20

Article 250 (Pages 188–189)

(1.) $960.90; $7.85 (2.) $710.92; $12.54 (3.) $2888.88; $76.92 (4.) $7725.36; $90.99 (5.) $511.49; $16.41 (6.) $838.03; $7.09 (7.) $647.17; $46.33 (8.) $357.48; $21.17 (9.) $26.13 (10.) $25.72 (11.) $3556.10 (12.) Of A; gain $.98 (13.) $390.30 (14.) $.05 (15.) $1808.16 (16.) $943.57

Article 256 (Pages 191–193)

(1.) Aug. 15, 1911; 64 da.; $5.30; $591.45 (2.) Mar. 25, 1912; 78 da.; $10.96; $831.94 (3.) Apr. 15, 1912; 37 da.; $1.32; $284 (4.) Mar. 3, 1912; 14 da.; $4.80; $1758.45 (5.) Nov. 18, 1910; 26 da.; $2.70; $931.95 (6.) June 30, 1912; 71 da.; $14.68; 2466.44 (7.) Dec. 23, 1911; 82 da.; $5.36; $667.49 (8.) May 8, 1911; 81 da.; $5.47; $481.03 (9.) June 19, 1912; 45 da.; $36.74; $3637.41 (10.) Dec. 12, 1911; 126 da.; $7.67; $390.81 (11.) Nov. 16, 1911; 74 da.; $8.95; $716.35 (12.) Nov. 30, 1911; 98 da.; $11.91; $474.37 (13.) $2546.81 (14.) $851.29 (15.) $721.72 (16.) $489.83 (17.) $868.56 (18.) $908.98 (19.) $2137.26 (20.) $466.34

Article 257 (Page 194)

(1.) $656.48	(2.) $965.97	(3.) $1393.70	(4.) $509.23
(5.) $6309.25	(6.) $4105.37	(7.) $698.56	(8.) $2565.86
(9.) $1293.64	(10.) $702.83	(11.) $492.41	(12.) $503.78

Article 258 (Page 197)

(1.) $3025.70 (2.) $1830.70 (3.) $146.22 (4.) $551.71 (5.) $6612.62 (6.) $5194.17 (7.) $646.42 (8.) $358.05

Article 259 (Pages 198–199)

(1.) $456.15	(2.) $813.09	(3.) $115.30	(4.) $114.70
(5.) $223.35	(6.) $21.54	(7.) $100.27	(8.) $317.96
(9.) $10.96			

Article 264 (Pages 201–205)

(1.) $18.77 (2.) $11.35 (3.) $4.69 (4.) $33.72 (5.) $23.84 (6.) $36.17 (7.) $854.04 (8.) $562.80 (9.) $2704.65 (10.) $8472.33 (11.) $7313.96 (12.) $808.86 (13.) $893.22 (14.) $2393.08 (15.) $18636.31 (16.) $9889 (18.) 4% (19.) 2% (20.) 3% (21.) $\frac{3}{8}$% (22.) 4% (23.) 5% (24.) $685.67 (25.) $427.60 (26.) $916 (27.) $17000 (28.) $6016 (29.) $976.66 (30.) $672 (31.) $5000 (32.) $957.99 (33.) $650 (34.) $500 (35.) $875.50 (36.) $468.75 (37.) $728 (38.) $263.50 (39.) $576 (40.) $852 (41.) $9.65 (42.) $13.39 (43.) $10.93 (44.) $470 (45.) $422.02 (46.) $739.66 (47.) 157 brls.; 57¢ bal. (48.) 82 brls.; $1.99 bal. (49.)297 bu. 4 lb. (50.) $765.67 (51.) $4.78 (52.) $6097 (53.) $4360.75 (54.) $379.57 (55.) 164 brls.; 25¢ bal. (56.) 145 brls.; $1.84 bal. (57.) 694 bu. 12 lb. (58.) 226 brls.; 9¢ bal.

Article 272 (Pages 208–211)

(1.) $235 (2.) $217 (3.) $1141 (4.) $83.13 (5.) $909 (6.) $2443.75 (7.) $4933.88 (8.) $3040.88 (9.) $12894.50 (10.) $3006.25 (11.) $11169.25 (12.) $7463 (13.) $5095.50 (14.) $11532 (15.) $8319.38 (16.) $1890 (17.) $3422.50 (18.) $2734.38 (19.) $1303.50 (20.) 4% (21.) 122$\frac{1}{2}$

(22.) 16½% (23.) 24½% (24.) 19¾% (25.) 7⅞% (26.) 4½% (27.) 91⅞
(28.) 16⅞% (29.) 13½% (30.) 17½% (31.) 105½% (32.) 14½%
(33.) 23¾% (34.) $1500 (35.) $11400 (36.) $23700 (37.) $900
(38.) $7100 (39.) $8900 (40.) $5800 (41.) $6200 (42) $2300
(43.) $4500 (44.) $3200 (45.) $13500 (46.) $1200 (47.) $4400
(48.) $8200 (49.) 72 shares (50.) $10237.50 (51.) $11076 (52.) $36.50
(53.) $5194 (54.) $940 (55.) $24981.25 (56.) 35¼%

Article 273 (Pages 212–214)

(1.) $87.50 (2.) $60 (3.) $125 (4.) $160 (5.) $112.50 (6.) 62⅜ (7.) 79⅞
(8.) 74⅞ (9.) 69⅞ (10.) 74⅞ (11.) 5% (12.) 8% (13.) 4% (14.) 5%
(15.) 5% (16.) 6⅜% (17.) 4% (18.) 4⅜% (19.) 6½% (20.) 4%
(21.) 3% (22.) 6% (23.) 10% (24.) 8% (25.) 5% (26.) 6% (27.) 4%
(28.) 4½% (29.) 8% (30.) 12% (31.) 7% (32.) 59⅞ (33.) 5% (34.)4%
stock yields $\frac{5}{11}$% more on inv. (35.) $46.25 (36.) $62.50 (37.) $258.75

Article 278 (Pages 216–218)

(2.) $631.58 (3.) $954 (4.) $569.01 (5.) $274.47
(7.) $837.42 (8.) $472.12 (9.) $622.70 (11.) $715.42
(12.) $634.40 (13.) $158.20 (15.) $872 (16.) $938.72
(17.) $1457.91 (18.) $721.91 (20.) $754.51 (21.) $1603.75
(22.) $3535.35 (23.) $881.98 (24.) $2796.39 (25.) $3266.40

Article 283 (Page 220)

(1.) $2026.27 (2.) $3536.10 (3.) $4505.25 (4.) $198.62
(5.) $459.59 (6.) $831.54 (7.) $901.90 (8.) $153.80
(9.) $274.79 (10.) $928.63 (11.) $4025.36 (12.) $1226.81

Article 284 (Page 221)

(1.) £375 1s. 3½d. (2.) £153 14s. 1½d. (3.) 4287.45 fr. (4.) 13384.79 fr. (5.) 3974.28 marks (6.) 26249.87 marks (7.) 1638.30 guilders (8.) 3054.07 guilders (9.) £750 10s. 6¾d. (10.) 1894.72 guilders

Article 285 (Pages 222–223)

(2.) 68%; $569.50 (3.) 76%; $1356.75 (4.) 65.28%; A $378.62; B $832.65; C $571.40

Article 286 (Page 224)

(2.) $3500 (3.) ½% (4.) $65.63 (5.) $564.75 (6.) $319.50
(7.) $1600 (8.) $3000; $1500 (9.) $2500; $3750; $5000

Article 287 (Page 225)

(2.) $141.75 (3.) $156.75 (4.) $184.50 (5.) $108.75 (6.) $405.92
(7.) $137.60 (8.) $40.75 (9.) $1.50 (10.) 15 mills

Article 288 (Pages 226–227)

(2.) $820.50 (3.) $791.10 (4.) $2768.40 (5.) $2360.75
(6.) $4138.40 (7.) $11928 (8.) $51.75 (9.) $42
(10.) $3480 (11.) $698 (12.) $3598

Article 289 (Pages 229–231)

(1.) $57 (2.) $282.20 (3.) $1725 (4.) 360 lb. (5.) 270 sheep (6.) $3980.40 (7.) $18.18 (8.) $16.40 (9.) $648 (10.) 152 da. (11.) 125 ft. (12.) 8 ft. (13.) 24 men (14.) 30 da. (15.) 525 lb. (16.) $56.25 (17.) A $1500; B $2100 (18.) 25% (19.) 35% (20.) $46.80

Article 290 (Page 232)

(1.) 90000 shingles (2.) $350 (3.) $425 (4.) $180. (5.) 15 da. (6.) $180

Article 292 (Pages 234–235)

(1.) 2 mo. 10 da. (2.) 1 mo. 29 da. (3.) 2 mo. 13 da. (4.) June 18, 1912 (5.) Jan. 3, 1912 (6.) 6 mo. (7.) Apr. 25, 1911 (8.) July 14, 1910

Article 293 (Pages 237–238)

(1.) May 19, 1910	(2.) May 2, 1911	(3.) Aug. 23, 1912
(4.) Aug. 4, 1911	(5.) Nov. 24, 1910	(6.) Apr. 25, 1912
(7.) Sept. 23, 1910	(8.) Oct. 28, 1911	(9.) May 12, 1912
(10.) Nov. 8, 1910	(11.) July 30, 1911	(12.) Dec. 5, 1912

Article 294 (Pages 240–241)

(1.) July 11, 1910	(2.) June 27, 1911	(3.) June 23, 1912
(4.) Dec. 24, 1911	(5.) Dec. 10, 1909	(6.) Sept. 16, 1912
(7.) Sept. 16, 1912	(8.) Dec. 2, 1911	(9.) Sept. 15, 1911
(10.) June 4, 1912		

Article 296 (Pages 244–246)

(1.) Oct. 20, 1910	(2.) Apr. 10, 1911	(3.) May 7, 1910
(4.) Sept. 15, 1911	(5.) Apr. 27, 1911	(6.) Aug. 2, 1910
(7.) June 14, 1912	(8.) Nov. 11, 1911	(9.) May 3, 1910
(10.) Sept. 16, 1910	(11.) June 21, 1912	(12.) Feb. 29, 1912
(13.) $606.60	(14.) Aug. 25, 1911	(15.) Jan. 30, 1912

Article 297 (Pages 247–248)

(1.) June 5, 1910 (2.) Oct. 29, 1911 (3.) Net pro. $1269.06; May 22, 1912

Article 299 (Pages 250–252)

(1.) $301.47 (2.) $353.40 (3.) $326.90 (4.) $521.41
(5.) $477.74

Article 301 (Pages 254–255)

(1.) $3060 gain (2.) $2589.88 loss (3.) $3786.90 loss (4.) $3996 gain (5.) $9901.75 loss (6.) $11304.85 gain (7.) $17543.75 cap. (8.) $5089.77 cap. (9.) $954.10 cap. (10.) $6899.40 insolv. (11.) $3012.45 insolv. (12.) $5458.25 insolv. (13.) $5292.60 cap. (14.) $2959.90 insolv. (15.) $11121.40 cap. (16.) $9868.65 cap. (17.) $5006.45 insolv. (18.) $5393.90 insolv. (19.) $1267 insolv. (20.) $3136.70 cap. (21.) $6508.68 (22.) A's $5884; B's $2942 (23.) A's $9640; B's $8140; C's $6640

Article 302 (Page 257)

(1.) A's $1920; B's $1680 (2.) A's $2400; B's $3000; C's $3600 (3.) A's $9310; B's $13965 (4.) A's $2240; B's $2800; C's $3360

Article 303 (Pages 260–261)

(1.) A's $2396.04; B's $1603.96 (2.) A's gain $2190.08; B's gain $1834.71; C's gain $2975.21; A's present int. $12190.08; B's present int. $9834.71; C's present int. $17975.21 (3.) A's $26137.72; B's $21868.26; C's $16994.01 (4.) A's gain $3043.87; A's present int. $18043.87; B's gain $2956.13; B's present int. $23956.13 (5.) A's $12935.37; B's $12064.63 (6.) A's $19437.35; B's $12562.65

Article 304 (Pages 265–266)

(1.) A's $16502.33; B's $21097.67 (2.) A's $19981.95; B's $24618.05 (3.) A's $21837.05; B's $21397.60; C's $21065.35 (4.) A's $11770.85; B's $20390.33; C's $11068.82 (5.) A's $16271.31; B's $9928.69

Article 306 (Page 267)

(1.) $64 (2.) $187 (3.) $93 (4.) $118 (5.) $121
(6.) $139 (7.) $18.25 (8.) $30.50 (9.) $145.75 (10.) $41.13
(11.) $418.50 (12.) $182.60

Article 307 (Page 268)

(1.) 1875 yd. (2.) 1746 gal. (3.) 592 lb. (4.) 172 bu.
(5.) 246 gal. (6.) 330 yd. (7.) 836¼ lb. (8.) 242⅞ doz.

Article 309 (Page 269)

(1.) $115.875 (2.) $64.20 (3.) $197.50 (4.) $39
(5.) $352 (6.) $164.70 (7.) $940.50 (8.) $646.30
(11.) $405 (12.) $81 (13.) $93.125 (14.) 463½ lb.
(15.) 142⅜ yd. (16.) 386⅜ bu. (17.) $52\frac{5}{16}$ gal. (18.) 425¼ yd.
(19.) 624 bu.

Article 310 (Page 270)

(1.) $247 (2.) $448 (3.) $324 (4.) $468.27
(5.) $382.20 (6.) $1472 (7.) $296.40 (8.) $1959.60
(9.) $4056 (12.) 346 (13.) 504¾ (14.) 461⅛
(15.) 362⅝ (16.) 307½ (17.) 232½ (18.) 1428⅜
(19.) 1284¼ (20.) 2375 (22.) 84875 (23.) 42174
(24.) 61632 (25.) 51740 (26.) 624232 (27.) 326697
(28.) 7118576 (29.) 250880 (30.) 67932800

Article 311 (Pages 271–272)

(1.) $276.26 (2.) $119.89 (3.) $207.48 (4.) $91.55 (5.) $74.84
(6.) $18.12 (7.) $66.15 (8.) $88.51 (9.) $27.70 (10.) $47.87
(11.) $35.99 (12.) $41.78 (13.) $471.05 (14.) $1253.32 (15.) $166.74

Article 312 (Page 273)

(1.) 64 (2.) 55 (3.) 607 (4.) 5306 (5.) 5294
(6.) 6376 (7.) 6603 (8.) 85495 (9.) 884017 (10.) 9589235
(11.) 7762236 (12.) 74366307

Article 313 (Page 274)

(1.) 1776 (2.) 3922 (3.) 2378 (4.) 5418 (5.) 1392 (6.) 3040
(7.) 945 (8.) 1495 (9.) 2814 (10.) 3942 (11.) 972 (12.) 3796

Article 319 (Pages 281–282)

(1.) 81 (2.) 784 (3.) 26244 (4.) 119025 (5.) 512 (6.) 3375
(7.) 14886936 (8.) 147197952 (9.) 625 (10.) 371293 (11.) 279936
(12.) 262144 (13.) $\frac{8}{27}$ (14.) $\frac{16}{25}$ (15.) $39\frac{1}{16}$ (16.) $144\frac{45}{64}$ (17.) .07776
(18.) .00531441 (19.) 2.097152 (20.) 188.2384 (21.) 74 (22.) 364
(23.) 139 (24.) 781 (25.) 412 (26.) 9103 (27.) 8643 (28.) 3816
(29.) 48937 (30.) 62534 (31.) 93 (32.) 752 (33.) 246 (34.) 999
(35.) 562 (36.) 832 (37.) 2106 (38.) 9372 (39.) 1001 (40.) 3475
(41.) 73 (42.) 642 (43.) 82 (44.) 643 (45.) 4 ft. 6.43 + in.
(46.) 15 ft. 10.653 + in. (47.) 466.69 + (48.) 7 cu. ft. 71 cu. in.
(49.) 67 sq. rd. 13 sq. yd. 2 sq. ft. 72 sq. in. (50.) 8.56 − ft. (51.) 32 yd. long; 24 yd. wide (52.) 129 ft. long; 43 ft. wide (53.) 48 ft. wide; 32 ft. long (54.) 522 ft. long; 87 ft. wide; 87 ft. high (55.) 228 ft. long; 57 ft. wide; 57 ft. deep (56.) 32 in. long; 32 in. high; 12 in. wide

ERRATA

In example 17, page 35, change 7 men to 9 men.
" " 10, " 110, " $\frac{13}{16}$A to $\frac{13}{16}$A.
" " 15, " 281, " $(2\frac{1}{12})^4$ to $(2\frac{1}{2})^4$.

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